KPK BOARD NOTES

MATHEMATICS

10TH CLASS

Presented by:

Urdu Books Whatsapp Group

STUDY GROUP

0333-8033313
راؤ ایاز

0343-7008883
پاکستان زندہ باد

0306-7163117
محمد سلمان سلیم

UNIT - 1:

QUADRATIC EQUATIONS:

Definition: An equation of the form $ax^2 + bx + c = 0$, $a \neq 0$ is called Quadratic equation .It is the standard form of Quadratic equation .For example

$3x^2 + 5x + 8 = 0$

Solution of Quadratic equation: A Quadratic equation

$ax^2 + bx + c = 0$, $a \neq 0$ can be solved by three methods.

(a) By factorization : In this method we find two terms in such away that its sum is equal to bx and its products is equal to acx^2.

(b) By Completing square : The following points should be Noted .

(i) Write the Quadratic equation in standard form .

(ii) Divided both sides by the coefficient of x^2 if it is other than 1.

(iii) Shift the constant term to the right hand side of the equation .

(iv) Take half the coefficient of x and then squaring it and added this square to both sides .

(v) Write the left hand side is a perfect square and simplify the right hand side .

(vi) Take the square root of sides of the equation and then find the values of x .

(c): By Quadratic formula : A Quadratic equation

$ax^2 + bx + c = 0 ....(i)$, $a \neq 0$

We can derived Quadratic formula by completing square.

Eq(i) divided by a

$$=> \quad x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$=> \quad x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Adding both sides $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

$\Rightarrow x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$

$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

Take square root of both sides.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$\Rightarrow x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .....(ii)

It is called Quadratic formula.

Example 1: Solve the equations by factorization.

(i) $2x^2 + 2x - 11 = 1$ (ii) $12t^2 = t + 1$

Solution: (i) $2x^2 + 2x - 11 = 1$ ......(i)

$\Rightarrow 2x^2 + 2x - 11 - 1 = 0$

$\Rightarrow 2x^2 + 2x - 12 = 0$

$\Rightarrow 2x^2 - 4x + 6x - 12 = 0$

$\Rightarrow 2x(x - 2) + 6(x - 2) = 0$

$\Rightarrow (x - 2)(2x + 6) = 0$

$\Rightarrow x-2=0 \quad or \quad 2x+6=0$

$\Rightarrow x=2 \quad or \quad 2x=-6$

$\Rightarrow x=2 \quad or \quad x=-\frac{6}{2}=-3$

Verification: Put $x=-3$ **in (i)**

$2(-3)^2+2(-3)-11=1$

$\Rightarrow 2(9)-6-11=1$

$\Rightarrow 18-17=1$

$\Rightarrow 1=1$ *which is true*

Now put $x=2$ **in (i)**

$2(2)^2+2(2)-11=1$

$\Rightarrow 2(4)+4-11=1$

$\Rightarrow 8-7=1$

$\Rightarrow 1=1$ *which is true*

Thus Solution set = { - 3 , 2} .

(ii) $12t^2=t+1$ ....(i)

$\Rightarrow 12t^2-t-1=0$

$\Rightarrow 12t^2-4t+3t-1=0$

$\Rightarrow 4t(3t-1)+1(3t-1)=0$

$\Rightarrow (3t-1)(4t+1)=0$

$\Rightarrow 3t-1=0 \quad or \quad 4t+1=0$

$\Rightarrow 3t=1 \quad or \quad 4t=-1$

$\Rightarrow t=\frac{1}{3} \quad or \quad t=-\frac{1}{4}$

Verification: **put** $t=\frac{1}{3}$ **in (i)**

$$12\left(\frac{1}{3}\right)^2=\frac{1}{3}+1$$

ختم نبوت ﷺ زندہ باد — عظمت صحابہ زندہ باد

السلام علیکم ورحمۃ اللہ وبرکاتہ:

معزز ممبران: آپ کا وٹس ایپ گروپ ایڈ من "**اردو بکس**" آپ سے مخاطب ہے۔

آپ تمام ممبران سے گزارش ہے کہ:

- گروپ میں صرف PDF کتب پوسٹ کی جاتی ہیں لہذا کتب کے متعلق اپنے کمنٹس / ریویوز ضرور دیں۔ گروپ میں بغیر ایڈ من کی اجازت کے کسی بھی قسم کی (اسلامی وغیر اسلامی، اخلاقی، تحریری) پوسٹ کرنا سختی سے منع ہے۔
- گروپ میں معزز، پڑھے لکھے، سلجھے ہوئے ممبرز موجود ہیں اخلاقیات کی پابندی کریں اور گروپ رولز کو فالو کریں بصورت دیگر معزز ممبرز کی بہتری کی خاطر ریموو کر دیا جائے گا۔
- کوئی بھی ممبر کسی بھی ممبر کو انباکس میں میسیج، مس کال، کال نہیں کرے گا۔ رپورٹ پر فوری ریموو کر کے کاروائی عمل میں لائے جائے گی۔
- ہمارے کسی بھی گروپ میں سیاسی و فرقہ واریت کی بحث کی قطعاً کوئی گنجائش نہیں ہے۔
- اگر کسی کو بھی گروپ کے متعلق کسی قسم کی شکایت یا تجویز کی صورت میں ایڈ من سے رابطہ کیجئے۔
- سب سے اہم بات:

گروپ میں کسی بھی قادیانی، مرزائی، احمدی، گستاخِ رسول، گستاخِ امہات المؤمنین، گستاخِ صحابہ و خلفائے راشدین حضرت ابو بکر صدیق، حضرت عمر فاروق، حضرت عثمان غنی، حضرت علی المرتضیٰ، حضرت حسنین کریمین رضوان اللہ تعالیٰ اجمعین، گستاخ اہلبیت یا ایسے غیر مسلم جو اسلام اور پاکستان کے خلاف پراپیگنڈا میں مصروف ہیں یا ان کے روحانی و ذہنی سپورٹرز کے لئے کوئی گنجائش نہیں ہے لہذا ایسے اشخاص بالکل بھی گروپ جوائن کرنے کی زحمت نہ کریں۔ معلوم ہونے پر فوراً ریموو کر دیا جائے گا۔

- تمام کتب انٹرنیٹ سے تلاش / ڈاؤنلوڈ کر کے **فری آف کاسٹ** وٹس ایپ گروپ میں شیئر کی جاتی ہیں۔ جو کتاب نہیں ملتی اس کے لئے معذرت کر لی جاتی ہے۔ جس میں محنت بھی صَرف ہوتی ہے لیکن ہمیں آپ سے صرف دعاؤں کی درخواست ہے۔
- عمران سیریز کے شوقین کیلئے علیحدہ سے عمران سیریز گروپ موجود ہے۔
- **لیڈیز کے لئے الگ گروپ کی سہولت موجود ہے جس کے لئے ویریفکیشن ضروری ہے۔**
- اردو کتب / عمران سیریز یا سٹڈی گروپ میں ایڈ ہونے کے لئے ایڈ من سے وٹس ایپ پر بذریعہ میسیج رابطہ کریں اور جواب کا انتظار فرمائیں۔ برائے مہربانی اخلاقیات کا خیال رکھتے ہوئے موبائل پر کال یا ایم ایس کرنے کی کوشش ہرگز نہ کریں۔ ورنہ گروپس سے تو ریموو کیا ہی جائے گا بلاک بھی کیا جائے گا۔

نوٹ: ہمارے کسی گروپ کی کوئی فیس نہیں ہے۔ سب فی سبیل اللہ ہے

محمد سلمان سلیم	پاکستان زندہ باد	راؤ ایاز
0306-7163117	0343-7008883	0333-8033313

پاکستان زندہ باد — پاکستان پائندہ باد

اللہ تبارک تعالیٰ ہم سب کا حامی وناصر ہو

$$\Rightarrow \quad 12\left(\frac{1}{9}\right) = \frac{1+3}{3}$$

$$\Rightarrow \quad \frac{4}{3} = \frac{4}{3} \quad \text{which is true.}$$

put $t = -\frac{1}{4}$ in (i)

$$12\left(-\frac{1}{4}\right)^2 = -\frac{1}{4} + 1$$

$$\Rightarrow \quad 12\left(\frac{1}{16}\right) = \frac{-1+4}{4}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{3}{4} \quad \text{which is true.}$$

Thus solution set = $\left\{-\frac{1}{4}, \frac{1}{3}\right\}$

Example2:Solve the equation $x^2 - 8x + 9 = 0$ by completing Square.

Solution: $x^2 - 8x + 9 = 0$

$\Rightarrow \quad x^2 - 8x = -9$

Adding both sides $(4)^2$

$\Rightarrow x^2 - 8x + (4)^2 = -9 + (4)^2$

$\Rightarrow (x-4)^2 = -9 + 16$

$\Rightarrow (x-4)^2 = 7$

Take square roots of both sides

$\Rightarrow \sqrt{(x-4)^2} = \pm\sqrt{7}$

$\Rightarrow x - 4 = \pm\sqrt{7}$

$\Rightarrow x = 4 \pm \sqrt{7}$

$\Rightarrow x = 4 + \sqrt{7}$ *or* $x = 4 - \sqrt{7}$

Thus Solution set = $\{4 + \sqrt{7}, 4 - \sqrt{7}\}$

Example. Solve $4x^2+3x-2=0$ by completing square.

Solution: $4x^2+3x-2=0$ divided both sides by 4

$$\Rightarrow x^2+\frac{3}{4}x-\frac{2}{4}=0$$

$$\Rightarrow x^2+\frac{3}{4}x=\frac{1}{2}$$

Adding both sides $\left(\frac{3}{8}\right)^2$

$$\Rightarrow x^2+\frac{3}{4}x+\left(\frac{3}{8}\right)^2=\left(\frac{3}{8}\right)^2+\frac{1}{2}$$

$$\Rightarrow \left(x+\frac{3}{8}\right)^2=\frac{9}{64}+\frac{1}{2}$$

$$\Rightarrow \left(x+\frac{3}{8}\right)^2=\frac{9+32}{64}$$

$$\Rightarrow \left(x+\frac{3}{8}\right)^2=\frac{41}{64}$$

Take square roots of both sides

$$\Rightarrow \sqrt{\left(x+\frac{3}{8}\right)^2}=\pm\sqrt{\frac{41}{64}}$$

$$\Rightarrow x+\frac{3}{8}=\pm\frac{\sqrt{41}}{8}$$

$$\Rightarrow x=-\frac{3}{8}\pm\frac{\sqrt{41}}{8}$$

$$\Rightarrow x=\frac{-3\pm\sqrt{41}}{8}$$

Thus Solution set = $\left\{\frac{-3+\sqrt{41}}{8}, \frac{-3-\sqrt{41}}{8}\right\}$

Example : Use the quadratic formula to solve

$$3x^2 - 6x + 2 = 0$$

Solution : Given $3x^2 - 6x + 2 = 0$ ......(i) .comparing (i) with the general quadratic equation

$$ax^2 + bx + c = 0 \quad .....(ii)$$

Then $a = 3$, $b = -6$, $c = 2$

Now by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad ....(iii)$$ .Put values in (iii)

$$\Rightarrow x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$\Rightarrow x = \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$\Rightarrow x = \frac{6 \pm \sqrt{12}}{6}$$

$$\Rightarrow x = \frac{6 \pm 2\sqrt{3}}{6}$$

$$\Rightarrow x = \frac{2(3 \pm \sqrt{3})}{6}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{3}}{3}$$

$$\Rightarrow x = \frac{3 + \sqrt{3}}{3} \quad ,or \quad x = \frac{3 - \sqrt{3}}{3}$$

Thus solution set = $\{\frac{3 + \sqrt{3}}{3} \; , \; \frac{3 - \sqrt{3}}{3}\}$

Exercise 1.1

Q.1: Solve each of the following equations by factorization:

(i): $x^2 + 5x + 4 = 0$

Solution: $x^2 + 5x + 4 = 0$

=> $x^2 + x + 4x + 4 = 0$

=> $x(x+1) + 4(x+1) = 0$

=> $(x+1)(x+4) = 0$

=> $x+1 = 0$ *or* $x+4 = 0$

=> $x = -1$ *or* $x = -4$

Thus solution set = { -1 , -4}.

(ii): $(x-3)^2 = 4$

Solution: $(x-3)^2 = 4$

=> $x^2 - 6x + 9 = 4$

=> $x^2 - 6x + 9 - 4 = 0$

=> $x^2 - 6x + 5 = 0$

=> $x^2 - x - 5x + 5 = 0$

=> $x(x-1) - 5(x-1) = 0$

=> $(x-1)(x-5) = 0$

=> $x-1 = 0$ *or* $x-5 = 0$

=> $x = 1$ *or* $x = 5$

Thus solution set = { 1 , 5}.

(iii) : $x^2 + 3x - 10 = 0$

Solution: $x^2 + 3x - 10 = 0$

=> $x^2 - 2x + 5x - 10 = 0$

=> $x(x-2) + 5(x-2) = 0$

=> $(x-2)(x+5) = 0$

=> $x-2 = 0$ *or* $x+5 = 0$

=> $x = 2$ *or* $x = -5$

Thus solution set = { -5 , 2} .

(iv): $6x^2 - 13x + 5 = 0$

=> $6x^2 - 3x - 10x + 5 = 0$

=> $3x(2x-1) - 5(2x-1) = 0$

=> $(2x-1)(3x-5) = 0$

=> $2x - 1 = 0$ or $3x - 5 = 0$

=> $2x = 1$ or $3x = 5$

=> $x = \frac{1}{2}$ or $x = \frac{5}{3}$

Thus solution set = $\{\frac{1}{2}, \frac{5}{3}\}$

(v): $3(x^2 - 1) = 4(x+1)$

Solution: $3(x^2 - 1) = 4(x+1)$

=> $3x^2 - 3 = 4x + 4$

=> $3x^2 - 4x - 3 - 4 = 0$

=> $3x^2 - 4x - 7 = 0$

=> $3x^2 + 3x - 7x - 7 = 0$

=> $3x(x+1) - 7(x+1) = 0$

=> $(x+1)(3x-7) = 0$

=> $x + 1 = 0$ or $3x - 7 = 0$

=> $x = -1$ or $3x = 7$

=> $x = \frac{7}{3}$

Thus solution set = $\{-1, \frac{7}{3}\}$.

(vi): $x(3x-5) = (x-6)(x-7)$

Solution: $x(3x-5) = (x-6)(x-7)$

=> $3x^2 - 5x = x^2 - 7x - 6x + 42$

=> $3x^2 - x^2 - 5x + 7x + 6x - 42 = 0$

=> $2x^2 + 8x - 42 = 0$

=> $2(x^2 + 4x - 21) = 0$ *Divided both sides by* 2

=> $x^2 + 4x - 21 = 0$

=> $x^2 - 3x + 7x - 21 = 0$

=> $x(x-3) + 7(x-3) = 0$

=> $(x-3)(x+7) = 0$

=> $x - 3 = 0$ *or* $x + 7 = 0$

=> $x = 3$ *or* $x = -7$

Thus solution set = { -7 , 3}.

Q.2: Solve each of the following equations by completing the square

(i): $x^2 + 6x - 40 = 0$

Solution: $x^2 + 6x - 40 = 0$

=> $x^2 + 6x = 40$

Adding both sides $(3)^2 = 9$

=> $x^2 + 6x + (3)^2 = 40 + 9$

=> $(x+3)^2 = 49$

Take square root of both sides

$\sqrt{(x+3)^2} = \pm\sqrt{49}$

=> $x + 3 = \pm 7$

=> $x + 3 = 7$, $x + 3 = -7$

=> $x = 7 - 3$, $x = -7 - 3$

=> $x = 4$, $x = -10$

Thus solution set = { -10 , 4}.

(ii): $x^2 - 10x + 11 = 0$

Solution: $x^2 - 10x + 11 = 0$

=> $x^2 - 10x = -11$ adding both sides $(5)^2 = 25$.

=> $x^2 - 10x + (5)^2 = (5)^2 - 11$

=> $(x-5)^2 = 25 - 11$

=> $(x-5)^2 = 14$

Take square root of both sides

=> $\sqrt{(x-5)^2} = \pm\sqrt{14}$

=> $x - 5 = \pm\sqrt{14}$

=> $x = 5 \pm \sqrt{14}$

=> $x = 5 + \sqrt{14}$, $x = 5 - \sqrt{14}$

Thus solution set = $\{5+\sqrt{14} \ , 5-\sqrt{14}\}$

(iii): $4x^2 + 12x = 0$

Solution: $4x^2 + 12x = 0$ Divided by 4 both sides

=> $\frac{4x^2}{4} + \frac{12x}{4} = \frac{0}{4}$

=> $x^2 + 3x = 0$ adding both sides $\left(\frac{3}{2}\right)^2$

=> $x^2 + 3x + \left(\frac{3}{2}\right)^2 = 0 + \left(\frac{3}{2}\right)^2$

=> $\left(x + \frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$

Take square root of both sides

=> $\sqrt{\left(x + \frac{3}{2}\right)^2} = \pm\sqrt{\left(\frac{3}{2}\right)^2}$

=> $x + \frac{3}{2} = \pm\frac{3}{2}$

$=> x = -\frac{3}{2} \pm \frac{3}{2}$

$=> x = -\frac{3}{2} + \frac{3}{2}$, $x = -\frac{3}{2} - \frac{3}{2}$

$=> x = 0$, $x = \frac{-3-3}{2} = -\frac{6}{2} = -3$

Thus solution set = { -3 , 0 }.

(iv): $5x^2 - 10x - 840 = 0$

Solution: $5x^2 - 10x - 840 = 0$ divided by 5 both sides

$=> \frac{5x^2}{5} - \frac{10x}{5} - \frac{840}{5} = 0$

$=> x^2 - 2x - 168 = 0$

$=> x^2 - 2x = 168$

Adding both sides 1

$=> x^2 - 2x + 1 = 168 + 1$

$=> (x-1)^2 = 169$ Take square root of both sides

$=> \sqrt{(x-1)^2} = \pm\sqrt{169}$

$=> x - 1 = \pm 13$

$=> x = 1 \pm 13$

$=> x = 1 + 13$, $x = 1 - 13$

$=> x = 14$, $x = -12$

Thus solution set = { -12 , 14}.

(v): $9x^2 - 6x + \frac{5}{9} = 0$

Solution: $9x^2 - 6x + \frac{5}{9} = 0$ Divided by 9 both sides

=> $\frac{9x^2}{9} - \frac{6x}{9} + \frac{5}{81} = 0$

=> $x^2 - \frac{2}{3}x + \frac{5}{81} = 0$

=> $x^2 - \frac{2}{3}x = -\frac{5}{81}$

Adding both sides $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

=> $x^2 - \frac{2}{3}x + \left(\frac{1}{3}\right)^2 = \frac{1}{9} - \frac{5}{81}$

=> $\left(x - \frac{1}{3}\right)^2 = \frac{9-5}{81}$

=> $\left(x - \frac{1}{3}\right)^2 = \frac{4}{81}$

Take square root of both sides

=> $\sqrt{\left(x - \frac{1}{3}\right)^2} = \pm\sqrt{\frac{4}{81}}$

=> $x - \frac{1}{3} = \pm\frac{2}{9}$

=> $x = \frac{1}{3} \pm \frac{2}{9}$

=> $x = \frac{1}{3} + \frac{2}{9}$, $x = \frac{1}{3} - \frac{2}{9}$

=> $x = \frac{3+2}{9}$, $x = \frac{3-2}{9}$

=> $x = \frac{5}{9}$, $x = \frac{1}{9}$

Thus solution set = $\{\frac{1}{9}, \frac{5}{9}\}$.

(vi): $(x-1)(x+3)=5(x+2)-3$

Solution: $(x-1)(x+3)=5(x+2)-3$

$\Rightarrow x^2+3x-x-3=5x+10-3$

$\Rightarrow x^2+2x-5x-3-10+3=0$

$\Rightarrow x^2-3x-10=0$

$\Rightarrow x^2-3x=10$

Adding both sides $\left(\frac{3}{2}\right)^2$

$\Rightarrow x^2-3x+\left(\frac{3}{2}\right)^2=10+\left(\frac{3}{2}\right)^2$

$\Rightarrow \left(x-\frac{3}{2}\right)^2=10+\frac{9}{4}$

$\Rightarrow \left(x-\frac{3}{2}\right)^2=\frac{40+9}{4}$

$\Rightarrow \left(x-\frac{3}{2}\right)^2=\frac{49}{4}$

Take square roots of both sides

$\Rightarrow \sqrt{\left(x-\frac{3}{2}\right)^2}=\pm\sqrt{\frac{49}{4}}$

$\Rightarrow x-\frac{3}{2}=\pm\frac{7}{2}$

$\Rightarrow x=\frac{3}{2}\pm\frac{7}{2}$

$\Rightarrow x=\frac{3}{2}+\frac{7}{2}$ *or* $x=\frac{3}{2}-\frac{7}{2}$

$\Rightarrow x=\frac{3+7}{2}$ *or* $x=\frac{3-7}{2}$

$\Rightarrow x = \frac{10}{2}$ or $x = -\frac{4}{2}$

$\Rightarrow x = 5$ or $x = -2$

Thus Solution set = $\{-2, 5\}$

Q.3: Solve each of the following equations by quadratic formula

(i): $x^2 - 8x + 15 = 0$

Solution: $x^2 - 8x + 15 = 0$

As by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad .....(i)$$

Here $a = 1$, $b = -8$, $c = 15$ put in (i)

$$\Rightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

$$\Rightarrow x = \frac{8 \pm \sqrt{64 - 60}}{2} = \frac{8 \pm \sqrt{4}}{2}$$

$$\Rightarrow x = \frac{8 \pm 2}{2} = \frac{2(4 \pm 1)}{2}$$

$\Rightarrow x = 4 \pm 1$

$\Rightarrow x = 4 + 1$, $x = 4 - 1$

$\Rightarrow x = 5$, $x = 3$

Thus solution set = $\{3, 5\}$.

(ii): $x^2 - 2x - 4 = 0$

Solution: $x^2 - 2x - 4 = 0$ As by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad .....(i)$$

Here $a = 1$, $b = -2$, $c = -4$ put in (i)

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$\Rightarrow \quad x = \frac{2 \pm \sqrt{4+16}}{2}$$

$$\Rightarrow \quad x = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2}$$

$$\Rightarrow \quad x = \frac{2(1 \pm \sqrt{5})}{2} = 1 \pm \sqrt{5}$$

Thus solution set = $\{1+\sqrt{5}\ ,\ 1-\sqrt{5}\}$

(iii): $4x^2 + 3x = 0$

Solution: $4x^2 + 3x = 0$ As by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad .....(i)$$

Here $a = 4$, $b = 3$, $c = 0$ put in (i)

$$\Rightarrow \quad x = \frac{-3 \pm \sqrt{(3)^2 - 4(4)(0)}}{2(4)}$$

$$\Rightarrow \quad x = \frac{-3 \pm \sqrt{9-0}}{8} = \frac{-3 \pm \sqrt{9}}{8}$$

$$\Rightarrow \quad x = \frac{-3 \pm 3}{8}$$

$$\Rightarrow \quad x = \frac{-3+3}{8}\ , \qquad x = \frac{-3-3}{8} = \frac{-6}{8}$$

$$\Rightarrow \quad x = \frac{0}{8} = 0\ , \quad x = -\frac{3}{4}$$

Thus solution set = $\{0\ , -\frac{3}{4}\}$

(iv): $3x(x-2)+1=0$

Solution: $3x(x-2)+1=0$

$\Rightarrow \; 3x^2 - 6x + 1 = 0$ As by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad .....(i)$$

Here $a = 3$, $b = -6$, $c = 1$ put in (i)

$$\Rightarrow \quad x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)}$$

$$\Rightarrow \quad x = \frac{6 \pm \sqrt{36 - 12}}{6} = \frac{6 \pm \sqrt{24}}{6}$$

$$\Rightarrow \quad x = \frac{6 \pm 2\sqrt{6}}{6} = \frac{2\left(3 \pm \sqrt{6}\right)}{6}$$

$$\Rightarrow \quad x = \frac{3 \pm \sqrt{6}}{3} = \frac{3}{3} \pm \frac{\sqrt{6}}{3}$$

$$\Rightarrow \quad x = 1 \pm \frac{\sqrt{6}}{3}$$

$$\Rightarrow \quad x = 1 + \frac{\sqrt{6}}{3} \quad , \quad x = 1 - \frac{\sqrt{6}}{3}$$

Thus solution set $= \left\{1 + \frac{\sqrt{6}}{3} \; , \; 1 - \frac{\sqrt{6}}{3}\right\}$

(v): $6x^2 - 17x + 12 = 0$

Solution: $6x^2 - 17x + 12 = 0$ As by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad .....(i)$$

Here $a = 6$, $b = -17$, $c = 12$ put in (i)

$$\Rightarrow \quad x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(6)(12)}}{2(6)}$$

$$\Rightarrow \quad x = \frac{17 \pm \sqrt{289 - 288}}{12} = \frac{17 \pm \sqrt{1}}{12}$$

$$=> \quad x = \frac{17 \pm 1}{12}$$

$$=> \quad x = \frac{17+1}{12} \quad , \quad x = \frac{17-1}{12}$$

$$=> \quad x = \frac{18}{12} = \frac{3}{2} \quad , x = \frac{16}{12} = \frac{4}{3}$$

Thus solution set $= \left\{\frac{3}{2}, \frac{4}{3}\right\}$

(vi: $\frac{x^2}{3} - \frac{x}{12} = \frac{1}{24}$

Solution: $\frac{x^2}{3} - \frac{x}{12} = \frac{1}{24}$ both sides multiplied by 24

$$=> \quad 24\left(\frac{x^2}{3} - \frac{x}{12}\right) = 24\left(\frac{1}{24}\right)$$

$$=> \quad 24\left(\frac{x^2}{3}\right) - 24\left(\frac{x}{12}\right) = 1$$

$$=> \quad 8x^2 - 2x - 1 = 0$$

As by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad .....(i)$$

Here $a = 8$, $b = -2$, $c = -1$ put in (i)

$$=> \quad x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(8)(-1)}}{2(8)}$$

$$=> \quad x = \frac{2 \pm \sqrt{4+32}}{16} = \frac{2 \pm \sqrt{36}}{16}$$

$$=> \quad x = \frac{2 \pm 6}{16}$$

$=> \quad x = \dfrac{2+6}{16} \quad , \quad x = \dfrac{2-6}{8}$

$=> \quad x = \dfrac{8}{16} = \dfrac{1}{2} \quad , x = -\dfrac{4}{16} = -\dfrac{1}{4}$

Thus solution set $= \left\{ \dfrac{1}{2} \; , -\dfrac{1}{4} \right\}$

Q.4:Find all the solutions of the following equations .

(i) $t^2 - 8t + 7 = 0$

Solution: $t^2 - 8t + 7 = 0$

$=> \; t^2 - t - 7t + 7 = 0$

$=> \; t(t-1) - 7(t-1) = 0$

$=> \; (t-1)(t-7) = 0$

$=> \; t - 1 = 0 \quad or \quad t - 7 = 0$

$=> \; t = 1 \quad or \quad t = 7$

Thus solution set = { 1 , 7} .

(ii) : $72 + 6x = x^2$

Solution: $72 + 6x = x^2$

$=> \; 0 = x^2 - 6x - 72$

$=> \; x^2 - 6x - 72 = 0$

$=> \; x^2 + 6x - 12x - 72 = 0$

$=> \; x(x+6) - 12(x+6) = 0$

$=> \; (x+6)(x-12) = 0$

$=> \; x + 6 = 0 \quad or \quad x - 12 = 0$

$=> \; x = -6 \quad or \quad x = 12$

Thus solution set = { - 6 , 12 } .

(iii) $r^2 + 4r + 1 = 0$

Solution: $r^2 + 4r + 1 = 0$

As by quadratic formula

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad ......(i)$$

Put $a=1$, $b=4$, $c=1$ **in (i)**

$$r=\frac{-4\pm\sqrt{(4)^2-4(1)(1)}}{2(1)}$$

$$\Rightarrow\ r=\frac{-4\pm\sqrt{16-4}}{2}$$

$$\Rightarrow\ r=\frac{-4\pm\sqrt{12}}{2}$$

$$=\frac{-4\pm\sqrt{4\times3}}{2}$$

$$\Rightarrow\ r=\frac{-4\pm2\sqrt{3}}{2}\quad\Rightarrow\ r=\frac{2\left(-2\pm\sqrt{3}\right)}{2}$$

$$\Rightarrow\ r=-2\pm\sqrt{3}$$

Thus solution set = $\left\{2+\sqrt{3}\ ,\ 2-\sqrt{3}\right\}$

(iv) : $x(x+10)=10(-10-x)$

Solution: $x(x+10)=10(-10-x)$

$$\Rightarrow\ x^2+10x=-100-10x$$

$$\Rightarrow\ x^2+10x+10x+100=0$$

$$\Rightarrow\ x(x+10)+10(x+10)=0$$

$$\Rightarrow\ (x+10)(x+10)=0$$

$$\Rightarrow\ x+10=0\quad or\quad x+10=0$$

$$\Rightarrow\ x=-10\ ,\quad x=-10$$

Thus solution set = { - 10 }

Q.5: The equation $(y+13)(y+a)$ **has no linear term . find value of** a **.**

Solution: $(y+13)(y+a)\ =\ y^2+13y+ay+13a$

$$=y^2+(13+a)y+13a\ \ldots\text{(i)}$$

> **Note: Now eq (i) has no linear term when there is no y . If the coefficient of y is zero .Then there is y term will be vanish . i.e. there is no linear term .**

In eq (i) there is no linear term if $13 + a = 0$

=> $a = -13$

Thus for $a = -13$ the eq(i) has no linear term .

Q.6: The equation $ax^2 + 5x = 3$ has $x = 1$ is a solution . what is the other solution .

Solution: $ax^2 + 5x = 3$ ....(i)

Since $x = 1$ is the solution of (i) . Then put $x = 1$ in (i)

$a(1)^2 + 5(1) = 3$

=> $a + 5 = 3$ => $a = 3 - 5$ => $a = -2$

Put $a = -2$ in (i)

$-2x^2 + 5x = 3$ => $0 = 2x^2 - 5x + 3$

Or $2x^2 - 5x + 3 = 0$

=> $2x^2 - 2x - 3x + 3 = 0$

=> $2x(x-1) - 3(x-1) = 0$

=> $(x-1)(2x-3) = 0$

=> $x - 1 = 0$ *or* $2x - 3 = 0$

=> $x = 1$ *or* $2x = 3$

=> $x = \frac{3}{2}$

Thus other solution is $x = \frac{3}{2}$

Q.7: What is the positive difference of the roots $x^2 - 7x - 9 = 0$

Solution: $x^2 - 7x - 9 = 0$

As by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad ......(i)$$

Put $a = 1$, $b = -7$, $c = -9$ in (i)

$$x = \frac{-(-7) \pm \sqrt{(7)^2 - 4(1)(-9)}}{2(1)}$$

=> $x = \frac{7 \pm \sqrt{49+36}}{2}$

=> $x = \frac{7 \pm \sqrt{85}}{2}$

=> $x = \frac{7+\sqrt{85}}{2}$ or $x = \frac{7-\sqrt{85}}{2}$

Thus the roots are $\frac{7+\sqrt{85}}{2}$, $\frac{7-\sqrt{85}}{2}$

The positive difference of the roots is

$$\frac{7+\sqrt{85}}{2} - \frac{7-\sqrt{85}}{2} = \frac{7+\sqrt{85}-7+\sqrt{85}}{2}$$

$$= \frac{2\sqrt{85}}{2} = \sqrt{85}$$

<u>Activity</u>; Solve the following quadratic equation

$x^2 - 8x + 15 = 0$ **, by**

(i) By factorization (ii) By completing square

<u>Solution</u> : (i) $x^2 - 8x + 15 = 0$

=> $x^2 - 3x - 5x + 15 = 0$

=> $x(x-3) - 5(x-3) = 0$

=> $(x-3)(x-5) = 0$

=> $x - 3 = 0$ or $x - 5 = 0$

=> $x = 3$ or $x = 5$

Thus solution is = { 3 , 5} .

(ii) $x^2 - 8x + 15 = 0$

=> $x^2 - 8x = -15$, adding both sides $(4)^2$

=> $x^2 - 8x + (4)^2 = (4)^2 - 15$

=> $(x-4)^2 = 16 - 15$

=> $(x-4)^2 = 1$, *Take square root of both sides*

=> $\sqrt{(x-4)^2} = \pm\sqrt{1}$

=> $x-4=\pm 1$ => $x=4\pm 1$

=> $x=4+1$ *or* $x=4-1$

=> $x=5$ *or* $x=3$

Thus solution is = { 3 , 5} .

Solution of equations reducible to quadratic form:

Type I: The equation of the form $ax^4+bx^2+c=0$ **...(i)**

Method: let $x^2=y$ => $x^4=y^2$ put in (i)

=> $ay^2+by+c=0$ .Then solve for y and put the value of y in $x^2=y$ find the value of x.

Example: Solve $12x^4-11x^2+2=0$

Solution: $12x^4-11x^2+2=0$ .....(i)

Let $x^2=y$.....(ii) *Then* $x^4=y^2$.....(iii)

Put (ii) and (iii) in (i) we get

$12y^2-11y+2=0$ *by factorization*

=> $12y^2-8y-3y+2=0$

=> $4y(3y-2)-1(3y-2)=0$

=> $(3y-2)(4y-1)=0$

=> $3y-2=0$ *or* $4y-1=0$

=> $3y=2$ *or* $4y=1$

=> $y=\frac{2}{3}$ *or* $y=\frac{1}{4}$

Now put $y=\frac{2}{3}$ in (ii) then $x^2=\frac{2}{3}$ => $x=\pm\sqrt{\frac{2}{3}}$

Now put $y=\frac{1}{4}$ *in (ii) then* $x^2=\frac{1}{4}$ => $x=\pm\frac{1}{2}$

Thus solution set = $\left\{-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}, -\frac{1}{2}, \frac{1}{2}\right\}$

Type II: The equation of the form $a\,p(x)+\dfrac{b}{p(x)}=c$

Method : Both sides multiplied by p(x) or p(x) = y the given equation and get quadratic equation and solve by factorization or by quadratic formula.

Example: Solve $2x+\dfrac{4}{x}=9$

Solution: $2x+\dfrac{4}{x}=9$ Both sides multiplied by x .

$\Rightarrow\ 2x^2+4=9x$

$\Rightarrow\ 2x^2-9x+4=0$

By factorization

$\Rightarrow 2x^2-x-8x+4=0$

$\Rightarrow\ x(2x-1)-4(2x-1)=0$

$\Rightarrow\ (2x-1)(x-4)=0$

$\Rightarrow\ 2x-1=0\quad or\quad x-4=0$

$\Rightarrow\ 2x=1\quad or\quad x=4$

$\Rightarrow\ x=\dfrac{1}{2}$

Thus solution set $=\left\{4\ ,\dfrac{1}{2}\right\}$

Type: III: Reciprocal equations of the form

$$a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0\quad or$$

$$a\left(x^2+\frac{1}{x^2}\right)+b\left(x-\frac{1}{x}\right)+c=0$$

Method: Let $x+\dfrac{1}{x}=y\quad or\quad x-\dfrac{1}{x}=y$

Then

$$(x+\frac{1}{x})^2 = y^2 \quad or \quad (x-\frac{1}{x})^2 = y^2$$

$$\Rightarrow x^2+\frac{1}{x^2} = y^2-2 \quad or \quad x^2+\frac{1}{x^2} = y^2+2$$

Put these values in the given equation we get a quadratic equation in y and solve for y and then find the value of x.

Example: (i) solve $2\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)+14=0$

Solution: $2\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)+14=0$ ....(i)

$$Let \quad x+\frac{1}{x}=y \quad .....(ii) \quad squaring\ both\ sides$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = y^2$$

$$\Rightarrow x^2+\frac{1}{x^2}+2=y^2 \Rightarrow x^2+\frac{1}{x^2}=y^2-2 \quad ....(iii)$$

Put (ii) ,(iii) in (i)

$$2(y^2-2)-9y+14=0$$

$$\Rightarrow 2y^2-4-9y+14=0$$

$$\Rightarrow 2y^2-9y+10=0$$

$$\Rightarrow 2y^2-4y-5y+10=0$$

$$\Rightarrow 2y(y-2)-5(y-2)=0$$

$$\Rightarrow (y-2)(2y-5)=0$$

$$\Rightarrow y-2=0 \quad or \quad 2y-5=0$$

$$\Rightarrow y=2 \quad or \quad 2y=5$$

$$\Rightarrow y=\frac{5}{2}$$

Now put y = 2 in (ii) . Then

$x+\frac{1}{x}=2$ *multiplied by x*

=> $x^2+1=2x$

=> $x^2-2x+1=0$

=> $(x-1)^2=0$

=> $x-1=0$, $x-1=0$

=> $x=1$, $x=1$

Now put $y=\frac{5}{2}$ in (ii)

$x+\frac{1}{x}=\frac{5}{2}$ *multiplied both sides by 2x*

=> $2x^2+2=5x$

=> $2x^2-5x+2=0$

=> $2x^2-4x-x+2=0$

=> $2x(x-2)-1(x-2)=0$

=> $(x-2)(2x-1)=0$

=> $x-2=0$ *or* $2x-1=0$

=> $x=2$ *or* $2x=1$

=> $x=\frac{1}{2}$

Thus solution set = $\left\{\frac{1}{2}\ ,1\ ,\ 2\right\}$.

Type IV: Equations that involves terms of the a^x , $a>0$, $a\neq 1$ are called exponential equations.

Example: **Solve** $4.2^{2x}-10.2^x+4=0$

Solution: $4.2^{2x}-10.2^x+4=0$ ……(i)

Let $2^x=y$ ….(ii) *then* $(2^x)^2=y^2$

$\Rightarrow \quad 2^{2x} = y^2 .....(iii)$

Put (ii) , (iii) in (i) .

$$4y^2 - 10y + 4 = 0$$
$$\Rightarrow \quad 2(2y^2 - 5y + 2) = 0$$
$$\Rightarrow \quad 2y^2 - 5y + 2 = 0$$
$$\Rightarrow \quad 2y^2 - 4y - y + 2 = 0$$
$$\Rightarrow \quad 2y(y-2) - 1(y-2) = 0$$
$$\Rightarrow \quad (y-2)(2y-1) = 0$$
$$\Rightarrow \quad y - 2 = 0 \quad or \quad 2y - 1 = 0$$
$$\Rightarrow \quad y = 2 \quad or \quad 2y = 1$$
$$\Rightarrow \quad y = \frac{1}{2}$$

Put y = 2 in (ii)

$2^x = 2 \quad \Rightarrow \quad 2^x = 2^1$ comparing powers

$$\Rightarrow \quad x = 1$$

$put \;\; y = \frac{1}{2} \;\; in \; (ii) \quad \Rightarrow \quad 2^x = \frac{1}{2}$

$$\Rightarrow \quad 2^x = 2^{-1} \quad \Rightarrow \quad x = -1$$

Thus solution set = { -1 , 1}.

<u>Type V</u>: Equations of the form

$(x+a)(x+b)(x+c)(x+d) = k$ **where** $a + b = c + d$

<u>Example</u>: Solve $(x+1)(x+3)(x-2)(x-4) = 24$

<u>Solution</u>: As 1- 2 = 3 – 4 .Then re- arranging the factors on the left side o the given equation.

$$[(x+1)(x-2)][(x+3)(x-4)] = 24$$
$$\Rightarrow (x^2 - 2x + x - 2)(x^2 - 4x + 3x - 12) = 24$$
$$\Rightarrow (x^2 - x - 2)(x^2 - x - 12) = 24 \; ....(i)$$

$Let \quad x^2 - x = y \; ......(ii)$ put (ii) in (i)

$$(y-2)(y-12) = 24$$

$$\Rightarrow y^2-12y-2y+24=24$$

$$\Rightarrow y^2-14y=24-24$$

$$\Rightarrow y^2-14y=0$$

$$\Rightarrow y(y-14)=0$$

$$\Rightarrow y=0 \quad or \quad y-14=0$$

$$\Rightarrow y=14$$

Put y = 0 in (ii)

$$x^2-x=0 \quad \Rightarrow x(x-1)=0$$

$$\Rightarrow x=0 \quad or \quad x-1=0$$

$$\Rightarrow x=1$$

Put y = 14 in (ii)

$$x^2-x=14$$

$$\Rightarrow x^2-x-14=0$$

By quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ ......(iv)

Put $a=1$, $b=-1$, $c=-14$ in (iv)

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-14)}}{2(1)}=\frac{1\pm\sqrt{1+56}}{2}$$

$$\Rightarrow x=\frac{1\pm\sqrt{57}}{2}$$

Thus solution set = $\left\{0\ ,1\ ,\dfrac{1+\sqrt{57}}{2}\ ,\ \dfrac{1-\sqrt{57}}{2}\right\}$

Exercise 1.2

Q.1:Solve the following equations.

(i): $x^4-5x^2+4=0$

Solution: $x^4-5x^2+4=0$ .....(i)

$Let \quad x^2=y$ .....(ii) $then \quad x^4=y^2$ ...(iii)

Put (ii) , (iii) in (i)

$$y^2-5y+4=0$$

$$\Rightarrow\ y^2-y-4y+4=0$$

$$\Rightarrow\ y(y-1)-4(y-1)=0$$

$$\Rightarrow\ (y-1)(y-4)=0$$

$$\Rightarrow\ y-1=0 \quad or \quad y-4=0$$

$$\Rightarrow\ y=1 \quad or \quad y=4$$

put $y=1$ *in (ii)*

$$x^2=1 \Rightarrow \sqrt{x^2}=\pm\sqrt{1}$$

$$\Rightarrow\ x=\pm1$$

Put $y=4$ in (ii)

$$x^2=4 \Rightarrow \sqrt{x^2}=\pm\sqrt{4}$$

$$\Rightarrow\ x=\pm2$$

Thus solution set = $\{\pm1, \pm2\}$

(ii): $x^4-7x^2+12=0$

Solution: $x^4-7x^2+12=0$ .....(i)

Let $x^2=y$ .....(ii) *then* $x^4=y^2$ ...(iii)

Put (ii), (iii) in (i)

$$y^2-7y+12=0$$

$$\Rightarrow\ y^2-3y-4y+12=0$$

$$\Rightarrow\ y(y-3)-4(y-3)=0$$

$$\Rightarrow\ (y-3)(y-4)=0$$

$$\Rightarrow\ y-3=0 \quad or \quad y-4=0$$

$$\Rightarrow\ y=3 \quad or \quad y=4$$

Put $y=3$ in (ii)

$$x^2=3 \Rightarrow \sqrt{x^2}=\pm\sqrt{3}$$

$$\Rightarrow\ x=\pm\sqrt{3}$$

Now put $y=4$ in (ii)

$x^2 = 4$

$\Rightarrow \sqrt{x^2} = \pm\sqrt{4} \quad \Rightarrow x = \pm 2$

Thus solution set = $\{\pm 2, \pm\sqrt{3}\}$

(iii): $6x^4 - 13x^2 + 5 = 0$

Solution: $6x^4 - 13x^2 + 5 = 0$ ....(i)

Let $x^2 = y$.......(ii) *then* $x^4 = y^2$ ....(iii)

Put (ii), (iii) in (i)

$6y^2 - 13y + 5 = 0$

$\Rightarrow 6y^2 - 3y - 10y + 5 = 0$

$\Rightarrow 3y(2y-1) - 5(2y-1) = 0$

$\Rightarrow (2y-1)(3y-5) = 0$

$\Rightarrow 2y - 1 = 0 \quad or \quad 3y - 5 = 0$

$\Rightarrow 2y = 1 \quad or \quad 3y = 5$

$\Rightarrow y = \frac{1}{2} \quad or \quad y = \frac{5}{3}$

Put $y = \frac{1}{2}$ in (ii)

$x^2 = \frac{1}{2} \Rightarrow \sqrt{x^2} = \pm\sqrt{\frac{1}{2}} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

Put $y = \frac{5}{3}$ in (ii).

$x^2 = \frac{5}{3} \Rightarrow \sqrt{x^2} = \pm\sqrt{\frac{5}{3}}$

$\Rightarrow x = \pm\sqrt{\frac{5}{3}}$

Thus solution set = $\left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{\frac{5}{3}}\right\}$

(iv): $x+2-\frac{1}{x+2}=\frac{3}{2}$

Solution: $x+2-\frac{1}{x+2}=\frac{3}{2}$ .....(i)

Let $x+2=y$....(ii) *then* $\frac{1}{x+2}=\frac{1}{y}$ ......(iii)

Put (ii) and (iii) in (i).

$y-\frac{1}{y}=\frac{3}{2}$ multiplied by 2y both sides

=> $2y^2-2=3y$

=> $2y^2-3y-2=0$

=> $2y^2+y-4y-2=0$

=> $y(2y+1)-2(2y+1)=0$

=> $(2y+1)(y-2)=0$

=> $2y+1=0$ *or* $y-2=0$

=> $2y=-1$ *or* $y=2$

=> $y=-\frac{1}{2}$

Put y = 2 in (ii) $x+2=2$

=> $x=2-2=0$

Put $y=-\frac{1}{2}$ in (ii).

$x+2=-\frac{1}{2}$ => $x=-2-\frac{1}{2}$

=> $x=\frac{-4-1}{2}=-\frac{5}{2}$

Thus solution set = $\left\{0 \quad ,-\frac{5}{2}\right\}$

(v): $x-\frac{4}{x}=2$

Solution: $x-\frac{4}{x}=2$ multiplied both sides by x .

=> $x^2-4=2x$

=> $x^2-2x-4=0$

Then by quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ ......(*i*)

Here $a=1$, $b=-2$, $c=-4$ put in (i) .

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}$$

=> $x=\frac{2\pm\sqrt{4+16}}{2}=\frac{2\pm\sqrt{20}}{2}$

=> $x=\frac{2\pm\sqrt{4\times5}}{2}=\frac{2\pm2\sqrt{5}}{2}$

=> $x=\frac{2(1\pm\sqrt{5})}{2}=1\pm\sqrt{5}$

Thus solution set = { $1\pm\sqrt{5}$ }.

(vi): $\frac{x+2}{x-2}-\frac{x-2}{x+2}=\frac{5}{6}$

Solution: $\frac{x+2}{x-2}-\frac{x-2}{x+2}=\frac{5}{6}$ .....(i)

Let $\frac{x+2}{x-2}=y$ ......(*ii*) *then* $\frac{x-2}{x+2}=\frac{1}{y}$ .....(*iii*)

Put (ii) , (iii) in (i) .

$y-\frac{1}{y}=\frac{5}{6}$ *multiplied both sides by* $6y$

=> $6y^2-6=5y$ => $6y^2-5y-6=0$

=> $6y^2-9y+4y-6=0$

=> $3y(2y-3)+2(2y-3)=0$

=> $(2y-3)(3y+2)=0$

$=> 2y-3=0 \quad or \quad 3y+2=0$

$=> 2y=3 \quad or \quad 3y=-2$

$=> y=\frac{3}{2} \quad or \quad y=-\frac{2}{3}$

$put \quad y=\frac{3}{2} \; in(ii)$

$\frac{x+2}{x-2}=\frac{3}{2} \quad => \quad 2(x+2)=3(x-2)$

$=> 2x+4=3x-6 \quad => \quad 4+6 = 3x-2x$

$=> \quad x=10$

Put $y=-\frac{2}{3}$ in (ii)

$\frac{x+2}{x-2}=-\frac{2}{3} \quad => \quad 3(x+2)=-2(x-2)$

$=> \quad 3x+6=-2x+4 \quad => \quad 3x+2x=4-6$

$=> \quad 5x=-2 \quad => \quad x=-\frac{2}{5}$

Thus solution set = $\{10 \;, -\frac{2}{5}\}$

<u>**(vii)**</u>: $3\left(x^2+\frac{1}{x^2}\right)-16\left(x+\frac{1}{x}\right)+26=0$

<u>**Solution**</u>: $3\left(x^2+\frac{1}{x^2}\right)-16\left(x+\frac{1}{x}\right)+26=0 \; ....(i)$

$Let \quad x+\frac{1}{x}=y \quad .....(ii) \quad squaring \;\; both \;\; sides$

$=> \left(x+\frac{1}{x}\right)^2=y^2$

$=> \quad x^2+\frac{1}{x^2}+2=y^2 \quad => \; x^2+\frac{1}{x^2}=y^2-2 \;\; ....(iii)$

Put (ii) , (iii) in (i)

$$3(y^2-2)-16y+26=0$$

$\Rightarrow 3y^2-6-16y+26=0$

$\Rightarrow 3y^2-16y+20=0$

$\Rightarrow 3y^2-6y-10y+20=0$

$\Rightarrow 3y(y-2)-10(y-2)=0$

$\Rightarrow (y-2)(3y-10)=0$

$\Rightarrow y-2=0 \quad or \quad 3y-10=0$

$\Rightarrow y=2 \quad or \quad 3y=10$

$\Rightarrow y=\frac{10}{3}$

Put y = 2 in (ii) then

$x+\frac{1}{x}=2$ *multiplied by* x

$\Rightarrow x^2+1=2x$

$\Rightarrow x^2-2x+1=0$

$\Rightarrow (x-1)^2=0$

$\Rightarrow x-1=0 \quad , \quad x-1=0$

$\Rightarrow x=1 \ , \ 1$

Now put $y=\frac{10}{3}$ in (ii)

$x+\frac{1}{x}=\frac{10}{3}$ multiplied by 3x .

$\Rightarrow 3x^2+3=10x$

$\Rightarrow 3x^2-10x+3=0$

$\Rightarrow 3x^2-9x-x+3=0$

$\Rightarrow 3x(x-3)-1(x-3)=0$

$\Rightarrow (x-3)(3x-1)=0$

$=> \quad x-3=0 \quad or \quad 3x-1=0$

$=> \quad x=3 \quad or \quad 3x=1$

$=> \quad x=\frac{1}{3}$

Thus solution set = $\left\{ \frac{1}{3}, 1, 3 \right\}$

(viii) : $\left(x+\frac{1}{x}\right)^2 - 10\left(x+\frac{1}{x}\right)+16=0$

Solution: $\left(x+\frac{1}{x}\right)^2 - 10\left(x+\frac{1}{x}\right)+16=0$ ....(i)

Let $x+\frac{1}{x}=y$ .....(ii) *squaring both sides*

$=> \left(x+\frac{1}{x}\right)^2 = y^2$ ....(iii)

Put (ii) , (iii) in (i)

$y^2-10y+16=0$

$=> y^2-2y-8y+16=0$

$=> y(y-2)-8(y-2)=0$

$=> (y-2)(y-8)=0$

$=> y-2=0 \quad or \quad y-8=0$

$=> y=2 \quad or \quad y=8$

Put y = 2 in (ii) then

$x+\frac{1}{x}=2$ *multiplied by* x

$=> x^2+1=2x$

$=> x^2-2x+1=0$

=> $(x-1)^2 = 0$

=> $x-1=0 \quad , \quad x-1=0$

=> $x = 1 \ , \ 1$

Now put $y = 8$ in (ii)

$x + \frac{1}{x} = 8$ multiplied by x .

=> $x^2 + 1 = 8x$

=> $x^2 - 8x + 1 = 0$

Then by quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ....(i)

Put $a=1$, $b=-8$, $c=1$ **in (i)**

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1(1))}}{2(1)}$$

=> $x = \frac{8 \pm \sqrt{64-4}}{2}$ => $x = \frac{8 \pm \sqrt{60}}{2}$

=> $x = \frac{8 \pm \sqrt{4 \times 15}}{2}$ => $x = \frac{8 \pm 2\sqrt{15}}{2}$

=> $x = \frac{2(4 \pm \sqrt{15})}{2}$ => $x = 4 \pm \sqrt{15}$

Thus solution set = $\{4-\sqrt{15} \ , \ 4+\sqrt{15}\}$

(ix): $\left(x^2 + \frac{1}{x^2}\right) - \left(x - \frac{1}{x}\right) - 4 = 0$

Solution: $\left(x^2 + \frac{1}{x^2}\right) - \left(x - \frac{1}{x}\right) - 4 = 0$ ....(i)

Let $x - \frac{1}{x} = y$ .....*(ii)* *squaring both sides*

=> $\left(x - \frac{1}{x}\right)^2 = y^2$

$$\Rightarrow \quad x^2+\frac{1}{x^2}-2=y^2 \quad \Rightarrow \quad x^2+\frac{1}{x^2}=y^2+2 \quad ....(iii)$$

Put (ii) , (iii) in (i)

$$y^2+2-y-4=0$$

$$\Rightarrow \quad y^2-y-2=0$$

$$\Rightarrow \quad y^2+y-2y-2=0$$

$$\Rightarrow \quad y(y+1)-2(y+1)=0$$

$$\Rightarrow \quad (y+1)(y-2)=0$$

$$\Rightarrow \quad y+1=0 \quad or \quad y-2=0$$

$$\Rightarrow \quad y=-1 \quad or \quad y=2$$

Put y = -1 in (ii)

$$x-\frac{1}{x}=-1 \quad \text{multiplied by } x$$

$$\Rightarrow \quad x^2-1=-x$$

$$\Rightarrow \quad x^2+x-1=0$$

As by quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad ......(iv)$$

Put $a=1$, $b=1$, $c=-1$ in (iv)

$$x=\frac{-1\pm\sqrt{(1)^2-4(1)(-1)}}{2(1)}=\frac{-1\pm\sqrt{1+4}}{2}$$

$$\Rightarrow \quad x=\frac{-1\pm\sqrt{5}}{2}$$

Put y = 2 in (ii)

$$x-\frac{1}{x}=2 \quad \text{multiplied by x.}$$

$$\Rightarrow \quad x^2-1=2x$$

$$\Rightarrow \quad x^2-2x-1=0$$

Put a = 1 , b = -2 , c = -1 in (iv)

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}=\frac{2\pm\sqrt{4+4}}{2}$$

$$=> \quad x=\frac{2\pm\sqrt{8}}{2}=\frac{2\pm2\sqrt{2}}{2}=1\pm\sqrt{2}$$

Thus solution set = $\left\{\frac{-1\pm\sqrt{5}}{2}\ ,\ 1\pm\sqrt{2}\right\}$

(x): $3^{2x}-10.3^x+9=0$

Solution: $3^{2x}-10.3^x+9=0$ ....(i)

Let $3^x=y$ .....(ii) *then* $(3^x)^2=y^2$

$=> \ 3^{2x}=y^2$......(iii)

Put (ii) , (iii) in (i)

$y^2-10y+9=0$

$=> \quad y^2-y-9y+9=0$

$=> y(y-1)-9(y-1)=0$

$=> (y-1)(y-9)=0$

$=> \quad y-1=0 \quad or \quad y-9=0$

$=> \quad y=1 \quad or \quad y=9$

Put y = 1 in (ii)

$3^x=1 \quad => \ 3^x=3^0$ $As\ \ 3^0=1$

$=> \ x=0$

Put y = 9 in (ii)

$3^x=9 \quad => \ 3^x=3^2$

$=> x=2$

Thus solution set = { 0 , 2 }.

(xi): $3.3^{2x+1}-10.3^x+1=0$

Solution: $3.3^{2x+1}-10.3^x+1=0$

$=> 3.3.3^{2x} - 10.3^{x} + 1 = 0$

$=> 9.3^{2x} - 10.3^{x} + 1 = 0 ......(i)$

Let $3^{x} = y \; .....(ii)$ *then* $(3^{x})^{2} = y^{2}$

$=> 3^{2x} = y^{2} ......(iii)$

Put (ii) , (iii) in (i)

$9y^{2} - 10y + 1 = 0$

$=> 9y^{2} - 9y - y + 1 = 0$

$=> 9y(y-1) - 1(y-1) = 0$

$=> (y-1)(9y-1) = 0$

$=> y - 1 = 0 \quad or \quad 9y - 1 = 0$

$=> y = 1 \quad or \quad 9y = 1$

$=> y = \frac{1}{9}$

Put y = 1 in (ii)

$3^{x} = 1 \quad => 3^{x} = 3^{0}$ *As* $3^{0} = 1$

$=> x = 0$

put $y = \frac{1}{9}$ in (ii)

$3^{x} = \frac{1}{9} \quad => 3^{x} = \frac{1}{3^{2}}$

$=> 3^{x} = 3^{-2} \quad => x = -2$

Thus solution set = { 0 , -2 } .

(xii): $5^{x+1} + 5^{2-x} = 126$

Solution: $5^{x+1} + 5^{2-x} = 126$

$=> 5.5^{x} + 5^{2}.5^{-x} - 126 = 0$

$=> 5.5^{x} + \frac{25}{5^{x}} - 126 = 0 \; ......(i)$

$Let \quad 5^x = y \;.....(ii) \qquad then \quad \frac{1}{5^x} = \frac{1}{y} .....(iii)$

Put (ii) , (iii) in (i)

$5y + \frac{25}{y} - 126 = 0$ multiplied by y

$=> \; 5y^2 + 25 - 126y = 0$

$=> 5y^2 - 126y + 25 = 0$

$=> \; 5y^2 - 125y - y + 25 = 0$

$=> \; 5y(y-25) - 1(y-25) = 0$

$=> \; (y-25)(5y-1) = 0$

$=> y - 25 = 0 \quad or \;\; 5y - 1 = 0$

$=> \; y = 25 \qquad or \quad 5y = 1$

$=> \quad y = \frac{1}{5}$

Put y = 25 in (ii)

$5^x = 25 \; => \; 5^x = 5^2$

$=> \; x = 2$

$put \; y = \frac{1}{5}$ in (ii).

$5^x = \frac{1}{5} \qquad => 5^x = 5^{-1}$

$=> \; x = -1$

Thus solution set = {-1 , 2}.

(xiii): $(x-3)(x+9)(x+5)(x-7) = 385$

Solution: $(x-3)(x+9)(x+5)(x-7) = 385$

As - 3 + 5 = -7 + 9 .Then re- arranging the factors

$$[(x-3)(x+5)][(x-7)(x+9)]=385$$

$$\Rightarrow (x^2+5x-3x-15)(x^2+9x-7x-63)=385$$

$$\Rightarrow (x^2+2x-15)(x^2+2x-63)=385 \quad ....(i)$$

Let $x^2+2x=y$ ......(ii) put (ii) in (i)

$$\Rightarrow (y-15)(y-63)=385$$

$$\Rightarrow y^2-63y-15y+945=385$$

$$\Rightarrow y^2-78y+945-385=0$$

$$\Rightarrow y^2-78y+560=0$$

$$\Rightarrow y^2-8y-70y+560=0$$

$$\Rightarrow y(y-8)-70(y-8)=0$$

$$\Rightarrow (y-8)(y-70)=0$$

$$\Rightarrow y-8=0 \quad or \quad y-70=0$$

$$\Rightarrow y=8 \quad or \quad y=70$$

Put y = 8 in (i)

$$x^2+2x=8 \Rightarrow x^2+2x-8=0$$

$$\Rightarrow x^2-2x+4x-8=0$$

$$\Rightarrow x(x-2)+4(x-2)=0$$

$$\Rightarrow (x-2)(x+4)=0$$

$$\Rightarrow x-2=0 \quad or \quad x+4=0$$

$$\Rightarrow x=2 \quad or \quad x=-4$$

Put y = 70 in (ii)

$$x^2+2x=70 \Rightarrow x^2+2x-70=0$$

By quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ .....(iii)

Put $a=1$, $b=2$, $c=-70$ in (iii)

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-70)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 280}}{2}$$

$$\Rightarrow \; x = \frac{-2 \pm \sqrt{284}}{2} = \frac{-2 \pm \sqrt{4 \times 71}}{2}$$

$$\Rightarrow \; x = \frac{-2 \pm 2\sqrt{71}}{2} = \frac{2\left(-1 \pm \sqrt{71}\right)}{2}$$

$$\Rightarrow \; x = -1 \pm \sqrt{71}$$

Thus solution set = $\left\{-4 \;, 2 \;\; -1 \pm \sqrt{71}\right\}$

(xiv): $(x+1)(x+2)(x+3)(x+4)+1=0$

Solution: $(x+1)(x+2)(x+3)(x+4)+1=0$

As $2+3=1+4$. Then re –arranging the factors

$$[(x+2)(x+3)][(x+1)(x+4)]+1=0$$

$$\Rightarrow \left(x^2+3x+2x+6\right)\left(x^2+4x+x+4\right)+1=0$$

$$\Rightarrow \left(x^2+5x+6\right)\left(x^2+5x+4\right)+1=0 \; ....(i)$$

Let $x^2+5x=y \; ......(ii)$ put (ii) in (i).

$$\Rightarrow \; (y+6)(y+4)+1=0$$

$$\Rightarrow \; y^2+4y+6y+24+1=0$$

$$\Rightarrow \; y^2+10y+25=0$$

$$\Rightarrow \; y^2+5y+5y+25=0$$

$$\Rightarrow \; y(y+5)+5(y+5)=0$$

$$\Rightarrow \; (y+5)(y+5)=0$$

$$\Rightarrow \; y+5=0 \;, \quad y+5=0$$

$\Rightarrow$ y = - 5 , y = -5

Put y = - 5 in (ii)

$$x^2+5x=-5 \quad \Rightarrow \quad x^2+5x+5=0$$

Then by quadratic formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} \; ......(iii)$

Put $a=1$, $b=5$, $c=5$ put in (iii) .

$$x=\frac{-5\pm\sqrt{(5)^2-4(1)(5)}}{2(1)}=\frac{-5\pm\sqrt{25-20}}{2}$$

$$\Rightarrow x=\frac{-5\pm\sqrt{5}}{2}$$

Thus solution set $=\{\frac{-5\pm\sqrt{5}}{2}\}$.

<u>**(xv)**</u>: $(x+1)(x+3)(x+5)(x+7)+16=0$

<u>**Solution**</u>: $(x+1)(x+3)(x+5)(x+7)+16=0$

As $1+7=3+5$

$$((x+1)(x+7))((x+3)(x+5))+16=0$$

$$\Rightarrow (x^2+7x+x+7)(x^2+5x+3x+15)+16=0$$

$$\Rightarrow (x^2+8x+7)(x^2+8x+15)+16=0 \;.......(i)$$

let $x^2+8x=y \;.....(ii)$ put (ii) in (i)

$$(y+7)(y+15)+16=0$$

$$\Rightarrow y^2+15y+7y+105+16=0$$

$$\Rightarrow y^2+22y+121=0$$

$$\Rightarrow (y+11)^2=0$$

$$\Rightarrow y+11=0 \quad , \quad y+11=0$$

$$\Rightarrow y=-11 \quad , \quad y=-11$$

put $y=-11$ *in* (ii)

$$x^2+8x=-11 \quad \Rightarrow \quad x^2+8x+11=0$$

As by quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\;....(iii)$

Put $a=1$, $b=8$, $c=11$ in (iii)

$$x=\frac{-8\pm\sqrt{(8)^2-4(1)(11)}}{2(1)}=\frac{-8\pm\sqrt{64-44}}{2}$$

$$=> \quad x=\frac{-8\pm\sqrt{20}}{2}=\frac{-8\pm\sqrt{4\times5}}{2}$$

$$=> \quad x=\frac{-8\pm2\sqrt{5}}{2}=\frac{2\left(-4\pm\sqrt{5}\right)}{2}$$

$$=> \quad x=-4\pm\sqrt{5}$$

Thus solution set = $\{-4\pm\sqrt{5}\}$

Q.2: Solve the equation $x^4-2x^3-2x^2+2x+1=0$

Solution: $x^4-2x^3-2x^2+2x+1=0$

$$=> \quad x^4+1-2x^3+2x-2x^2=0$$

Divided by x^2

$$\frac{x^4}{x^2}+\frac{1}{x^2}-\frac{2x^3}{x^2}+\frac{2x}{x^2}-\frac{2x^2}{x^2}=\frac{0}{x^2}$$

$$=> \quad x^2+\frac{1}{x^2}-2x+\frac{2}{x}-2=0$$

$$=> \quad x^2+\frac{1}{x^2}-2\left(x-\frac{1}{x}\right)-2=0 \quad ......(i)$$

Let $x-\frac{1}{x}=y \quad .....(ii)$ *squaring both sides*

$$=> \quad \left(x-\frac{1}{x}\right)^2=y^2$$

$$=> \quad x^2+\frac{1}{x^2}-2=y^2 \quad => \quad x^2+\frac{1}{x^2}=y^2+2 \quad ....(iii)$$

Put (ii) , (iii) in (i)

$$y^2+2-2y-2=0$$

$$=> \quad y^2-2y=0$$

=> $y(y-2)=0$

=> $y=0 \quad or \quad y-2=0$

=> $y=0 \quad or \quad y=2$

Put y = 0 in (ii)

$x-\frac{1}{x}=0$ multiplied by x

=> $x^2-1=0$

=> $x^2=1$ => $x=\pm 1$

Put y = 2 in (ii)

$x-\frac{1}{x}=2$ multiplied by x.

=> $x^2-1=2x$

=> $x^2-2x-1=0$

Put a = 1 , b = -2 , c = -1 in (iv)

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}=\frac{2\pm\sqrt{4+4}}{2}$$

=> $x=\frac{2\pm\sqrt{8}}{2}=\frac{2\pm 2\sqrt{2}}{2}=1\pm\sqrt{2}$

Thus solution set = $\{\pm 1,\ 1\pm\sqrt{2}\}$

Radical Equations: An equation in which the variable appears in one or more radicands is called Radical equations.

For example $\sqrt{x+2}=3$, $\sqrt{x+5}=\sqrt{2x-1}$

Note: A solution of the transformed equation that does not Satisfy the original Radical equation is called Extraneous root. By checking the extraneous root will be excluded from the solution set .

Type I: Equation of the form $\sqrt{ax+b}=cx+d$

Method: Squaring both sides we get quadratic equation.

Example: *Solve* $\sqrt{27-3x}=x-3$

Solution: $\sqrt{27-3x}=x-3$ ……(i) Squaring both sides

=> $\left(\sqrt{27-3x}\right)^2 = (x-3)^2$

=> $27-3x = x^2 - 6x + 9$

=> $0 = x^2 - 6x + 3x + 9 - 27$

=> $x^2 - 3x - 18 = 0$

=> $x^2 + 3x - 6x - 18 = 0$

=> $x(x+3) - 6(x+3) = 0$

=> $(x+3)(x-6) = 0$

=> $x+3=0$ *or* $x-6=0$

=> $x=-3$ *or* $x=6$

Check: put $x = 6$ in (i)

$\sqrt{27-3(6)} = 6-3$ => $\sqrt{27-18} = 3$

=> $\sqrt{9} = 3$ => $3 = 3$ (*True*)

Now put $x = -3$ in (i)

$\sqrt{27-3(-3)} = -3-3$ => $\sqrt{27+9} = -6$

=> $\sqrt{36} = -6$ => $6 = -6$ *which is not true*

Thus solution set = $\{6\}$.

Type II: Equation of the form $\sqrt{x+a} + \sqrt{x+b} = \sqrt{x+c}$

Method: Two times taken the square of the given equation then get a quadratic equation.

Example: Solve $\sqrt{x+2} + \sqrt{x+7} = \sqrt{x+23}$

Solution: $\sqrt{x+2} + \sqrt{x+7} = \sqrt{x+23}$ ....(i)

Squaring both sides

=> $\left(\sqrt{x+2} + \sqrt{x+7}\right)^2 = \left(\sqrt{x+23}\right)^2$

=> $\left(\sqrt{x+2}\right)^2 + \left(\sqrt{x+7}\right)^2 + 2\left(\sqrt{x+2}\right)\left(\sqrt{x+7}\right) = x+23$

=> $x+2+x+7+2\sqrt{(x+2)(x+7)} = x+23$

=> $2x+9+2\sqrt{x^2+7x+2x+14}=x+23$

=> $2\sqrt{x^2+9x+14}=x-2x+23-9$

=> $2\sqrt{x^2+9x+14}=-x+14$

=> $2\sqrt{x^2+9x+14}=14-x$ Take square of both sides

=> $\left(2\sqrt{x^2+9x+14}\right)^2=(14-x)^2$

=> $4\left(x^2+9x+14\right)=196+x^2-28x$

=> $4x^2+36x+56=196+x^2-28x$

=> $4x^2-x^2+36x+28x+56-196=0$

=> $3x^2+64x-140=0$

=> $3x^2-6x+70x-140=0$

=> $3x(x-2)+70(x-2)=0$

=> $(x-2)(3x+70)=0$

=> $x-2=0$ *or* $3x+70=0$

=> $x=2$ *or* $3x=-70$

=> $x=-\frac{70}{3}$

On checking $x=-\frac{70}{3}$ does not satisfy the eq(i) .

Thus $x=-\frac{70}{3}$ is extraneous root.

Thus solution set = { 2 }.

<u>Type III</u>: Equation of the form

$$\sqrt{x^2+px+m}+\sqrt{x^2+px+n}=q$$

Method: Transformed one radicand to the right hand side and then squaring two times and get quadratic equation.

<u>Example</u>: $\sqrt{x^2+3x+5}+\sqrt{x^2+3x+1}=2$

Solution: $\sqrt{x^2+3x+5}+\sqrt{x^2+3x+1}=2$

=> $\sqrt{x^2+3x+5}=2-\sqrt{x^2+3x+1}$ *Take square*

=> $\left(\sqrt{x^2+3x+5}\right)^2=\left(2-\sqrt{x^2+3x+1}\right)^2$

=> $x^2+3x+5=(2)^2+\left(\sqrt{x^2+3x+1}\right)^2-2(2)(\sqrt{x^2+3x+1})$

=> $x^2+3x+5=4+x^2+3x+1-4\sqrt{x^2+3x+1}$

=> $4\sqrt{x^2+3x+1}=x^2-x^2+3x-3x+5-5$

=> $4\sqrt{x^2+3x+1}=0$

Squaring both sides

=> $\left(4\sqrt{x^2+3x+1}\right)^2=(0)^2$

=> $16(x^2+3x+1)=0$ **divided both sides by 16**

=> $x^2+3x+1=0$

As by quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ ...(*ii*)

Put $a=1$, $b=3$, $c=1$ in (ii).

$$x=\frac{-3\pm\sqrt{(3)^2-4(1)(1)}}{2(1)}=\frac{-3\pm\sqrt{9-4}}{2}$$

=> $x=\dfrac{-3\pm\sqrt{5}}{2}$

Thus solution set = $\left\{\dfrac{-3\pm\sqrt{5}}{2}\right\}$

Exercise 1.3

Q.1: Solve the following equations:

(i): $\sqrt{5x+21}=x+3$

Solution: $\sqrt{5x+21} = x+3$ ....(i) squaring both sides

$$\Rightarrow \left(\sqrt{5x+21}\right)^2 = (x+3)^2$$
$$\Rightarrow 5x+21 = x^2+6x+9$$
$$\Rightarrow 0 = x^2+6x-5x+9-21$$
$$\Rightarrow x^2+x-12=0$$
$$\Rightarrow x^2-3x+4x-12=0$$
$$\Rightarrow x(x-3)+4(x-3)=0$$
$$\Rightarrow (x-3)(x+4)=0$$
$$\Rightarrow x-3=0 \quad or \quad x+4=0$$
$$\Rightarrow x=3 \quad or \quad x=-4$$

Check: put x = 3 in (i)

$$\sqrt{5(3)+21} = 3+3 \quad \Rightarrow \quad \sqrt{15+21} = 6$$
$$\Rightarrow \sqrt{36} = 6 \qquad \Rightarrow \quad 6 = 6 \quad (True)$$

Put x = - 4 in (i) .

$$\Rightarrow \sqrt{5(-4)+21} = -4+3 \quad \Rightarrow \quad \sqrt{-20+21} = -1$$
$$\Rightarrow \sqrt{1} = -1 \qquad \Rightarrow 1 = -1 \quad (False)$$

Thus solution set = { 3 } .

<u>(ii)</u>: $\sqrt{2x-1} = x-2$

Solution: $\sqrt{2x-1} = x-2$ .....(i) squaring both sides

$$\Rightarrow \left(\sqrt{2x-1}\right)^2 = (x-2)^2$$
$$\Rightarrow 2x-1 = x^2-4x+4$$
$$\Rightarrow 0 = x^2-4x-2x+4+1$$
$$\Rightarrow x^2-6x+5=0$$
$$\Rightarrow x^2-x-5x+5=0$$
$$\Rightarrow x(x-1)-5(x-1)=0$$

$\Rightarrow \quad (x-1)(x-5)=0$

$\Rightarrow \quad x-1=0 \quad or \quad x-5=0$

$\Rightarrow \quad x=1 \quad or \quad x=5$

Check: Put x = 1 in (i)

$\sqrt{2(1)-1}=1-2 \quad \Rightarrow \sqrt{2-1}=-1$

$\Rightarrow \sqrt{1}=-1 \quad \Rightarrow \quad 1=-1 \quad (false)$

Put x = 5 in (i)

$\sqrt{2(5)-1}=5-2 \quad \Rightarrow \sqrt{10-1}=3$

$\Rightarrow \sqrt{9}=3 \quad \Rightarrow \quad 3=3 \quad (True)$

Thus solution set = { 5 }.

(iii): $\sqrt{4x+5}=2x-5$

Solution: $\sqrt{4x+5}=2x-5$ ....(i) squaring both sides

$\Rightarrow \left(\sqrt{4x+5}\right)^2=(2x-5)^2$

$\Rightarrow 4x+5=4x^2-20x+25$

$\Rightarrow \quad 0=4x^2-20x-4x+25-5$

$\Rightarrow \quad 4x^2-24x+20=0$

$\Rightarrow \quad 4(x^2-6x+5)=0 \qquad divided\ by\ 4$

$\Rightarrow \quad x^2-6x+5=0$

$\Rightarrow \quad x^2-x-5x+5=0$

$\Rightarrow \quad x(x-1)-5(x-1)=0$

$\Rightarrow \quad (x-1)(x-5)=0$

$\Rightarrow \quad x-1=0 \quad or \quad x-5=0$

$\Rightarrow \quad x=1 \quad or \quad x=5$

Check: Put x = 1 in (i)

$\sqrt{4(1)+5}=2(1)-5 \quad \Rightarrow \sqrt{4+5}=2-5$

$\Rightarrow \sqrt{9}=-3 \quad \Rightarrow \quad 3=-3 \quad (false)$

Put x = 5 in (i)

$\sqrt{4(5)+5} = 2(5)-5 \quad => \sqrt{20+5} = 10-5$

$=> \sqrt{25} = 5 \quad => 5 = 5 \quad (True)$

Thus solution set = { 5 }.

(iv): $\sqrt{29-4x} = 2x+3$

Solution: $\sqrt{29-4x} = 2x+3$ .....(i) squaring both sides

$=> \left(\sqrt{29-4x}\right)^2 = (2x+3)^2$

$=> 29-4x = 4x^2+12x+9$

$=> 0 = 4x^2+12x+4x+9-29$

$=> 4x^2+16x-20 = 0$

$=> 4(x^2+4x-5) = 0$ divided by 4

$=> x^2+4x-5 = 0$

$=> x^2-x+5x-5 = 0$

$=> x(x-1)+5(x-1) = 0$

$=> (x-1)(x+5) = 0$

$=> x-1 = 0 \quad or \quad x+5 = 0$

$=> x = 1 \quad or \quad x = -5$

By checking -5 is an extraneous root.

Thus solution set = { 1}.

(v): $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Solution: $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$ ...(i)

squaring both sides

$=> \left(\sqrt{x+7}+\sqrt{x+2}\right)^2 = \left(\sqrt{6x+13}\right)^2$

$=> \left(\sqrt{x+7}\right)^2 + \left(\sqrt{x+2}\right)^2 + 2(\sqrt{x+7})(\sqrt{x+2}) = 6x+13$

$=> x+7+x+2+2\left(\sqrt{(x+7)(x+2)}\right) = 6x+13$

=> $2x+9+2\sqrt{x^2+2x+7x+14}=6x+13$

=> $2\sqrt{x^2+9x+14}=6x-2x+13-9$

=> $2\sqrt{x^2+9x+14}=4x+4$

=> $\sqrt{x^2+9x+14}=2x+2$ *divided by 2*

=> $\left(\sqrt{x^2+9x+14}\right)^2=(2x+2)^2$ *squaring both sides*

=> $x^2+9x+14=4x^2+4+8x$

=> $x^2-4x^2+9x-8x+14-4=0$

=> $-3x^2+x+10=0$

=> $-(3x^2-x-10)=0$ *multiplied by −1 both sides*

=> $3x^2-x-10=0$

=> $3x^2-6x+5x-10=0$

=> $3x(x-2)+5(x-2)=0$

=> $(x-2)(3x+5)=0$

=> $x-2=0$ or $3x+5=0$

=> $x=2$ or $x=-\frac{5}{3}$

As $-\frac{5}{3}$ is an extraneous root.

Thus solution set = { 2 }.

(vi): $\sqrt{x}+\sqrt{3x+1}=\sqrt{5x+1}$

Solution: $\sqrt{x}+\sqrt{3x+1}=\sqrt{5x+1}$ .....(i)

Squaring both sides

=> $\left(\sqrt{x}+\sqrt{3x+1}\right)^2=\left(\sqrt{5x+1}\right)^2$

=> $x+3x+1+2\left(\sqrt{x}\right)\left(\sqrt{3x+1}\right)=5x+1$

$=> 4x+1+2\left(\sqrt{x(3x+1)}\right)=5x+1$

$=> 2\left(\sqrt{3x^2+x}\right)=5x-4x+1-1$

$=> 2\left(\sqrt{3x^2+x}\right)=x$

Squaring both sides

$=> \left(2\sqrt{3x^2+x}\right)^2=x^2$

$=> 4(3x^2+x)=x^2$

$=> 12x^2+4x-x^2=0$

$=> 11x^2+4x=0 \quad => \quad x(11x+4)=0$

$=> x=0 \quad , \quad 11x+4=0$

$=> 11x=-4 \quad => \quad x=-\frac{4}{11}$

By checking $-\frac{4}{11}$ is an extraneous root .

Thus solution set = { 0 } .

(vii): $\sqrt{6x+40}-\sqrt{x+21}=\sqrt{x+5}$

Solution: $\sqrt{6x+40}-\sqrt{x+21}=\sqrt{x+5}$ squaring both sides

$=> \left(\sqrt{6x+40}-\sqrt{x+21}\right)^2=\left(\sqrt{x+5}\right)^2$

$=> \left(\sqrt{6x+40}\right)^2+\left(\sqrt{x+21}\right)^2-2\left(\sqrt{6x+40}\right)\left(\sqrt{x+21}\right)=x+5$

$=> 6x+40+x+21-2\left(\sqrt{(6x+40)(x+21)}\right)=x+5$

$=> 7x+61-2\sqrt{6x^2+126x+40x+840}=x+5$

$=> 7x-x+61-5=2\sqrt{6x^2+166x+840}$

$=> 6x+56=2\sqrt{6x^2+166x+840}$

$=> 2(3x+28)=2\sqrt{6x^2+166x+840}$ *divided by* 2

$\Rightarrow 3x+28=\sqrt{6x^2+166x+840}$ *squaring both sides*

$\Rightarrow (3x+28)^2=\left(\sqrt{6x^2+166x+840}\right)^2$

$\Rightarrow 9x^2+784+168x=6x^2+166x+840$

$\Rightarrow 9x^2-6x^2+168x-166x+784-840=0$

$\Rightarrow 3x^2+2x-56=0$

$\Rightarrow 3x^2-12x+14x-56=0$

$\Rightarrow 3x(x-4)+14(x-4)=0$

$\Rightarrow (x-4)(3x+14)=0$

$\Rightarrow x-4=0 \quad , \quad 3x+14=0$

$\Rightarrow x=4 \quad , \quad 3x=-14$

$\Rightarrow x=-\frac{14}{3}$

By checking $-\frac{14}{3}$ is an extraneous root.

Thus solution set = { 4 }.

(viii): $\sqrt{2x-3}+\sqrt{2x+4}=\sqrt{6x+13}$

Solution: $\sqrt{2x-3}+\sqrt{2x+4}=\sqrt{6x+13}$ squaring both sides

$\Rightarrow \left(\sqrt{2x-3}+\sqrt{2x+4}\right)^2=\left(\sqrt{6x+13}\right)^2$

$\Rightarrow \left(\sqrt{2x-3}\right)^2+\left(\sqrt{2x+4}\right)^2+2\left(\sqrt{2x-3}\right)\left(\sqrt{2x+4}\right)=6x+13$

$\Rightarrow 2x-3+2x+4+2\left(\sqrt{(2x-3)(2x+4)}\right)=6x+13$

$\Rightarrow 4x+1+2\left(\sqrt{4x^2+8x-6x-12}\right)=6x+13$

$\Rightarrow 2\sqrt{4x^2+2x-12}=6x-4x+13-1$

$\Rightarrow 2\sqrt{4x^2+2x-12}=2x+12$

$\Rightarrow 2\sqrt{4x^2+2x-12}=2(x+6)$

$=> \sqrt{4x^2+2x-12} = x+6$ *squaring both sides*

$=> \left(\sqrt{4x^2+2x-12}\right)^2 = (x+6)^2$

$=> 4x^2+2x-12 = x^2+12x+36$

$=> 4x^2-x^2+2x-12x-12-36=0$

$=> 3x^2-10x-48=0$

$=> 3x^2-18x+8x-48-0$

$=> 3x(x-6)+8(x-6)=0$

$=> (x-6)(3x+8)=0$

$=> x-6=0 \quad , \quad 3x+8=0$

$=> x=6 \quad , \quad 3x=-8$

$=> x=-\frac{8}{3}$

By checking $-\frac{8}{3}$ is an extraneous root.

Thus solution set = { 6 } .

(ix): $\sqrt{x^2+2x+4}+\sqrt{x^2+2x+9}=5$

Solution: $\sqrt{x^2+2x+4}+\sqrt{x^2+2x+9}=5$

$=> \sqrt{x^2+2x+4} = 5-\sqrt{x^2+2x+9}$ *squaring both sides*

$=> \left(\sqrt{x^2+2x+4}\right)^2 = \left(5-\sqrt{x^2+2x+9}\right)^2$

$=> x^2+2x+4 = 25+x^2+2x+9-2(5)(\sqrt{x^2+2x+9})$

$=> 10\sqrt{x^2+2x+9} = x^2-x^2+2x-2x+25+9-4$

$=> 10\sqrt{x^2+2x+9} = 30$ *divided by 10*

$=> \sqrt{x^2+2x+9} = 3$

$\Rightarrow \left(\sqrt{x^2+2x+9}\right)^2 = (3)^2$

$\Rightarrow x^2+2x+9=9$

$\Rightarrow x^2+2x+9-9=0$

$\Rightarrow x^2+2x=0$

$\Rightarrow x(x+2)=0$

$\Rightarrow x=0 \quad , \quad x+2=0 \Rightarrow x=-2$

Thus solution set = { 0 , -2 } .

(x): $\sqrt{2x^2+3x+5}+\sqrt{2x^2+3x+1}=2$

Solution: $\sqrt{2x^2+3x+5}+\sqrt{2x^2+3x+1}=2$

$\Rightarrow \sqrt{2x^2+3x+5}=2-\sqrt{2x^2+3x+1}$

$\Rightarrow \left(\sqrt{2x^2+3x+5}\right)^2=\left(2-\sqrt{2x^2+3x+1}\right)^2$

$\Rightarrow 2x^2+3x+5=4+\left(\sqrt{2x^2+3x+1}\right)^2-2(2)\left(\sqrt{2x^2+3x+1}\right)$

$\Rightarrow 2x^2+3x+5=4+2x^2+3x+1-4\sqrt{2x^2+3x+1}$

$\Rightarrow 4\sqrt{2x^2+3x+1}=2x^2-2x^2+3x-3x+4+1-5$

$\Rightarrow 4\sqrt{2x^2+3x+1}=0$ *divided by 4*

$\Rightarrow \sqrt{2x^2+3x+1}=0$ *squaring both sides*

$\Rightarrow \left(\sqrt{2x^2+3x+1}\right)^2=(0)^2$

$\Rightarrow 2x^2+3x+1=0$

$\Rightarrow 2x^2+2x+x+1=0$

$\Rightarrow 2x(x+1)+1(x+1)=0$

$\Rightarrow (x+1)(2x+1)=0$

=> $x+1=0$, $2x+1=0$

=> $x=-1$, $2x=-1$

=> $x=-\frac{1}{2}$

Thus solution set = $\{-1,-\frac{1}{2}\}$.

Q.2: Find $2x+5$, if x satisfy

$\sqrt{40-9x}-2\sqrt{7-x}=\sqrt{-x}$

Solution : $\sqrt{40-9x}-2\sqrt{7-x}=\sqrt{-x}$

=> $\sqrt{40-9x}-\sqrt{-x}=2\sqrt{7-x}$ .....(i)

Squaring both sides

$\left(\sqrt{40-9x}-\sqrt{-x}\right)^2=\left(2\sqrt{7-x}\right)^2$

=> $\left(\sqrt{40-9x}\right)^2+\left(\sqrt{-x}\right)^2-2\left(\sqrt{40-9x}\right)\left(\sqrt{-x}\right)=4(7-x)$

=> $40-9x+(-x)-2\sqrt{(40-9x)(-x)}=28-4x$

=> $40-9x-x-2\sqrt{-40x+9x^2}=28-4x$

=> $40-28-10x+4x=2\sqrt{9x^2-40x}$

=> $12-6x=2\sqrt{9x^2-40x}$

=> $2(6-3x)=2\sqrt{9x^2-40x}$ **both sides divided by 2 .**

=> $6-3x=\sqrt{9x^2-40x}$ **squaring both sides**

=> $(6-3x)^2=\left(\sqrt{9x^2-40x}\right)^2$

=> $36+9x^2-36x=9x^2-40x$

=> $9x^2-9x^2-36x+40x=-36$

=> $4x=-36$ **divided by 4**

=> $x=-\frac{36}{4}=-9$

Now $2x+5=2(-9)+5$

$=-18+5=-13$

Thus $2x+5=-13$ **Ans .**

Review Exercise 1:

At the end of each question , four circles are given . Fill in the correct circle only .

(i) If $(x+1)(x-5)=0$, then the solution are

(a) $x=1\ ,\ -5$ (b) $x=1\ ,\ 5$

(c) $x=-1\ ,\ -5$ (d) $x=-1\ ,\ 5$

(ii) If $x^2-x-1=0$, then $x=$

(a) $\frac{-1\pm\sqrt{5}}{2}$ (b) $-1\pm\frac{\sqrt{5}}{2}$

(c) $\frac{1\pm\sqrt{5}}{2}$ (d) $1\pm\frac{\sqrt{5}}{2}$

(iii) $\frac{-1\pm\sqrt{5}}{2}$ in simplified form is

(a) $1\pm\sqrt{24}$ (b) $1\pm\sqrt{6}$

(c) $2\pm\sqrt{6}$ (d) cannot be simplified

(iv) To apply quadratic formula to $2x^2-x=3$

(a) $a=2\ ,\ b=-1\ ,c=3$ (b) $a=2\ ,\ b=1\ ,c=3$

(c) $a=2\ ,\ b=-1\ ,c=-3$ (d) $a=2\ ,\ b=-1\ ,c=0$

(v) If $x^2-3x-4=0$, then the solution are

(a) $x=4\ ,\ -1$ (b) $x=-4\ ,\ 1$

(c) $x=4\ ,\ 1$ (d) $x=-4,\ -1$

(vi) If $2x^2+4x-9=0$, the solution are

(a) $x=\frac{2\pm\sqrt{22}}{2}$ (b) $x=\frac{-2\pm\sqrt{22}}{2}$

(c) $x=2\pm\frac{\sqrt{22}}{2}$ (d) $x=-2\pm\frac{\sqrt{22}}{2}$

(vii) $x^2-\frac{1}{4}=0$, the solution are

(a) $x=\pm\frac{1}{2}$ (b) $x=\pm\frac{1}{4}$

(c) $x=\pm\frac{1}{8}$ (d) $x=\pm\frac{1}{16}$

(viii) What are the solutions of the equation $x^2+7x-18=0$

(a) 2 or -9 (b) -2 or 9

(c) -2 or -9 (d) 2 or 9

(ix) Which of the following values of x are roots of the equation $x^2-8x+15=0$

(a) $x=1$ *or* $x=-7$ (b) $x=2$ *or* $x=4$

(c) $x=-2$, *or* $x=4$ (d) $x=3$ *or* $x=5$

Answers: (i) d (ii) c (iii) d (iv) c (v) a (vi) b (vii) a (viii) a (ix) d

Q.2: Solve $2w^4-5w^2+2=0$

Solution: $2w^4-5w^2+2=0$ ....(i)

Let $w^2=x$ ....(ii) Then $w^4=x^2$ ....(iii)

Put (ii), (iii) in (i).

$$2x^2-5x+2=0$$

$$\Rightarrow 2x^2-4x-x+2=0$$

$$\Rightarrow 2x(x-2)-1(x-2)=0$$

$$\Rightarrow (x-2)(2x-1)=0$$

$$\Rightarrow x-2=0 \quad or \quad 2x-1=0$$

$$\Rightarrow x=2 \quad or \quad 2x=1$$

$$\Rightarrow x=\frac{1}{2}$$

Put $x=2$ in (ii) => $w^2=2$ take square root

$$\Rightarrow \sqrt{w^2}=\pm\sqrt{2} \quad \Rightarrow w=\pm\sqrt{2}$$

Put $x=\frac{1}{2}$ in (ii) => $w^2=\frac{1}{2}$ take square root

$$\Rightarrow \sqrt{w^2}=\pm\sqrt{\frac{1}{2}} \quad \Rightarrow w=\pm\sqrt{\frac{1}{2}}=\pm\frac{1}{\sqrt{2}}$$

Solution set = $\left\{\pm\sqrt{2},\ \pm\frac{1}{\sqrt{2}}\right\}$

Q.3: **Find the constants** a **and b such that** $x=-1$ **and** $x=1$ **are both solutions to the equation** $ax^2+bx+2=0$

Solution : $ax^2+bx+2=0$ .....(i)

Put $x=-1$ in (i) => $a(-1)^2+b(-1)+2=0$

=> $a-b+2=0$ ....(iii)

Put $x=1$ in (i) => $a(1)^2+b(1)+2=0$

=> $a+b+2=0$ ....(iv)

Adding (iii) and (iv)

$$a-b+2=0$$
$$a+b+2=0$$
$$2a+4=0$$

=> $2a=-4$ => $a=-\frac{4}{2}$

=> $a=-2$ put $a=-2$ in (iv)

$-2+b+2=0$ => $b=0$

Thus $a=-2$, $b=0$ Ans.

Q.4: **Find all values of x such that** $x^2+5x+6=0$ **and** $x^2+19x+34=0$ **are equal.**

Solution: Since $x^2+5x+6=0$ and $x^2+19x+34=0$ are equal.

Then $x^2+5x+6=x^2+19x+34$

=> $x^2-x^2+5x-19x=34-6$

=> $-14x=28$ => $x=-\frac{28}{14}$

=> $x=-2$ Ans.

Q.5: Challenge : Find the solutions to the equation

$$49x^2-316x+132=0$$

Solution: $49x^2-316x+132=0$

=> $49x^2-294x-22x+132=0$

$=> 49x(x-6)-22(x-6)=0$

$=> (x-6)(49x-22)=0$

$=> x-6=0 \quad or \quad 49x-22=0$

$=> x=6 \quad or \quad 49x=22$

$=> x=\frac{22}{49}$

Thus solution set = $\left\{6, \frac{22}{49}\right\}$

Activity: **Juwaria tried to solve the quadratic equation $x^2+5x-2=0$ by completing the square, but she made mistake. In which line of her working shown below, did she make the mistake.**

Solution: $x^2+5x-2=0$

Line -1 : $=> x^2+5x=2$ **divided both sides by** $\left(\frac{5}{2}\right)^2$

Line – 2 : $=> x^2+5x+\left(\frac{5}{2}\right)^2=2+\left(\frac{5}{2}\right)^2$

Line -3 : $=> \left(x+\frac{5}{2}\right)^2=2\div\frac{25}{4}$

Line -4 : $=> \left(x+\frac{5}{2}\right)^2=\frac{8+25}{4}$

Line -5 : $=> \left(x+\frac{5}{2}\right)^2=\frac{33}{4}$

Line -6: $=> \sqrt{\left(x+\frac{5}{2}\right)^2}=\pm\sqrt{\frac{33}{4}}$

Line – 7 $=> x+\frac{5}{2}=\pm\sqrt{\frac{33}{4}}$

Line – 8 : => $x = -\frac{5}{2} \pm \frac{\sqrt{33}}{2}$

Line – 9 : => $x = \frac{-5+\sqrt{33}}{2}$ *or* $x = \frac{-5-\sqrt{33}}{2}$

Juwaria made mistake in line – 3 . She write $\left(\frac{5}{2}\right)^2 = \frac{10}{2}$

It is wrong .

Unit 2:

Theory of quadratic Equation:

Discriminant : As $ax^2+bx+c=0$ .....(*i*) *where* $a \neq 0$ is a quadratic equation. Then b^2-4ac is called discriminant of the quadratic equation.

Example : Find discriminant of quadratic equation

$$x^2+9x+2=0$$

Solution: $x^2+9x+2=0$

Here $a=1$, $b=9$, $c=2$.

Then discriminant

$$b^2-4ac=(9)^2-4(1)(2)$$
$$=81-8=73$$

Nature of the quadratic equation:

(i) if $b^2-4ac=0$ then the roots are real ,equal, and rational .

(ii) if $b^2-4ac>0$ then the roots are real , unequal . If b^2-4ac is a perfect square then the roots are rational .

If b^2-4ac is not a perfect square so the roots are irrational.

(iii)if $b^2-4ac<0$ then the roots are unequal and imaginary

Example: Examine the nature of the roots of the following equations.

(*i*) $x^2-8x+16=0$ (*ii*) $x^2+9x+2=0$

(*iii*) $6x^2-x-15=0$ (*iv*) $4x^2+x+1=0$

Solution: (i) $x^2-8x+16=0$

Here $a=1$, $b=-8$, $c=16$ Then discriminant

$$b^2-4ac=(-8)^2-4(1)(16)$$
$$=64-64=0$$

Then the roots are real , equal and rational .

(ii) $x^2+9x+2=0$

Here $a=1$, $b=9$, $c=2$. Then discriminant

$b^2 - 4ac = (9)^2 - 4(1)(2)$
$= 81 - 8 = 73$

=> $b^2 - 4ac = 73 > 0$

Then the roots are real , unequal and irrational .

(iii) $6x^2 - x - 15 = 0$

Here $a = 6$; $b = -1$, $c = -15$. Then disciminant

$b^2 - 4ac = (-1)^2 - 4(6)(-15)$
$= 1 + 360 = 361$

=> $b^2 - 4ac = (19)^2$

Since the discriminant is a perfect square . Then the roots are real , unequal and rational .

(iv) $4x^2 + x + 1 = 0$

Here $a = 4$, $b = 1$, $c = 1$. Then discriminant

$b^2 - 4ac = (1)^2 - 4(4)(1)$
$= 1 - 16 = -15$

=> $b^2 - 4ac = -15 < 0$

Since the discriminant is negative .Then the roots are unequal and imaginary .

Exercise 2.1:

Q.1:Find the discriminant of the following quadratic equations.

(i) $x^2 - 4x + 13 = 0$ (ii) $4x^2 - 5x + 1 = 0$

(iii) $x^2 + x + 1 = 0$

Solution: (i) $x^2 - 4x + 13 = 0$

Here $a = 1$, $b = -4$, $c = 13$. Then the discriminant

$b^2 - 4ac = (-4)^2 - 4(1)(13)$
$= 16 - 52$

=> $b^2 - 4ac = -36$

(ii) $4x^2 - 5x + 1 = 0$

Here $a = 4$, $b = -5$, $c = 1$. Then discriminant

$$b^2 - 4ac = (-5)^2 - 4(4)(1)$$
$$= 25 - 16$$
$$= 9$$

(iii) $x^2 + x + 1 = 0$

Here $a = 1$, $b = 1$, $c = 1$. Then discriminant

$$b^2 - 4ac = (1)^2 - 4(1)(1)$$
$$= 1 - 4$$
$$= -3$$

Q.2: Examine the nature of the roots of the following equations

(i) $3x^2 - 5x + 1 = 0$ **(ii)** $6x^2 + x - 2 = 0$

(iii) $3x^2 + 2x + 1 = 0$

Solution: (i) $3x^2 - 5x + 1 = 0$

Here $a = 3$, $b = -5$, $c = 1$. Then discriminant

$$b^2 - 4ac = (-5)^2 - 4(3)(1)$$
$$= 25 - 12 = 13$$

As discriminant is positive and not a perfect square .Then the roots are real , unequal and irrational .

(ii) $6x^2 + x - 2 = 0$

Here $a = 6$, $b = 1$, $c = -2$. Then the discriminant

$$b^2 - 4ac = (1)^2 - 4(6)(-2)$$
$$= 1 + 48 = 49$$

=> $b^2 - 4ac = (7)^2$

As discriminant is a perfect square .Then the roots are real , unequal and rational .

(iii) $3x^2 + 2x + 1 = 0$

Here $a = 3$, $b = 2$, $c = 1$. Then discriminant

$b^2 - 4ac = (2)^2 - 4(3)(1)$

$= 4 - 12 = -8$

As discriminant is negative then the roots are unequal and imaginary .

Q.3: For what value of k , the roots of the following equations are equal .

(i) $x^2 + kx + 9 = 0$ **(ii)** $12x^2 + kx + 3 = 0$

(iii) $x^2 - 5x + k = 0$

Solution: (i) $x^2 + kx + 9 = 0$

As roots are equal then $b^2 - 4ac = 0$ ......(*i*)

Put $a = 1$, $b = k$, $c = 9$ in (i)

$k^2 - 4(1)(9) = 0$ => $k^2 - 36 = 0$

=> $k^2 = 36$ => $\sqrt{k^2} = \pm\sqrt{36}$

=> $k = \pm 6$

(ii) $12x^2 + kx + 3 = 0$

As roots are equal then $b^2 - 4ac = 0$ ......(*i*)

Put $a = 12$, $b = k$, $c = 3$ in (i)

$k^2 - 4(12)(3) = 0$ => $k^2 - 144 = 0$

=> $k^2 = 144$ => $\sqrt{k^2} = \pm\sqrt{144}$

=> $k = \pm 12$

(iii) $x^2 - 5x + k = 0$

As roots are equal then $b^2 - 4ac = 0$ ......(*i*)

Put $a = 1$, $b = -5$, $c = k$ in (i)

$(-5)^2 - 4(1)(k) = 0$

=> $25 - 4k = 0$

=> $25 = 4k$

=> $k = \frac{25}{4}$

Q.4: Determine whether the following quadratic equations have real roots and if so find the roots.

(i) $x^2 + 5x + 5 = 0$

(ii) $4x^2 + 12x + 9 = 0$ (iii) $6x^2 + x - 2 = 0$

Solution:

(i) $x^2 + 5x + 5 = 0$

Here $a = 1$, $b = 5$, $c = 5$. Then discriminant is

$$b^2 - 4ac = (5)^2 - 4(1)(5)$$
$$= 25 - 20 = 5$$

Since discriminant is positive thus the roots are real , unequal And irrational .

Now we find the roots by quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad ....(i) \text{ put values in (i)}$$

$$=> x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(5)}}{2(1)} = \frac{-5 \pm \sqrt{25 - 20}}{2}$$

$$=> x = \frac{-5 \pm \sqrt{5}}{2}$$

Thus the real roots are $= \frac{-5 \pm \sqrt{5}}{2}$

(ii) $4x^2 + 12x + 9 = 0$

Here $a = 4$, $b = 12$, $c = 9$ .Then discriminant is

$$b^2 - 4ac = (12)^2 - 4(4)(9)$$
$$= 144 - 144 = 0$$

Since discriminant is equal to zero . Thus the roots are equal.

Now $4x^2 + 12x + 9 = 0$

$$=> 4x^2 + 6x + 6x + 9 = 0$$

$$=> 2x(2x + 3) + 3(2x + 3) = 0$$

=> $(2x+3)(2x+3)=0$

=> $2x+3=0$, $2x+3=0$

=> $2x=-3$, $2x=-3$

=> $x=-\frac{3}{2}$, $-\frac{3}{2}$

Thus the roots are $-\frac{3}{2}$, $-\frac{3}{2}$.

(iii) $6x^2+x-2=0$

Here $a=6$, $b=1$, $c=-2$ then discriminant

$b^2-4ac=(1)^2-4(6)(-2)=1+48$

=> $b^2-4ac=49=(7)^2$

Since discriminant is positive then the roots are real. Now

$6x^2+x-2=0$ => $6x^2-3x+4x-2=0$

=> $3x(2x-1)+2(2x-1)=0$

=> $(2x-1)(3x+2)=0$

=> $2x-1=0$ *or* $3x+2=0$

=> $2x=1$ *or* $3x=-2$

=> $x=\frac{1}{2}$ *or* $x=-\frac{2}{3}$

Thus the real roots are $=\frac{1}{2}$, $-\frac{2}{3}$

<u>Q.5</u>: Determine the nature of roots of the following quadratic equations and verify the results by solving them.

(i) $3x^2-10x+3=0$

(ii) $x^2-6x+4=0$ **(iii)** $x^2-3=0$

<u>Solution</u>: (i) $3x^2-10x+3=0$

Here $a=3$, $b=-10$, $c=3$ then discriminant is

$b^2 - 4ac = (-10)^2 - 4(3)(3) = 100 - 36$

$\Rightarrow b^2 - 4ac = 64 = (8)^2$

Since the discriminant is a perfect square then the roots are Real , unequal and rational . Now we find the roots.

$3x^2 - 10x + 3 = 0 \quad \Rightarrow \quad 3x^2 - 9x - x + 3 = 0$

$\Rightarrow 3x(x-3) - 1(x-3) = 0 \quad \Rightarrow \quad (x-3)(3x-1) = 0$

$\Rightarrow \quad x - 3 = 0 \quad or \quad 3x - 1 = 0$

$\Rightarrow \quad x = 3 \quad or \quad 3x = 1$

$\Rightarrow \quad x = \frac{1}{3}$

Thus the roots are = 3, $\frac{1}{3}$. Which are real , unequal and rational.

(ii) $x^2 - 6x + 4 = 0$

Here $a = 1$, $b = -6$, $c = 4$ then discriminant is

$b^2 - 4ac = (-6)^2 - 4(1)(4)$

$= 36 - 16 = 20$

As discriminant is positive and not a perfect square .Then the roots are real , unequal and irrational. Now we find the roots.

Then by quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .....(i)

Put values in (i)

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)} = \frac{6 \pm \sqrt{36-16}}{2}$$

$$\Rightarrow \quad x = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm \sqrt{4 \times 5}}{2}$$

$$\Rightarrow x = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$$

Thus roots are $3+\sqrt{5}$, $3-\sqrt{5}$ which are real,unequal and irrational.

(iii) $x^2-3=0$

Here $a=1$, $b=0$, $c=-3$ then discriminant is

$b^2-4ac=(0)^2-4(1)(-3)=12$

As discriminant is positive and not a perfect square .Then the roots are real , unequal and irrational. Now we find the roots.

$$x^2-3=0 \quad => \quad x^2=3$$

$$=> \sqrt{x^2}=\pm\sqrt{3} \quad => \quad x=\pm\sqrt{3}$$

Thus the roots are = $-\sqrt{3}$, $\sqrt{3}$ which are real , unequal and irrational.

Q.6: For what value of k the roots of the following equations are: (a) real (b) imaginary

(i) $2x^2+3x+k=0$ (ii) $kx^2+2x+1=0$

(iii) $x^2+5x+k=0$

Solution: (i) $2x^2+3x+k=0$

(a) Since roots are real then $b^2-4ac \geq 0$ ......(i)

Here $a=2$,$b=3$,$c=k$ put in(i)

$$(3)^2-4(2)(k) \geq 0 \quad => \quad 9-8k \geq 0$$

$$=> 9 \geq 8k \quad => \quad k \leq \frac{9}{8}$$

(b) Since the roots are imaginary then $b^2-4ac<0$ ....(ii)

Put values in (ii) .

$$(3)^2-4(2)(k)<0 \quad => \quad 9-8k<0$$

$$=> 9<8k \quad => \quad \frac{9}{8}<k$$

$$=> k>\frac{9}{8}$$

(ii) $kx^2 + 2x + 1 = 0$

(a) Since roots are real then $b^2 - 4ac \geq 0$ ......(i)

Here $a = k$, $b = 2$, $c = 1$ put (i)

$(2)^2 - 4(k)(1) \geq 0 \quad => \quad 4 - 4k \geq 0$

$=> \quad 4 \geq 4k \quad => \quad \frac{4}{4} \geq \frac{4k}{4}$

$=> 1 \geq k \quad => k \leq 1$

(b) Since the roots are imaginary then $b^2 - 4ac < 0$ ....(ii)

Put values in (ii) .

$(2)^2 - 4(k)(1) < 0 \quad => \quad 4 - 4k < 0$

$=> 4 < 4k \quad => \quad \frac{4}{4} < \frac{4k}{4}$

$=> 1 < k \quad => k > 1$

(iii) $x^2 + 5x + k = 0$

(a) Since roots are real then $b^2 - 4ac \geq 0$ ......(i)

Here $a = 1$, $b = 5$, $c = k$ in (i)

$(5)^2 - 4(1)(k) \geq 0 \quad => \quad 25 - 4k \geq 0$

$=> \quad 25 \geq 4k \quad => \frac{25}{4} \geq \frac{4}{4}k$

$=> \quad \frac{25}{4} \geq k \quad => \quad k \leq \frac{25}{4}$

(b) Since the roots are imaginary then $b^2 - 4ac < 0$ ....(ii)

Put values in (ii) .

$(5)^2 - 4(k)(1) < 0 \quad => \quad 25 - 4k < 0$

$=> 25 < 4k \quad => \quad \frac{25}{4} < \frac{4k}{4}$

$=> \frac{25}{4} < k \quad => \quad k > \frac{25}{4}$

Cube roots of unity and their property:

Cube roots of unity: Let x be a cube root of unity.

$$Then \quad x = \sqrt[3]{1} \quad => \quad x = (1)^{\frac{1}{3}}$$

$$=> x^3 = 1 \quad => \quad x^3 - 1 = 0$$

$$=> (x)^3 - (1)^3 = 0$$

$$=> (x-1)(x^2+x+1) = 0$$

$$=> x - 1 = 0 \quad or \quad x^2 + x + 1 = 0$$

$$=> x = 1 \quad or \quad x^2 + x + 1 = 0$$

Then by quadratic formula we solve $x^2 + x + 1 = 0$

$$\text{As } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad ....(i)$$

Put $a = 1$, $b = 1$, $c = 1$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$=> x = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

Thus cube roots of unity are 1 , $\frac{-1+i\sqrt{3}}{2}$, $\frac{-1-i\sqrt{3}}{2}$

$$let \quad w = \frac{-1+i\sqrt{3}}{2} \text{ ; } then \quad w^2 = \frac{-1-i\sqrt{3}}{2}$$

Thus cube roots of unity are 1, w , w^2.

Where $w = \frac{-1+i\sqrt{3}}{2}$, $w^2 = \frac{-1-i\sqrt{3}}{2}$.

Properties of cube roots of unity:

(i) The sum of cube roots of unity is zero i.e. $1 + w + w^2 = 0$

$$As \quad 1+w+w^2 = 1+\frac{-1+i\sqrt{3}}{2}+\frac{-1-i\sqrt{3}}{2}$$

$$= \frac{2-1+i\sqrt{3}-1-i\sqrt{3}}{2} = \frac{0}{2} = 0$$

$\Rightarrow \; 1+w+w^2 = 0$

(ii) The product of cube roots of unity is zero

i.e. $(1)(w)(w^2) = 1$ Or $w^3 = 1$

$$As \quad (1)(w)(w^2) = (1)\left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{-1-i\sqrt{3}}{2}\right)$$

$$= \frac{(-1)^2-(i\sqrt{3})^2}{4} = \frac{1-i^2 3}{4}$$

$$= \frac{1-(-1)(3)}{4} = \frac{1+3}{4} \qquad As \; i^2 = -1$$

$$\Rightarrow \; w^3 = \frac{4}{4} = 1$$

(iii) Each complex cube root of unity is reciprocal of the other. i.e.

$$w = \frac{1}{w^2} \qquad or \qquad w^2 = \frac{1}{w}$$

<u>Example</u> : Show that :

$$x^3 + y^3 = (x+y)(x+wy)(x+w^2y)$$

<u>Solution</u>: Take L . H . S.

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2) \ldots\ldots\ldots(i)$$

Take R.H.S.

$$(x+y)(x+wy)(x+w^2y) = (x+y)(x^2 + w^2xy + wxy + w^3y^2)$$

$$= (x+y)(x^2 + xy(w^2+w) + (1)y^2)$$

$$= (x+y)(x^2 + xy(-1) + y^2)$$

$$= (x+y)(x^2 - xy + y^2) \ldots\ldots\ldots(ii)$$

From (i) and (ii) $x^3 + y^3 = (x+y)(x+wy)(x+w^2y)$

Exercise 2.2 :

Q.1: Find the cube roots of the following numbers .

(i) -1 (ii) 8 (iii) -27

Solution: (i) - 1

Let x be a cube root of -1 .

$$Then \quad x=\sqrt[3]{-1} \quad => \quad x=(-1)^{\frac{1}{3}}$$

$$=> \quad x^3=-1 \quad => \quad x^3+1=0$$

$$=> (x)^3+(1)^3=0$$

$$=> (x+1)(x^2-x+1)=0$$

$$=> x+1=0 \quad or \quad x^2-x+1=0$$

$$=> x=-1 \quad or \quad x^2-x+1=0$$

Then by quadratic formula we solve $x^2-x+1=0$

As $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ ....(i)

Put $a=1$, $b=-1$, $c=1$ in (i)

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}=\frac{1\pm\sqrt{1-4}}{2}$$

$$=> x=\frac{1\pm\sqrt{-3}}{2}=\frac{1\pm i\sqrt{3}}{2}$$

$$=> x=\frac{1+i\sqrt{3}}{2} \quad , \quad \frac{1-i\sqrt{3}}{2}$$

$$=> x=-\left(\frac{-1-i\sqrt{3}}{2}\right) \quad , \quad -\left(\frac{-1+i\sqrt{3}}{2}\right)$$

$$=> x=-w^2 \quad , \quad x=-w$$

Thus cube roots of -1 are -1 , $-w$, $-w^2$

(ii) 8

Let x be the cube root of 8. Then

$x = \sqrt[3]{8} \quad => \quad x = (8)^{\frac{1}{3}}$

$=> \quad x^3 = 8 \quad => \quad x^3 - 8 = 0$

$=> x^3 - (2)^3 = 0 \quad => \quad (x-2)(x^2 + 2x + 4) = 0$

$=> \quad x - 2 = 0 \quad or \quad x^2 + 2x + 4 = 0$

$=> \quad x = 2$

Now we solve $x^2 + 2x + 4 = 0$ by quadratic formula

As $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ....(i)

Put $a = 1$, $b = 2$, $c = 4$ in (i)

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)} = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$=> x = \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm i\sqrt{4\times 3}}{2}$$

$$=> x = \frac{-2 \pm i2\sqrt{3}}{2} = 2\left(\frac{-1 \pm i\sqrt{3}}{2}\right)$$

$$=> x = 2\left(\frac{-1 + i\sqrt{3}}{2}\right) \quad , \quad x = 2\left(\frac{-1 - i\sqrt{3}}{2}\right)$$

$As \quad w = \dfrac{-1 + i\sqrt{3}}{2} \quad and \quad w^2 = \dfrac{-1 - i\sqrt{3}}{2}$

$Then \quad x = 2w \quad , \quad x = 2w^2$

Thus cube roots of 8 are 2 , $2w$, $2w^2$

(iii) - 27

Let x be the cube root of -27

Then $x = \sqrt[3]{-27}$ => $x = (-27)^{\frac{1}{3}}$

=> $x^3 = -27$ => $x^3 + 27 = 0$

=> $x^3 + (3)^3 = 0$ => $(x+3)(x^2 - 3x + 9) = 0$

=> $x + 3 = 0$ *or* $x^2 - 3x + 9 = 0$

=> $x = -3$.

Now we solve $x^2 - 3x + 9 = 0$ by quadratic formula .

As $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ....(i)

put $a = 1$, $b = -3$, $c = 9$ in (i)

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)} = \frac{3 \pm \sqrt{9 - 36}}{2}$$

=> $x = \dfrac{3 \pm \sqrt{-27}}{2} = \dfrac{3 \pm i\sqrt{9 \times 3}}{2}$

=> $x = \dfrac{3 \pm 3i\sqrt{3}}{2} = 3\left(\dfrac{1 \pm i\sqrt{3}}{2}\right)$

=> $x = 3\left(\dfrac{1 + i\sqrt{3}}{2}\right)$, $x = 3\left(\dfrac{1 - i\sqrt{3}}{2}\right)$

=> $x = -3\left(\dfrac{-1 - i\sqrt{3}}{2}\right)$, $x = -3\left(\dfrac{-1 + i\sqrt{3}}{2}\right)$

As $w = \dfrac{-1 + i\sqrt{3}}{2}$ *and* $w^2 = \dfrac{-1 - i\sqrt{3}}{2}$

=> $x = -3w$, $x = -3w^2$

Thus cube roots of 27 are -3 , $-3w$, $-3w^2$

Q.2: Evaluate:

(i) $w^{12} + w^{58} + w^{95}$

Solution:

$$w^{12} + w^{58} + w^{95} = (w^3)^4 + w^{57}.w + w^{93}.w^2$$
$$= (1)^4 + (w^3)^{19}.w + (w^3)^{31}.w^2 \quad As \quad w^3 = 1$$
$$= 1 + (1)^{19}.w + (1)^{31}.w^2$$
$$= 1 + w + w^2$$
$$= 0$$

(ii) $\left(1 + w - w^2\right)^7$

Solution: $\left(1 + w - w^2\right)^7 = \left(-w^2 - w^2\right)^7 \quad As \ 1 + w = -w^2$
$$= \left(-2w^2\right)^7$$
$$= (-2)^7 w^{14}$$
$$= -128.w^{12}.w^2 = -128(w^3)^4.w^2$$
$$= -128(1)^4.w^2$$
$$= -128w^2$$

(iii) $\left(1 + 3w - w^2\right)\left(1 + w - 2w^2\right)$

Solution:
$$\left(1 + 3w - w^2\right)\left(1 + w - 2w^2\right) = \left(1 + w + 2w - w^2\right)\left(1 + w - 2w^2\right)$$
$$= (-w^2 + 2w - w^2)(-w^2 - 2w^2) \quad As \ 1 + w = -w^2$$
$$= \left(2w - 2w^2\right)\left(-3w^2\right)$$
$$= -6w^3 + 6w^4$$
$$= -6(1) + 6w^3.w$$
$$= -6 + 6(1).w$$
$$= -6(1 - w)$$

Q.3: Prove that

(i) $(1 + 2w)(1 + 2w^2)(1 - w - w^2) = 6$

Solution: Take L . H . S.

$(1+2w)(1+2w^2)(1-w-w^2) = (1+2w)(1+2w^2)\left(1-(w+w^2)\right)$

$= (1+2w)(1+2w^2)(1-(-1))$

$= (1+2w)(1+2w^2)(1+1)$

$= 2(1+2w^2+2w+4w^3)$ $\quad$ $As\ \ w^3 = 1$

$= 2\left(1+2(w+w^2)+4(1)\right)$

$= 2\left(1+2(-1)+4\right)$ $\quad$ $As\ \ w+w^2 = -1$

$= 2\left(1-2+4\right) = 2(3)$

$= 6$

(ii) $\left(-1+\sqrt{3}\right)^4\left(-1-i\sqrt{3}\right)^5 = 512w^2$

Solution: Take L . H. S.

$As\ \ w = \dfrac{-1+i\sqrt{3}}{2}$ $\quad$ => $\ 2w = -1+i\sqrt{3}$

$w^2 = \dfrac{-1-i\sqrt{3}}{2}$ $\quad$ => $\ 2w^2 = -1-i\sqrt{3}$. Then

$\left(-1+i\sqrt{3}\right)^4\left(-1-i\sqrt{3}\right)^5 = (2w)^4(2w^2)^5$

$= 2^4.2^5(w^4.w^{10})$

$= 2^{4+5}.w^{14}$

$= 2^9\, w^2.w^{12}$

$= 512\, w^2(w^3)^4$

$= 512w^2(1)^4$

$= 512w^2$

Q.4: Show that :

(i) $x^3-y^3 = (x-y)(x-wy)(x-w^2y)$

Solution: Take L . H . S.

$x^3-y^3 = (x-y)(x^2+xy+y^2)..........(i)$

Take R.H.S.

$$(x-y)(x-wy)(x-w^2y)=(x-y)\left(x^2-w^2xy-wxy+w^3y^2\right)$$
$$=(x-y)\left(x^2-xy(w^2+w)+(1)y^2\right)$$
$$=(x-y)(x^2-xy(-1)+y^2)$$
$$=(x-y)(x^2+xy+y^2)........(ii)$$

From (i) and (ii) $x^3-y^3=(x-y)(x-wy)(x-w^2y)$

(ii) $(1+w)(1+w^2)(1+w^4)(1+w^8)=1$

Solution: $As \quad 1+w=-w^2 \;,\; 1+w^2=-w$

Take L . H . S.

$$(1+w)(1+w^2)(1+w^4)(1+w^8)=(-w^2)(-w)(1+w.w^3)(1+w^6.w^2)$$
$$=(w^3)(1+w.1)\left(1+(w^3)^2.w^2\right)$$
$$=(1)((1+w)((1+1.w^2)$$
$$=(1)(-w^2)(-w)$$
$$=w^3=1$$

Roots and co-efficients of a Quadratic equation.

As quadratic equation is $ax^2+bx+c=0 \quad , a\neq 0$

Let α, β *the roots of given quadratic equation.*

$$Then \quad \alpha=\frac{-b+\sqrt{b^2-4ac}}{2a} \quad , \quad \beta=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

The sum of the roots

$$\alpha+\beta=\frac{-b+\sqrt{b^2-4ac}}{2a}+\frac{-b-\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a}=-\frac{2b}{2a}$$

$$=> \alpha+\beta=-\frac{b}{a}$$

The product of the roots

$$\alpha\beta = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \left(\frac{(-b)^2-\left(\sqrt{b^2-4ac}\right)^2}{4a^2}\right) = \frac{b^2-(b^2-4ac)}{4a^2}$$

$$= \frac{b^2-b^2+4ac}{4a^2} = \frac{4ac}{4a^2}$$

$$\Rightarrow \quad \alpha\beta = \frac{c}{a}$$

Thus sum of the roots $= \alpha+\beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

Thus product of the roots $= \alpha\beta = \frac{c}{a} = \frac{\text{constant}}{\text{coefficient of } x^2}$

Example: With out solving the equation , find the sum and product of the roots of the equation .

(i) $2x^2-3x-4=0$ (ii) $3x^2+6x-2=0$

Solution: (i) $2x^2-3x-4=0$

Here $a=2$, $b=-3$, $c=-4$

As sum of the roots $= -\frac{b}{a} = -\frac{(-3)}{2} = \frac{3}{2}$

As product of the roots $= \frac{c}{a} = -\frac{4}{2} = -2$

(ii) $3x^2+6x-2=0$

Here $a=3$, $b=6$, $c=-2$

As sum of the roots $= -\frac{b}{a} = -\frac{6}{3} = -2$

As product of the roots = $\frac{c}{a} = -\frac{2}{3}$

Exercise 2.3:

Q.1: Without solving the equation, find the sum and products of the following quadratic equations.

(*i*) $4x^2 - 4x - 3 = 0$

Solution: $4x^2 - 4x - 3 = 0$

Here $a = 4$, $b = -4$, $c = -3$

Sum of the roots $= -\frac{b}{a} = -\frac{(-4)}{4} = 1$

Product of the roots $= \frac{c}{a} = -\frac{3}{4}$

(ii) $2x^2 + 5x + 6 = 0$

Solution: Here $a = 2$, $b = 5$, $c = 6$

Sum of the roots $= -\frac{b}{a} = -\frac{5}{2}$

Product of the roots $= \frac{c}{a} = \frac{6}{2} = 3$

(iii) $3x^2 + 2x - 5 = 0$

Solution: Here $a = 3$, $b = 2$, $c = -5$

Sum of the roots $= -\frac{b}{a} = -\frac{2}{3}$

Product of the roots $= \frac{c}{a} = -\frac{5}{2}$

Q.2: Find the value of "k" if the sum of the roots of $2x^2 + kx + 6 = 0$ is equal to the product of its roots.

Solution: $2x^2 + kx + 6 = 0$ ....(i)

Here $a = 2$, $b = k$, $c = 6$

Sum of the roots $= -\frac{b}{a} = -\frac{k}{2}$ ......(*ii*)

Product of the roots $= \frac{c}{a} = \frac{6}{2} = 3$ ....(iii)

Since sum of the roots = Product of the roots . Then

$-\frac{k}{2} = 3 \quad \Rightarrow \quad -k = 6$

$\Rightarrow \ k = -6$

Q.3: Find the value of k if the sum of the square of the Roots of $x^2 - 5kx + 6k^2 = 0$ is equal to 13.

Solution: $x^2 - 5kx + 6k^2 = 0$ ......(i)

Note: $a^2 + b^2 = (a+b)^2 - 2ab$

Here $a = 1$, $b = -5k$, $c = 6k^2$

Let α , β *be the roots of* (*i*)

Sum of the roots $= \alpha + \beta = -\frac{b}{a} = -\frac{(-5k)}{1} = 5k$

Product of the roots $= \alpha\beta = \frac{c}{a} = \frac{6k^2}{1} = 6k^2$

According to the given condition

$\alpha^2 + \beta^2 = 13 \quad \Rightarrow \quad (\alpha+\beta)^2 - 2\alpha\beta = 13$ *put values*

$\Rightarrow (5k)^2 - 2(6k^2) = 13$

$\Rightarrow 25k^2 - 12k^2 = 13 \quad \Rightarrow 13k^2 = 13$

$\Rightarrow \ k^2 = \frac{13}{13} \quad \Rightarrow \ k^2 = 1$

$\Rightarrow \ k = \pm 1$

Q.4: Find the value of k if the roots of $x^2 - 5x + k = 0$ differ by unity.

Solution: $x^2 - 5x + k = 0$ .....(i)

Let α be the one root of (i) then the second root is $\alpha + 1$.

Here $a = 1$, $b = -5$, $c = k$. Then

Sum of the roots $= \alpha + \alpha + 1 = -\frac{b}{a}$

$$=> \ 2\alpha + 1 = -\frac{(-5)}{1}$$

$$=> \ 2\alpha = 5 - 1 \qquad => \ 2\alpha = 4$$

$$=> \ \alpha = \frac{4}{2} = 2$$

product of the roots $= \alpha(\alpha + 1) = \frac{c}{a}$

$$=> \ \alpha^2 + \alpha = \frac{k}{1} \qquad => \ \alpha^2 + \alpha = k .....(ii)$$

put $\alpha = 2$ *in* (ii)

$$(2)^2 + 2 = k \qquad => \ k = 4 + 2$$

$$=> \ k = 6$$

Q.5: Find the value of k if the roots of $x^2 - 9x + k + 2 = 0$ is differ by "3".

Solution: $x^2 - 9x + k + 2 = 0 ......(i)$

Let α be the one root of (i) then the second root is $\alpha + 3$.

Here $a = 1$, $b = -9$, $c = k + 2$

Sum of the roots $= \alpha + \alpha + 3 = -\frac{b}{a}$

$$=> \ 2\alpha + 3 = -\frac{(-9)}{1} \qquad => \ 2\alpha = 9 - 3$$

$$=> \ 2\alpha = 6 \qquad => \ \alpha = \frac{6}{2}$$

$$=> \ \alpha = 3$$

Product of the roots $= \alpha(\alpha + 3) = \frac{c}{a}$

=> $\alpha^2 + 3\alpha = \frac{k+2}{1}$

=> $\alpha^2 + 3\alpha = k + 2$ ......(ii) $put\ \alpha = 3\ \ in(ii)$

=> $(3)^2 + 3(3) = k + 2$ => $9 + 9 = k + 2$

=> $k + 2 = 18$ => $k = 18 - 2$

=> $k = 16$

Q.6: If α, β are the roots of $x^2 - 5x + k = 0$. Find k such that $3\alpha + 2\beta = 12$

Solution: $x^2 - 5x + k = 0$ ...(i)

$3\alpha + 2\beta = 12$ ......(ii) and α, β are the roots of (i).

Here $a = 1$, $b = -5$, $c = k$

Sum of the roots $= \alpha + \beta = -\frac{b}{a}$

=> $\alpha + \beta = -\frac{(-5)}{1}$ => $\alpha + \beta = 5$ ......(iii)

Product of the roots $= \alpha\beta = \frac{c}{a}$

=> $\alpha\beta = \frac{k}{1}$ => $\alpha\beta = k$ ....(iv)

Eq(iii) multiplied by 2 => $2\alpha + 2\beta = 10$.....(v)

Eq(v) is subtracted from eq(ii).

$$\begin{array}{r} 3\alpha + 2\beta = \ 12 \\ \underline{\pm 2\alpha \pm \beta = \pm 10} \\ \alpha = 2 \end{array}$$

Now $put\ \ \alpha = 2\ \ in(iii)$

$2 + \beta = 5$ => $\beta = 5 - 2$

=> $\beta = 3$

Now put $\alpha = 2$, $\beta = 3$ $in(iv)$

$(2)(3) = k$ => $k = 6$

Q.7: Find the value of m and n if both sum and product

roots of the quadratic equation $mx^2 - 3x - n = 0$ are equal to $\frac{3}{5}$.

Solution: $mx^2 - 3x - n = 0$ .....(i)

Here $a = m$, $b = -3$, $c = -n$

Sum of the roots $= -\frac{b}{a} = -\frac{(-3)}{m} = \frac{3}{m}$ ......(ii)

Product of the roots $= \frac{c}{a} = -\frac{n}{m}$ ......(iii)

According to the given condition

$\frac{3}{m} = \frac{3}{5} \quad \Rightarrow 15 = 3m$

$\Rightarrow \quad m = \frac{15}{3} = 5$

Now $-\frac{n}{m} = \frac{3}{5} \quad \Rightarrow -n = \frac{3m}{5} \quad$ *put* $m = 5$

$\Rightarrow \; n = -\frac{3\times 5}{5} = -3$

Thus m = 5 and n = -3

Symmetric functions of roots of a quadratic equation:

Let α , β be the roots of a quadratic equation. Then the expression $\alpha + \beta$, $\alpha\beta$ $\alpha^2 + \beta^2$ are called the functions of the roots of the quadratic equation. By symmetric function of the roots of an equation, we mean that the function remains unchanged in values when the roots are interchanged. For example $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are symmetric functions of α , β .

Example: If α , β are the roots of $ax^2 + bx + c = 0$ find the values of the symmetric functions of the roots of a given quadratic equation in terms of its coefficient.

(i) $\alpha+\beta$ (ii) $\alpha\beta$ (iii) $\alpha^2+\beta^2$ (iv) $\alpha^3+\beta^3$

(v) $\frac{1}{\alpha}+\frac{1}{\beta}$ (vi) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

Solution: $ax^2+bx+c=0$ .....(i)

As α , β are the roots of (i).

(i) $\alpha+\beta$

As sum of the roots $=\alpha+\beta=-\frac{b}{a}$ .....(i)

(ii) $\alpha\beta$:

As product of the roots $=\alpha\beta=\frac{c}{a}$ ......(ii)

(iii) $\alpha^2+\beta^2$:

$As \quad \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ .....(iii)

Put (i) , (ii) in (iii)

$$\alpha^2+\beta^2=\left(-\frac{b}{a}\right)^2-2\left(\frac{c}{a}\right)$$

$$=\frac{b^2}{a^2}-\frac{2c}{a}$$

$$=\frac{b^2-2ac}{a^2}$$

(iv) $\alpha^3+\beta^3$

$As \quad \alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ ....(iv)

Put (i) and (ii) in (iv)

$$\alpha^3+\beta^3=\left(-\frac{b}{a}\right)^3-3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)$$

$$=\frac{b^3}{a^3}+\frac{3bc}{a^2}$$

$$= \frac{b^3 + 3abc}{a^3}$$

(v) $\frac{1}{\alpha} + \frac{1}{\beta}$:

$$As \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{\alpha + \beta}{\alpha\beta} \text{......}(v)$$

Put (i) and (ii) in (v) .

$$\Rightarrow \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{-\frac{b}{a}}{\frac{c}{a}} = -\frac{b}{a} \times \frac{a}{c}$$

$$= -\frac{b}{c}$$

(vi) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$:

$$As \quad \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \text{......}(vi)$$

Put (i) , (ii) in (vi)

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)}{\left(\frac{c}{a}\right)^2} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$$

$$= \frac{b^2 - 2ac}{a^2} \times \frac{a^2}{c^2}$$

$$= \frac{b^2 - 2ac}{c^2}$$

Formation of quadratic equation whose roots are given:

Let α , β are the roots of quadratic equation.

Then sum of the roots $S = \alpha + \beta = -\frac{b}{a}$

And product of the roots $P = \alpha\beta = \frac{c}{a}$

As quadratic equation is $ax^2 + bx + c = 0$

$$=> \quad x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$=> \quad x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$$

Put $S = -\frac{b}{a}$ *and* $P = \frac{c}{a}$

Then required quadratic equation is

$$x^2 - Sx + P = 0$$

Example: Form a quadratic equation whose roots are $1+\sqrt{5}$ and $1-\sqrt{5}$

Solution: Given $1+\sqrt{5}$ and $1-\sqrt{5}$ are the roots.

Then $S = 1+\sqrt{5} + 1-\sqrt{5} = 2$

And $P = (1+\sqrt{5})(1-\sqrt{5}) = (1)^2 - (\sqrt{5})^2$

$= 1 - 5 = -4$

AS required quadratic equation is

$x^2 - Sx + P = 0$ Put values

$=> \quad x^2 - 2x - 4 = 0$

EXERCISE: 2.4:

Q.1: **If** α , β are the roots of $ax^2 + bx + c = 0$, **find the Values of** (i) $\alpha^3\beta + \alpha\beta^3$ (ii) $(\alpha - \beta)^2$

Solution:

As α , β are the roots of $ax^2 + bx + c = 0$

Sum of the roots $= \alpha + \beta = -\dfrac{b}{a}$

Product of the roots $= \alpha\beta = \dfrac{c}{a}$ ......(ii)

(i) $\alpha^3\beta + \alpha\beta^3$:

$$\alpha^3\beta + \alpha\beta^3 = \alpha\beta\left(\alpha^2 + \beta^2\right)$$

$$= \alpha\beta\left((\alpha + \beta)^2 - 2\alpha\beta\right)........(iii)$$

Put (i) and (ii) in (iii).

$$\alpha^3\beta + \alpha\beta^3 = \left(\frac{c}{a}\right)\left((-\frac{b}{a})^2 - 2(\frac{c}{a})\right)$$

$$= \frac{c}{a}\left(\frac{b^2}{a^2} - \frac{2c}{a}\right) = \frac{c}{a}\left(\frac{b^2 - 2ac}{a^2}\right)$$

$$= \frac{c\left(b^2 - 2ac\right)}{a^3}$$

(ii) $(\alpha - \beta)^2$

As $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ ....(iv)

Put (i) and (ii) in (iv)

$$(\alpha - \beta)^2 = \left(-\frac{b}{a}\right)^2 - 4\left(\frac{c}{a}\right)$$

$$= \frac{b^2}{a^2} - \frac{4c}{a}$$

$$= \frac{b^2 - 4ac}{a^2}$$

Q.2: Find the quadratic equation whose roots are

(i) 1 , $\dfrac{1}{2}$ (ii) -3 , 4 (iii) $3 + \sqrt{2}$, $3 - \sqrt{2}$

(iv) $a - 2a$

Solution: (i) $1, \frac{1}{2}$

Sum of the roots $S = 1 + \frac{1}{2} = \frac{2+1}{2} = \frac{3}{2}$

Product of the roots $P = (1)(\frac{1}{2}) = \frac{1}{2}$

Now required quadratic equation is

$x^2 - Sx + P = 0$ ....(*i*) put values in (i)

=> $x^2 - \frac{3}{2}x + \frac{1}{2} = 0$ *multiplied by* 2

=> $2x^2 - 3x + 1 = 0$

(ii) - 3 , 4

Sum of the roots = S = - 3 + 4 = 1

Product of the roots = P = (-3)(4) = -12

Now required quadratic equation is

$x^2 - Sx + P = 0$ ....(*i*) put values in (i)

$x^2 - 1.x - 12 = 0$

=> $x^2 - x - 12 = 0$

(iii) $3 + \sqrt{2}$, $3 - \sqrt{2}$

Sum of the roots $= S = 3 + \sqrt{2} + 3 - \sqrt{2} = 6$

Product of the roots $= P = (3 + \sqrt{2})(3 - \sqrt{2})$

=> $P = (3)^2 - (\sqrt{2})^2$

$= 9 - 2 = 7$

Now required quadratic equation is

$x^2 - Sx + P = 0$ ....(*i*) put values in (i)

=> $x^2 - 6x + 7 = 0$

(iv) a , $-2a$

Sum of the roots $= S = a - 2a = -a$

Product of the roots $= P = (a)(-2a) = -2a^2$

Now required quadratic equation is

$x^2 - Sx + P = 0$ ....(i) put values in (i)

$\Rightarrow x^2 - (-a)x - 2a^2 = 0$

$\Rightarrow x^2 + ax - 2a^2 = 0$

Q.3: Form a quadratic equation whose roots are square of the roots of the equation $ax^2 + bx + c = 0$, $a \neq 0$.

Solution: Let α, β be the roots of $ax^2 + bx + c = 0$.....(i).

Then sum of the roots of eq(i) is

$$\alpha + \beta = -\frac{b}{a} \quad ....(ii)$$

Product of the roots of eq(i) is

$$\alpha\beta = \frac{c}{a} \quad ....(iii)$$

Now according to the given condition then α^2 *and* β^2 are the roots of required quadratic equation.

Sum of the roots of required equation is

$$S = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta ....(iv)$$

Put (ii) and (iii) in (iv)

$$S = \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$$

$$\Rightarrow S = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2} \quad ....(v)$$

Product of the roots of the required equation is

$$P = \alpha^2\beta^2 = (\alpha\beta)^2 ....(vi) \quad put\ (iii)\ in\ (vi)$$

$$\Rightarrow P = \left(\frac{c}{a}\right)^2 = \frac{c^2}{a^2} \quad ......(vii)$$

Now required quadratic equation is

$x^2 - Sx + P = 0$ ....(viii)

put values in (v) , (vii) in (viii)

$$=> x^2 - \left(\frac{b^2 - 2ac}{a^2}\right)x + \frac{c^2}{a^2} = 0$$

$$=> a^2x^2 - (b^2 - 2ac)x + c^2 = 0$$

Q.4: If α , β are the roots of $2x^2 + 3x + 1 = 0$.

Then find the Values of (i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ **(iii)** $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Solution: $2x^2 + 3x + 1 = 0$ ......(i)

As α , β are the roots of (i)

As $a = 2$, $b = 3$, $c = 1$ .Then

Sum of the roots of given equation is $\alpha + \beta = -\frac{b}{a}$

$$=> \alpha + \beta = -\frac{3}{2} \quad ....(ii)$$

Product of the roots of given equation is $\alpha\beta = \frac{c}{a}$

$$=> \alpha\beta = \frac{1}{2} \quad ......(iii)$$

(i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$:

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} \quad .....(iv)$$

$$=> \quad \frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\left(-\frac{3}{2}\right)^2-2\left(\frac{1}{2}\right)}{\frac{1}{2}}=\frac{\frac{9}{4}-1}{\frac{1}{2}}$$

$$=\frac{\frac{9-4}{4}}{\frac{1}{2}}=\frac{5}{4}\times\frac{2}{1}=\frac{5}{2}$$

(ii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$:

$$As \ \frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\beta^2+\alpha^2}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2} \quad .....(v)$$

Put (ii) and (iii) in (v) .

$$=> \quad \frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\left(-\frac{3}{2}\right)^2-2\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^2}=\frac{\frac{9}{4}-1}{\frac{1}{4}}$$

$$=\frac{\frac{9-4}{4}}{\frac{1}{4}}=\frac{5}{4}\times\frac{4}{1}=5$$

(iii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta} \quad ....(vi)$$

Put (ii) and (iii) in (vi).

$$\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\left(-\frac{3}{2}\right)^3-3\left(\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{\frac{1}{2}}=\frac{-\frac{27}{8}+\frac{9}{4}}{\frac{1}{2}}$$

$$=\frac{\frac{-27+18}{8}}{\frac{1}{2}}=-\frac{9}{8}\times\frac{2}{1}$$

$$=-\frac{9}{4}$$

Q.5: If α , β are the roots of $3x^2-2x+5=0$, find the equation whose roots are $\frac{\alpha}{\beta}$, $\frac{\beta}{\alpha}$

Solution: $3x^2-2x+5=0$ ....(i)

Here $a=3$, $b=-2$, $c=5$

As α , β are the roots of (i).

Sum of the roots of given equation is

$$\alpha+\beta=-\frac{b}{a}=-\frac{(-2)}{3}$$

$$=> \alpha+\beta=\frac{2}{3} \quad ....(ii)$$

Product of the roots of given equation is

$$\alpha\beta=\frac{c}{a}=\frac{5}{3} \quad ....(iii)$$

Since $\frac{\alpha}{\beta}$, $\frac{\beta}{\alpha}$ are the roots of required equation.

$$S=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$=> S=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} \quad ......(iv)$$

Put (ii) and (iii) in (iv) .

$$S=\frac{\left(\frac{2}{3}\right)^2-2\left(\frac{5}{3}\right)}{\frac{5}{3}}=\frac{\frac{4}{9}-\frac{10}{3}}{\frac{5}{3}}$$

$$\Rightarrow \quad S=\frac{\frac{4-30}{9}}{\frac{5}{3}}=-\frac{26}{9}\times\frac{3}{5}$$

$$\Rightarrow \quad S=-\frac{26}{15}$$

Now product of the roots is

$$P=\left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)=1$$

Required quadratic equation is

$x^2-Sx+P=0$ ......(v) Put values of S and P .

$$\Rightarrow \quad x^2-\left(-\frac{26}{15}\right)x+1=0$$

$$\Rightarrow \quad 15x^2+26x+15=0$$

<u>Q.6</u>: If α , β are the roots of $x^2-4x+2=0$, find the equation whose roots are $\alpha+\frac{1}{\alpha}$, $\beta+\frac{1}{\beta}$.

<u>Solution</u>: $x^2-4x+2=0$ .....(i)

Here $a=1$, $b=-4$, $c=2$

As α , β are the roots of (i) .

Sum of the roots of eq(i) is

$$\alpha+\beta=-\frac{b}{a}=-\frac{(-4)}{1}$$

$$\Rightarrow \quad \alpha+\beta=4 \text{ ....}(ii)$$

Now product of the roots of eq(i) is

$$\alpha\beta = \frac{c}{a} = \frac{2}{1} = 2 \;......(iii)$$

As $\alpha + \frac{1}{\alpha}$, $\beta + \frac{1}{\beta}$ are the roots of required quadratic equation . Then

$$S = \alpha + \frac{1}{\alpha} + \beta + \frac{1}{\beta} = \alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta}$$

$$\Rightarrow S = \alpha + \beta + \frac{\beta + \alpha}{\alpha\beta} \;....(iv)$$

Put (ii) and (iii) in (iv) .

$$S = 4 + \frac{4}{2} = 4 + 2 = 6$$

$$P = \left(\alpha + \frac{1}{\alpha}\right)\left(\beta + \frac{1}{\beta}\right) = \alpha\beta + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{1}{\alpha\beta}$$

$$= \alpha\beta + \frac{\alpha^2 + \beta^2}{\alpha\beta} + \frac{1}{\alpha\beta}$$

$$\Rightarrow \quad P = \alpha\beta + \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + \frac{1}{\alpha\beta} \;.....(v)$$

Put (ii) and (iii) in (v)

$$P = 2 + \frac{(4)^2 - 2(2)}{2} + \frac{1}{2}$$

$$\Rightarrow \quad P = 2 + \frac{16 - 4}{2} + \frac{1}{2} = \frac{4 + 12 + 1}{2}$$

$$\Rightarrow \quad P = \frac{17}{2}$$

Now required quadratic equation is

$$x^2 - Sx + P = 0 \;.....(vi) \quad \text{put values}$$

$$\Rightarrow \quad x^2 - 6x + \frac{17}{2} = 0$$

$$\Rightarrow \quad 2x^2 - 12x + 17 = 0$$

Synthetic division: The process of synthetic division can be explained by the following example.

Example: $2x^3 - x^2 + 3$ divided by $x - 2$.

Solution:

2	2	−1	0	3
		4	6	12
	2	3	6	15

Here quotient is $Q(x) = 2x^2 + 3x + 6$ and remainder = 15

The above process is called synthetic division. The process can be described as below.

(i) To divide by x -2 , use 2 as the synthetic divisor .

(ii) write down the coefficients of the dividend polynomial in row from left to right in descending order of powers of x . write a zero for each missing term , including a missing constant.

(iii) Bring down the first coefficient.

(iv) First number multiplying by the divisor 2 , then adding with the second number of second row . Then multiplying this number by the divisor 2 and then adding with third term and continuous this process up to end .

(v) The last number in the third row is called remainder and the others are the coefficients of the quotient.

Example: Use synthetic division to find the quotient Q(x) and the remainder R when the polynomial $3x^3 - 2x^2 - 150$ is divided by x − 4 .

Solution: Let $P(x) = 3x^3 - 2x^2 - 150$

$\Rightarrow$ $P(x) = 3x^3 - 2x^2 + 0.x - 150$

4	3	−2	0	−150
		12	40	160
	3	10	40	10

Thus quotient $Q(x) = 3x^2 + 10x + 40$

And Remainder = 10 .

Exercise 2.5:

Q.1: Use synthetic division to find the quotient Q(x) and the remainder R when the first polynomial is divided by the second polynomial.

(i) $3x^3 + 2x^2 - x - 1$; $x + 3$

Solution: $P(x) = 3x^3 + 2x^2 - x - 1$

As P(x) is divided by x + 4 then

$x - a = x + 3$ => $x - a = x - (-3)$

=> $a = -3$

Now					
	−3	3	2	−1	−1
			−9	21	−60
		3	−7	20	−61

Thus $Q(x) = 3x^2 - 7x + 20$ and *remainder* $R = -61$

(ii) $2x^3 - 7x^2 + 12x - 27$; $x - 3$

Solution: *Let* $P(x) = 2x^3 - 7x^2 + 12x - 27$

As P(x) is divided by x − 3 then

$x - a = x - 3$ => $a = 3$

3	2	−7	12	−27
		6	−3	27
	2	−1	9	0

Thus $Q(x) = 2x^2 - x + 9$ and remainder $R = 0$

(iii) $2x^4 - 3x^2 + 5x - 7$; $x + 2$

Solution: let $P(x) = 2x^4 + 0.x^3 - 3x^2 + 5x - 7$

As x + 2 is divisor of P(x) then

$x-a=x+2 \quad => x-a=x-(-2)$

$=> \quad a=-2$

-2	2	0	-3	5	-7
		-4	8	-10	10
	2	-4	5	-5	3

Thus $Q(x)=2x^3-4x^2+5x-5 \quad and \quad R=3$.

Q.2: Use synthetic division to find the values of k if - 2 is a Zero of the polynomial x^3+4x^2+kx+8.

Solution: $P(x)=x^3+4x^2+kx+8$

As $P(-2)=0$ *.Then* $x+2$ *is one root of* $P(x)$.

Now by synthetic division

-2	1	4	k	8
		-2	-4	$-2k+8$
	1	2	$k-4$	$-2k+16$

Since P(-2) = 0 . Then remainder is equal to zero .

$-2k+16=0 \quad => -2k=-16$

$=> \quad k=\frac{-16}{-2} \quad => \quad k=8$

Q.3:Use synthetic division to find the values of p and q if x + 1 and x – 2 are the factors of x^3+px^2+qx+6

Solution: Let $P(x)=x^3+px^2+qx+6$

As $x+1$ *and* $x-2$ *are the roots of* $P(x)$.

Then -1 and 2 are the divisors of P(x).

$$\begin{array}{c|cccc}
-1 & 1 & p & q & 6 \\
 & & -1 & -p+1 & -q+p-1 \\
\hline
 & 1 & p-1 & q-p+1 & -q+p+5 \text{ remainder} \\
2 & & & & \\
 & & 2 & 2p+2 & \\
\hline
 & 1 & p+1 & p+q+3 \text{ remainder} &
\end{array}$$

Since x + 1 and x -2 are the roots of P(x) then remainders
Are equal to zeros . Then

$-q+p+5=0$ ....(i)

$p+q+3=0$ ......(ii)

Now adding (i) and (ii) .

$$\begin{aligned}
-q+p+5&=0 \\
q+p+3&=0 \\
\hline
2p+8&=0
\end{aligned}$$

=> $2p=-8$ => $p=-\frac{8}{2}$

=> $p=-4$

Put p = -4 in (ii)

$-4+q+3=0$ => $q-1=0$

=> $q=1$

Thus p = -4 and q = 1 .

Q.4: If x + 1 and x – 2 are factors of the polynomial x^3+ax^2+bx+2, then using synthetic division , find the Values of a and b.

Solution: $P(x)=x^3+ax^2+bx+2$

As x + 1 and x - 2 are the factors of P(x).
Then x = -1 and x = 2 are the roots of P(x) . Then

$$\begin{array}{r|cccc} -1 & 1 & a & b & 2 \\ & & -1 & -a+1 & a-b-1 \\ \hline & 1 & a-1 & -a+b+1 & a-b+1 \\ 2 & & 2 & 2a+2 & \\ \hline & 1 & a+1 & a+b+3 & \end{array}$$

As $x+1$ and $x-2$ are the factors of $P(x)$.
Then remainders are equal to zero.

$a-b+1=0$ ....(i)

$a+b+3=0$ ....(ii)

Adding (i) and (ii).

$$\begin{array}{r} a-b+1=0 \\ a+b+3=0 \\ \hline 2a+4=0 \end{array}$$

$\Rightarrow 2a+4=0 \quad \Rightarrow 2a=-4$

$\Rightarrow a=-\dfrac{4}{2} \quad \Rightarrow a=-2$

Put $a=-2$ in (ii)

$-2+b+3=0 \quad \Rightarrow b+1=0$

$\Rightarrow b=-1$

Thus $a=-2$ *and* $b=-1$.

<u>Q.5</u>: One root of the cubic equation $x^3-7x-6=0$ is 3. Use Synthetic division to find the other roots.

<u>Solution</u>: $P(x)=x^3+0.x^2-7x-6=0$

As 3 is the root of P(x).

$$\begin{array}{r|cccc} 3 & 1 & 0 & -7 & -6 \\ & & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

Other factors is $Q(x)=x^2+3x+2$

Now other roots are the roots of the equation $x^2+3x+2=0$.

$x^2+3x+2=0 \quad => \quad x^2+x+2x+2=0$

$=> \quad x(x+1)+2(x+1)=0$

$=> \quad (x+1)(x+2)=0$

$=> \quad x+1=0 \quad or \quad x+2=0$

$=> \quad x=-1 \quad or \quad x=-2$

Thus other roots are -1 and -2 .

Q.6: If -1 and 2 are roots of the quadratic equation $x^4-5x^3+3x^2+7x-2=0$. Use synthetic division to find other .

Solution: $P(x)=x^4-5x^3+3x^2+7x-2$

As -1 and 2 are the roots of P(x).

-1	1	-5	3	7	-2
		-1	6	-9	2
	1	-6	9	-2	0
2					
		2	-8	2	
	1	-4	1	0	

Now other factor is $Q(x)=x^2-4x+1$.

Now other roots are the roots of the equation $x^2-4x+1=0$.

Now by quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ .....(i)

Put $a=1$, $b=-4$, $c=1$ in (i)

$=> \quad x=\dfrac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}=\dfrac{4\pm\sqrt{16-4}}{2}$

$=> \quad x=\dfrac{4\pm\sqrt{12}}{2}=\dfrac{4\pm\sqrt{4\times 3}}{2}$

=> $x = \frac{4 \pm 2\sqrt{3}}{2} = \frac{2(2 \pm \sqrt{3})}{2}$

=> $x = 2 \pm \sqrt{3}$

Thus other roots are $2 \pm \sqrt{3}$.

Simultaneous Equations: More than one equation which are satisfied by the same values of the variables involved are called simultaneous equations or system of equations.

Case 1: When one equation is linear and other is quadratic.

Example: Solve the system

$2x + y = 10$

$4x^2 + y^2 = 68$

Solution: $2x + y = 10$ ......(i)

$4x^2 + y^2 = 68$ ......(ii)

From (i) => $y = 10 - 2x$ ....(iii)

Put (iii) in (ii).

$4x^2 + (10 - 2x)^2 = 68$

=> $4x^2 + 100 + 4x^2 - 40x - 68 = 0$

=> $8x^2 - 40x + 32 = 0$

Divided by 8 both sides

=> $x^2 - 5x + 4 = 0$

=> $x^2 - x - 4x + 4 = 0$

=> $x(x-1) - 4(x-1) = 0$

=> $(x-1)(x-4) = 0$

=> $x - 1 = 0$ or $x - 4 = 0$

=> $x = 1$ or $x = 4$

Put $x = 1$ in (iii)

$y = 10 - 2(1) = 10 - 2$

=> $y = 8$

Put $x = 4$ in (iii).

$y = 10 - 2(4) = 10 - 8$

=> $y = 2$

Thus solution set = $\{(1\ ,8)\ ,\ (4\ ,\ 2)\}$.

<u>System 2</u>: **When both the equations are quadratic:**

<u>Example</u>: **Solve the system**

$$x^2 + y^2 = 4$$

$$2x^2 - y^2 = 8$$

Solution: $x^2 + y^2 = 4 \ldots\ldots.(i)$

$2x^2 - y^2 = 8 \ \ldots\ldots\ldots(ii)$

Adding (i) and (ii)

$$\begin{array}{r} x^2 + y^2 = 4 \\ \underline{2x^2 - y^2 = 8} \\ 3x^2 = 12 \end{array}$$

=> $x^2 = \frac{12}{3}$ => $x^2 = 4$

=> $\sqrt{x^2} = \pm\sqrt{4}$ => $x = \pm 2$

Put $x = 2$ in (i)

$(2)^2 + y^2 = 4$ => $4 + y^2 = 4$

=> $y^2 = 4 - 4$ => $y^2 = 0$

=> $y = 0$

Put $x = -2$ in (i)

$(-2)^2 + y^2 = 4$ => $4 + y^2 = 4$

=> $y^2 = 4 - 4$ => $y^2 = 0$

=> $y = 0$

Thus solution set = { (-2 , 0) , (2 , 0) } .

<u>Exercise 2.6</u>:

<u>Q.1</u>: **Solve the following system of equations.**

<u>(i)</u>: $2x - y = 3$, $x^2 + y^2 = 2$

Solution:

$2x - y = 3 .....(i) \quad and \quad x^2 + y^2 = 2 ......(ii)$

From (i) $-y = 3 - 2x \quad => \quad y = 2x - 3 ....(iii)$

Put (iii) in (ii) .

$x^2 + (2x-3)^2 = 2$

$=> \quad x^2 + 4x^2 - 12x + 9 = 2$

$=> \quad 5x^2 - 12x + 7 = 0$

$=> \quad 5x^2 - 5x - 7x + 7 = 0$

$=> \quad 5x(x-1) - 7(x-1) = 0$

$=> \quad (x-1)(5x-7) = 0$

$=> \quad x - 1 = 0 \quad or \quad 5x - 7 = 0$

$=> \quad x = 1 \quad or \quad 5x = 7$

$=> \quad x = \frac{7}{5}$

Put x = 1 in (iii) .

$y = 2(1) - 3 \quad => \quad y = 2 - 3$

$=> \quad y = -1$

Put $x = \frac{7}{5}$ in (iii).

$y = 2\left(\frac{7}{5}\right) - 3 = \frac{14-15}{5}$

$=> \quad y = -\frac{1}{5}$

Thus solution set $= \{(1, -1), (\frac{7}{5}, -\frac{1}{5})\}$.

(ii): $x + 2y = 0 \quad , \quad x^2 + 4y^2 = 32$

Solution: $x + 2y = 0 ......(i) \quad , \quad x^2 + 4y^2 = 32 ...(ii)$

From (i) $=> \quad x = -2y \; ......(iii)$

Put (iii in (ii) .

$(-2y)^2 + 4y^2 = 32 \quad => \quad 4y^2 + 4y^2 = 32$

$=> \quad 8y^2 = 32 \quad => \quad y^2 = \frac{32}{8}$

$=> \quad y^2 = 4 \quad => \quad \sqrt{y^2} = \pm\sqrt{4}$

$=> \quad y = \pm 2$

Put $y = 2$ in (iii)

$x = -2(2) \quad => \quad x = -4$

Put $y = -2$ in (iii)

$x = -2(-2) \quad => \quad x = 4$

Thus solution set = { (- 4 , 2) , (4 , - 2) } .

(iii): $2x - y = -8 \quad , \quad x^2 + 4x = y$

Solution: $2x - y = -8 ......(i) \quad , \quad x^2 + 4x = y.....(ii)$

From (i) $2x + 8 = y \quad => \quad y = 2x + 8 .....(iii)$

Put (iii) in (ii) .

$x^2 + 4x = 2x + 8 \quad => \quad x^2 + 4x - 2x - 8 = 0$

$=> \quad x^2 + 2x - 8 = 0$

$=> \quad x^2 - 2x + 4x - 8 = 0$

$=> \quad x(x-2) + 4(x-2) = 0$

$=> \quad (x-2)(x+4) = 0$

$=> \quad x - 2 = 0 \quad or \quad x + 4 = 0$

$=> \quad x = 2 \quad or \quad x = -4$

Put $x = 2$ in (iii) .

$y = 2(2) + 8 \quad => \quad y = 4 + 8$

$=> \quad y = 12$

Put $x = -4$ in (iii) .

$y = 2(-4) + 8 \quad => \quad y = -8 + 8$

$=> \quad y = 0$

∴ Thus solution set = { (- 4 , 0) , (2 , 12)}.

(iv): $2x + y = 4 \quad , \quad x^2 - 2x + y^2 = 3$

Solution: $2x + y = 4 \ldots\ldots(i) \quad , \quad x^2 - 2x + y^2 = 3 \ldots\ldots(ii)$

From (i) $y = 4 - 2x \ldots\ldots(iii)$

Put (iii) in (ii)

$$x^2 - 2x + (4 - 2x)^2 = 3$$

$$\Rightarrow x^2 - 2x + 16 + 4x^2 - 16x - 3 = 0$$

$$\Rightarrow 5x^2 - 18x + 13 = 0$$

$$\Rightarrow 5x^2 - 5x - 13x + 13 = 0$$

$$\Rightarrow 5x(x - 1) - 13(x - 1) = 0$$

$$\Rightarrow (x - 1)(5x - 13) = 0$$

$$\Rightarrow x - 1 = 0 \quad or \quad 5x - 13 = 0$$

$$\Rightarrow x = 1 \quad or \quad 5x = 13$$

$$\Rightarrow x = \frac{13}{5}$$

Put x = 1 in (iii)

$$y = 4 - 2(1) = 4 - 2$$

$$\Rightarrow y = 2$$

Put $x = \dfrac{13}{5}$

$$y = 4 - 2\left(\frac{13}{5}\right) = \frac{20 - 26}{5}$$

$$\Rightarrow y = -\frac{6}{5}$$

Thus solution set $= \{(1 \ , \ 2) \ , \ (\frac{13}{5} \ , -\frac{6}{5})\}$

(v): $4x^2 + 5y^2 = 4 \quad , \quad 3x^2 + y^2 = 3$

Solution: $4x^2 + 5y^2 = 4 \ldots\ldots(i), \quad 3x^2 + y^2 = 3 \ldots\ldots(ii)$

Eq(ii) is multiplied by 5

$15x^2+5y^2=15.....(iii)$ subtract (i) from (iii).

$$\begin{array}{r} 15x^2+5y^2=15 \\ \underline{\pm 4x^2 \pm 5y^2=\pm 4} \\ 11x^2=11 \end{array}$$

$\Rightarrow \quad x^2=\frac{11}{11} \qquad \Rightarrow \quad x^2=1$

$\Rightarrow \sqrt{x^2}=\pm\sqrt{1} \qquad \Rightarrow \quad x=\pm 1$

Put x = 1 in (ii)

$3(1)^2+y^2=3 \qquad \Rightarrow \quad 3+y^2=3$

$\Rightarrow \quad y^2=3-3 \qquad \Rightarrow \quad y^2=0$

$\Rightarrow \quad y=0$

Put x = -1 in (ii)

$3(-1)^2+y^2=3 \qquad \Rightarrow \quad 3+y^2=3$

$\Rightarrow \quad y^2=3-3 \qquad \Rightarrow \quad y^2=0$

$\Rightarrow \quad y=0$

Thus solution set = { (- 1 , 0) , (1 , 0) }.

(vi): $5x^2=y^2+9 \quad , \qquad x^2=-y^2+45$

Solution: $5x^2=y^2+9......(i), \quad x^2=-y^2+45......(ii)$

Adding (i) and (ii)

$$\begin{array}{r} 5x^2=y^2+9 \\ \underline{x^2=-y^2+45} \\ 6x^2=54 \end{array}$$

$\Rightarrow \quad x^2=\frac{54}{6} \qquad \Rightarrow \quad x^2=9$

$\Rightarrow \sqrt{x^2}=\pm\sqrt{9} \qquad \Rightarrow \quad x=\pm 3$

Put x = 3 in (ii)

$(3)^2 = -y^2 + 45 \quad => \quad y^2 = 45 - 9$

$=> \quad y^2 = 36 \quad => \quad \sqrt{y^2} = \pm\sqrt{36}$

$=> \quad y = \pm 6$

Put x = -3 in (ii)

$(-3)^2 = -y^2 + 45 \quad => \quad y^2 = 45 - 9$

$=> \quad y^2 = 36 \quad => \quad \sqrt{y^2} = \pm\sqrt{36}$

$=> \quad y = \pm 6$

Thus solution set $= \{(-3, -6), (-3, 6), (3, -6), (3, 6)\}$.

(vii): $4x^2 + 3y^2 - 5 = 0$

$2x^2 + 3y^2 - 4 = 0$

Note: in the book question is wrong.

Solution: $4x^2 + 3y^2 - 5 = 0$ ....(i)

$2x^2 + 3y^2 - 4 = 0$ ....(ii)

Subtract (ii) from (i)

$$\begin{array}{r} 4x^2 + 3y^2 - 5 = 0 \\ \underline{\pm 2x^2 \pm 3y^2 \mp 4 = 0} \\ 2x^2 - 1 = 0 \end{array}$$

$=> \quad 2x^2 = 1 \quad => \quad x^2 = \frac{1}{2}$

$=> \quad \sqrt{x^2} = \pm\sqrt{\frac{1}{2}} \quad => \quad x = \pm\frac{1}{\sqrt{2}}$

Put $x = \frac{1}{\sqrt{2}}$ in (ii)

$2\left(\frac{1}{\sqrt{2}}\right)^2 + 3y^2 - 4 = 0$ => $2\left(\frac{1}{2}\right) + 3y^2 - 4 = 0$

=> $1 + 3y^2 - 4 = 0$ => $3y^2 - 3 = 0$

=> $3y^2 = 3$ => $y^2 = \frac{3}{3}$

=> $y^2 = 1$ => $\sqrt{y^2} = \pm\sqrt{1}$

=> $y = \pm 1$

Put $x = -\frac{1}{\sqrt{2}}$ in (ii)

$2\left(-\frac{1}{\sqrt{2}}\right)^2 + 3y^2 - 4 = 0$ => $2\left(\frac{1}{2}\right) + 3y^2 - 4 = 0$

=> $1 + 3y^2 - 4 = 0$ => $3y^2 - 3 = 0$

=> $3y^2 = 3$ => $y^2 = \frac{3}{3}$

=> $y^2 = 1$ => $\sqrt{y^2} = \pm\sqrt{1}$

=> $y = \pm 1$

Thus solution set = $\{(-\frac{1}{\sqrt{2}}, -1), (-\frac{1}{\sqrt{2}}, 1), (\frac{1}{\sqrt{2}}, -1), (\frac{1}{\sqrt{2}}, 1)\}$

<u>Q.2</u>: <u>Challenge</u> : Solve the following system of equations.

(i) $x + y = 9$, $x^2 - 3xy + 2y^2 = 0$

<u>Solution</u>: $x + y = 9$ **....(i)** , $x^2 - 3xy + 2y^2 = 0$ **......(ii)**

From(i) $x = 9 - y$ **....(iii) , put (iii) in (ii)**

$(9 - y)^2 - 3(9 - y)y + 2y^2 = 0$

=> $81 - 18y + y^2 - 27y + 3y^2 + 2y^2 = 0$

=> $6y^2 - 45y + 81 = 0$ => $3(2y^2 - 15y + 27) = 0$

=> $2y^2 - 15y + 27 = 0$ **both sides divided by 3**

=> $2y^2 - 6y - 9y + 27 = 0$

$=> 2y(y-3)-9(y-3)=0$

$=> (y-3)(2y-9)=0$

$=> y-3=0 \quad or \quad 2y-9=0$

$=> y=3 \quad or \quad 2y=9$

$=> y=\frac{9}{2}$

put $y=3$ **in (i)**

$x-3=9 \quad => \quad x=9-3=6$

put $y=\frac{9}{2}$ **in eq(i)**

$x+\frac{9}{2}=9 \quad => \quad x=9-\frac{9}{2}$

$=> x=\frac{18-9}{2}=\frac{9}{2}$

Thus solution et = $\left\{\left(\frac{9}{2} \quad \frac{9}{2}\right), (6 \ , \ 3)\right\}$

(ii) $y-x=4$, $2x^2+xy+y^2=8$

Solution: $y-x=4$ **...(i)** , $2x^2+xy+y^2=8$ **.....(ii)**

From (i) $y=x+4$ **....(iii)** **, put (iii) in (ii)**

$2x^2+x(x+4)+(x+4)^2=8$

$=> 2x^2+x^2+4x+x^2+8x+16-8=0$

$=> 4x^2+12x+8=0 \quad => 4(x^2+3x+2)=0$

$=> x^2+3x+2=0$ **Divided both sides by 4**

$=> x^2+x+2x+2=0$

$=> x(x+1)+2(x+1)=0$

$=> (x+1)(x+2)=0$

$=> x+1=0 \quad or \quad x+2=0$

$=> x=-1 \quad or \quad x=-2$

Put $x=-1$ **in (i)**

$y-(-1)=4 \Rightarrow y+1=4 \Rightarrow y=4-1=3$

Put $x=-2$ in (i)

$y-(-2)=4 \Rightarrow y+2=4 \Rightarrow y=4-2=2$

Thus solution set = { (-1 , 3) , (- 2 , 2) } .

<u>Example</u>: Suppose a rectangular shed is being built that has in area of 120 square feet and is 7 feet longer than its width . Determined its dimensions .

<u>Solution</u>: Let width of the shed = x m

Then length of the shed = x + 7 m

Then according to the given condition

$$x(x+7)=120$$

$$\Rightarrow x^2+7x-120=0$$

$$\Rightarrow x^2+15x-8x-120=0$$

$$\Rightarrow x(x+15)-8(x+15)=0$$

$$\Rightarrow (x+15)(x-8)=0$$

$$\Rightarrow x+15=0 \quad or \quad x-8=0$$

$$\Rightarrow x=-15 \quad or \quad x=8$$

As the width is not negative then x = - 15 is not possible.

Thus width of rectangle = 8 m

Length of the rectangle = 8 + 7 = 15 m .

<u>Exercise 2.7</u>:

<u>Q.1</u>:Find two consecutive positive integers whose product is 72 .

<u>Solution</u>: Let x and x + 1 are two consecutive positive integers

Then $x(x+1)=72$

$$\Rightarrow x^2+x-72=0$$

$$\Rightarrow x^2-8x+9x-72=0$$

$$\Rightarrow x(x-8)+9(x-8)=0$$

$$\Rightarrow (x-8)(x+9)=0$$

$$\Rightarrow x-8=0 \quad or \quad x+9=0$$

$=> \quad x = 8 \quad or \quad x = -9$

Thus positive consecutive integers are x = 8
And x + 1 = 9.

Q.2: The sum of the squares of three consecutive integers is 50 . Find the integers .

Solution: Let x , x + 1 and x + 2 are three consecutive integers

$$Then \quad x^2 + (x+1)^2 + (x+2)^2 = 50$$

$$=> \quad x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 50$$

$$=> \quad 3x^2 + 6x + 5 - 50 = 0$$

$$=> \quad 3x^2 + 6x - 45 = 0$$

$$=> \quad 3(x^2 + 2x - 15) = 0$$

$$=> \quad x^2 + 2x - 15 = 0$$

$$=> \quad x^2 - 3x + 5x - 15 = 0$$

$$=> \quad x(x-3) + 5(x-3) = 0$$

$$=> \quad (x-3)(x+5) = 0$$

$$=> \quad x - 3 = 0 \quad or \quad x + 5 = 0$$

$$=> \quad x = 3 \quad or \quad x = -5$$

Then $x = 3$, $x + 1 = 4$, $x + 2 = 5$

As x = - 5 then $x + 1 = -4$, $x + 2 = -3$

Thus required numbers are 3 , 4 5
Or -5 , -4 , -3 .

Q.3: The length of a hall is 5 meters more than its width . If the area of the hall is 36 sq. m . Find the length and width of the hall.

Solution: Let the width of the hall = x m
Then length of the hall = x + 5 m

Then area of the hall

$$x(x+5)=36$$

$$\Rightarrow \quad x^2+5x-36=0$$

$$\Rightarrow \quad x^2-4x+9x-36=0$$

$$\Rightarrow \quad x(x-4)+9(x-4)=0$$

$$\Rightarrow \quad (x-4)(x+9)=0$$

$$\Rightarrow \quad x-4=0 \quad or \quad x+9=0$$

$$\Rightarrow \quad x=4 \quad or \quad x=-9$$

As width is not negative then x = - 9 is not possible.

Then x = 4

Thus width of rectangle = 4 m .

Length of rectangle = x + 5 = 4 + 5 = 9 m.

Q.4:The sum of two numbers is 11 and sum of their Squares is 65. Find the numbers.

Solution: Let x and y are two numbers.Then according to the given condition is

$$x+y=11 \quad ....(i)$$

$$x^2+y^2=65 \quad ....(ii)$$

From (i) $x=11-y$ ......(iii) . Put (iii) in (ii) .

$$(11-y)^2+y^2=65$$

$$\Rightarrow \quad 121+y^2-22y+y^2=65$$

$$\Rightarrow \quad 2y^2-22y+121-65=0$$

$$\Rightarrow \quad 2y^2-22y+56=0$$

$$\Rightarrow \quad 2\left(y^2-11y+28\right)=0$$

$$\Rightarrow \quad y^2-11y+28=0$$

$$\Rightarrow \quad y^2-4y-7y+28=0$$

$$\Rightarrow \quad y(y-4)-7(y-4)=0$$

$$\Rightarrow \quad (y-4)(y-7)=0$$

$\Rightarrow \quad y-4=0 \quad or \quad y-7=0$

$\Rightarrow \quad y=4 \quad or \quad y=7$

Put $y = 4$ in (iii).

$x=11-4=5$

Put $y = 7$ in (iii).

$x=11-7 \quad \Rightarrow \quad x=4$

Thus the required numbers are 4 , 7 .

Q.5: The sum of the squares of two numbers is 100. One Number is 2 more than the other. Find the numbers.

Solution: Let one number = x

Then the other number = x + 2 .

Then according to the given condition

$$x^2+(x+2)^2=100$$

$$\Rightarrow \quad x^2+x^2+4x+4-100=0$$

$$\Rightarrow \quad 2x^2+4x-96=0$$

$$\Rightarrow \quad 2(x^2+2x-48)=0$$

$$\Rightarrow \quad x^2+2x-48=0$$

$$\Rightarrow \quad x^2-6x+8x-48=0$$

$$\Rightarrow \quad x(x-6)+8(x-6)=0$$

$$\Rightarrow \quad (x-6)(x+8)=0$$

$$\Rightarrow \quad x-6=0 \quad or \quad x+8=0$$

$$\Rightarrow \quad x=6 \quad or \quad x=-8$$

If x = 6 then x + 2 = 8

If x = -8 then x + 2 = -6

Hence required numbers are 6 , 8 or - 6 , -8 .

Q.6: The area of rectangular field is 252 square meters. The length of its side is 9 meter longer than its width. Find its sides.

Solution: Let width of the rectangular field = x m

Then length of the rectangular field = x + 9 m

Then according to the given condition the area of rectangle is

$$x(x+9) = 252$$

$$\Rightarrow x^2 + 9x - 252 = 0$$

$$\Rightarrow x^2 - 12x + 21x - 252 = 0$$

$$\Rightarrow x(x-12) + 21(x-12) = 0$$

$$\Rightarrow (x-12)(x+21) = 0$$

$$\Rightarrow x - 12 = 0 \quad or \quad x + 21 = 0$$

$$\Rightarrow x = 12 \quad or \quad x = -21$$

As width of the rectangle is not negative then x = -21 is not Possible. Thus x = 12

Hence width of the rectangle = 12 m

Length of the rectangle = 12 + 9 = 21 m.

Q.7: One side of a rectangle is 3 centimeters less than twice the other. If the area of the rectangle is 54 square centimeters, then find the sides of the rectangle.

Solution: let one side of a rectangle = x cm

Then the other side = 2x − 3 cm.

According to the given condition the area of rectangle is

$$x(2x-3) = 54$$

$$\Rightarrow 2x^2 - 3x - 54 = 0$$

$$\Rightarrow 2x^2 - 12x + 9x - 54 = 0$$

$$\Rightarrow 2x(x-6) + 9(x-6) = 0$$

$$\Rightarrow (x-6)(2x+9) = 0$$

$$\Rightarrow x - 6 = 0 \quad or \quad 2x + 9 = 0$$

$$\Rightarrow x = 6 \quad or \quad 2x = -9$$

$$\Rightarrow x = -\frac{9}{2}$$

As width of the rectangle is not negative there fore x = 6

Thus width of the rectangle = 6 cm

And length of the rectangle = $2x-3=2(6)-3=12-3=9cm$

Q.8: The length of one side of a right triangle exceeds the Length of the other by 3 centimeters. If the hypotenuse is 15 centimeters, then find the length of the sides of the triangle.

Solution: let one side of a triangle = x cm

Then the other side = x + 3 cm

And hypotenuse = 15 cm.

Then by pathagourse theorem.

$$x^2+(x+3)^2=(15)^2$$

$$\Rightarrow x^2+x^2+6x+9=225$$

$$\Rightarrow 2x^2+6x+9-225=0$$

$$\Rightarrow 2x^2+6x-216=0$$

$$\Rightarrow 2(x^2+3x-108)=0$$

$$\Rightarrow x^2+3x-108=0$$

$$\Rightarrow x^2+12x-9x-108=0$$

$$\Rightarrow x(x+12)-9(x+12)=0$$

$$\Rightarrow (x+12)(x-9)=0$$

$$\Rightarrow x+12=0 \quad or \quad x-9=0$$

$$\Rightarrow x=-12 \quad or \quad x=9$$

As side of a triangle is not negative there fore we take x = 9

Thus one side of triangle = 9 cm

And other side = $x+3=9+3=12cm$.

Q.9: The sides of a triangle in cm are x – 1 , x and x + 1. Find the sides of the triangle.

Solution: let x – 1 and x are the sides of the triangle and x + 1 is the hypotenuse.

Then by pathagourse theorem

$$(x+1)^2 = (x-1)^2 + x^2$$

=> $x^2 + 2x + 1 = x^2 - 2x + 1 + x^2$

=> $x^2 + 2x + 1 = 2x^2 - 2x + 1$

=> $x^2 - 2x^2 + 2x + 2x + 1 - 1 = 0$

=> $-x^2 + 4x = 0$

=> $-x(x-4) = 0$

=> $-x = 0 \quad or \quad x - 4 = 0$

=> $x = 0 \quad or \quad x = 4$

As side of triangle is not zero then we take x = 4

Thus sides of triangle are x – 1 = 4 -1 = 3

And x = 4

And hypotenuse x + 1 = 4 + 1 = 5 .

<u>Q.10</u>: A shepared bought some goats for Rs 9000 . If he had Paid Rs. 100 less for each , he would have got 3 goats more for the same amount of money . How many goats did he buy , when the rate in each case is uniform.

<u>Solution</u>: Let the number of goats are bought = x

Let the amount paid per goat = y

Then $xy = Rs.\ 9000$ ......(*i*)

If the former had paid Rs. 100 per goat then amount

y – 100

Then number of goats are bought x + 3 .

Then according to the given condition ,

$$(x+3)(y-100) = 9000 \text{ ....}(ii)$$

From (i) $y = \dfrac{9000}{x}$ ....(*iii*) put (iii) in (ii) .

$(x+3)\left(\frac{9000}{x}-100\right)=9000$

$\Rightarrow (x+3)\left(\frac{9000-100x}{x}\right)=9000$

$\Rightarrow (x+3)(9000-100x)=9000x$

$\Rightarrow 9000x-100x^2+27000-300x-9000x=0$

$\Rightarrow -100x^2-300x+27000=0$

$\Rightarrow -100(x^2+3x-270)=0$ *divided by* -100

$\Rightarrow x^2+3x-270=0$

$\Rightarrow x^2+18x-15x-270=0$

$\Rightarrow x(x+18)-15(x+18)=0$

$\Rightarrow (x+18)(x-15)=0$

$\Rightarrow x+18=0$ *or* $x-15=0$

$\Rightarrow x=-18$ *or* $x=15$

As x = -18 is not admissible, so we neglect it.
Thus the Number of goats are bought = 15.

Review Exercise 2:

Q.1: Write the correct option in the given box.

(i) If the sum of the roots of $(a+1)x^2+(2a+3)x+3a+4=0$ is -1, then the product of the roots is

(a) 0 (b) 1 (c) 2 (d) 3

(ii) The sum of the roots of quadratic equation is 2 and the sum Of the cubes of the roots is 98. The equation is

(a) $x^2-2x-15=0$ (b) $x^2-2x+15=0$

(c) $x^2-4x+15=0$ (d) *Non of these*

(iii) If a, b, c are positive real numbers, then both the roots of the equation $ax^2+bx+c=0$, are always

(a) real and positive (b) real and negative

(c) rational and unequal (d) Non of these

(iv) If a and b are the roots of $4x^2 - 3x + 7 = 0$ then the value of $\frac{1}{a}+\frac{1}{b}$ is

$(a)\ -\frac{3}{4}$ $(b)\ \frac{3}{7}$ $(c)\ -\frac{3}{7}$ $(d)\ \frac{4}{7}$

Answers: (i) c (ii) a (iii) d (iv) b

Q.2: For what value of k the roots of the equation $3x^2 - 5x + k = 0$ **are equal.**

Solution: $3x^2 - 5x + k = 0$ ……(i)

Here $a = 3$, $b = -5$, $c = k$

Since roots are equal then discriminant is equal to zero . i.e.

$b^2 - 4ac = 0$ …….(ii) put values in (ii) .

$\Rightarrow (-5)^2 - 4(3)(k) = 0 \Rightarrow 25 = 12k$

$\Rightarrow k = \frac{25}{12}$

Q.3: Evaluate $\left(-1+\sqrt{-3}\right)^7 + \left(-1-\sqrt{-3}\right)^7$.

Solution: $\left(-1+\sqrt{-3}\right)^7 + \left(-1-\sqrt{-3}\right)^7$

$As \quad \frac{-1+i\sqrt{3}}{2} = w \quad and \quad \frac{-1-i\sqrt{3}}{2} = w^2$

$\Rightarrow -1+i\sqrt{3} = 2w \quad and \quad -1-i\sqrt{3} = 2w^2$

Then

$$\left(-1+i\sqrt{3}\right)^7 + \left(-1-i\sqrt{3}\right)^7 = (2w)^7 + \left(2w^2\right)^7$$

$$= 2^7 w^7 + 2^7 w^{14}$$

$$= 2^7\left(w^6.w + w^{12}.w^2\right)$$

$$= 2^7\left((w^3)^2.w + (w^3)^4.w^2\right)$$

As $w^3 = 1$

$$=> \left(-1+\sqrt{-3}\right)^7 + \left(-1-\sqrt{-3}\right)^7 = 2^7\left((1)^3.w + (1)^4.w^2\right)$$
$$= 2^7\left(w + w^2\right)$$
$$= 128(-1) \qquad AS\ w + w^2 = -1$$
$$= -128$$

Q.4: With out solving the equation , find the sum and products of the roots of the following equation quadratic equations .

$(i)\ 4x^2 - 1 = 0 \qquad (ii)\ \ 3x^2 + 4x = 0$

Solution: **(i)** $4x^2 + 0.x - 1 = 0$

Here $a = 4$, $b = 0$, $c = -1$

As sum of the roots $S = -\dfrac{b}{a}$ ……(i)

Put values in (i).

$$S = -\frac{0}{4} \quad => \quad S = 0$$

Now product of the roots $= P = \dfrac{c}{a}$……(ii) put values in (ii)

$$=> \quad P = -\frac{1}{4}$$

(ii) $3x^2 + 4x + 0 = 0$

Here $a = 3$, $b = 4$, $c = 0$

As sum of the roots $S = -\dfrac{b}{a}$ ……(i)

Put values in (i).

$$=> \ S = -\frac{4}{3}$$

Product of the roots $= P = \dfrac{c}{a}$ put values

$$=> \ P = \frac{0}{3} \quad => \quad P = 0$$

Q.5: Find the value of k so that the sum of the roots of the equation $3x^2+(2k+1)x+k-5=0$ is equal to the product of roots.

Solution: $3x^2+(2k+1)x+k-5=0$ ……(i)

Here $a=3$, $b=2k+1$, $c=k-5$

As sum of the roots $S=-\dfrac{b}{a}$ ……(i)

Put values in (i).

$$=> \; S=-\frac{2k+1}{3} \; ......(ii)$$

As product of the roots $=P=\dfrac{c}{a}$ ......(iii) put values in (iii)

$$P=\frac{k-5}{3} \; ......(iv) \; .$$

AS Sum of the roots = product of the roots

Then from (ii) and (iv)

$$-\frac{2k+1}{3}=\frac{k-5}{3} \quad => \quad -(2k+1)=k-5$$

$$=> \; -2k-2=k-5 \quad => \; -2+5=k+2k$$

$$=> \; 3k=3 \quad => k=\frac{3}{3}=1$$

Q.6: Find the value of k if the roots of $x^2-3x+k+1=0$ differ by unity .

Solution: $x^2-3x+k+1=0$ …….(i)

Let α *and* β are the roots of eq(i) .

Then according to the given condition

$$\alpha-\beta=1 \; ....(i)$$

Here $a=1$, $b=-3$, $c=k+1$

Sum of the roots $\alpha+\beta=-\dfrac{b}{a}$

$\alpha+\beta=-\frac{-3}{1}$ => $\alpha+\beta=3$ ....(ii)

Product of the roots $\alpha\beta=\frac{c}{a}$

=> $\alpha\beta=\frac{k+1}{1}$ => $\alpha\beta=k+1$ .....(iii)

Adding (i) and (ii).

$$\alpha-\beta=1$$
$$\underline{\alpha+\beta=3}$$
$$2\alpha=4$$

=> $\alpha=\frac{4}{2}$ => $\alpha=2$

Put $\alpha=2$ in (ii)

$2+\beta=3$ => $\beta=3-2$

=> $\beta=1$

Now Put $\alpha=2$ and $\beta=1$ in (iii)

$(2)(1)=k+1$ => $2-1=k$

=> $k=1$

Thus k = 1 .

Q.7:Find the quadratic equation whose roots multiplicative inverse of the roots of $12x^2-17x+6=0$.

Solution: $12x^2-17x+6=0$ ....(i)

Let α *and* β are the roots of eq(i).

Sum of the roots of eq(i) is $\alpha+\beta=-\frac{b}{a}$ .....(ii)

Product of the roots of eq(i) is $\alpha\beta=\frac{c}{a}$ ......(iii)

Here $a=12$, $b=-17$, $c=6$ put in (ii) and (iii).

Then $\alpha+\beta=-\frac{-17}{12}$ => $\alpha+\beta=\frac{17}{12}$ .....(iv)

and $\alpha\beta = \frac{6}{12}$ $\Rightarrow$ $\alpha\beta = \frac{1}{2}$ ....(v).

According to the given condition $\frac{1}{\alpha}$ *and* $\frac{1}{\beta}$ are the roots of required quadratic equation.

Now sum of the roots of required quadratic equation is

$$S = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} \text{ ....(vi)}$$

Put (iv) and (v) in (vi).

$$S = \frac{\frac{17}{12}}{\frac{1}{2}} = \frac{17}{12} \times \frac{2}{1} = \frac{17}{6}$$

Now product of the roots of required quadratic equation is

$$P = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right) = \frac{1}{\alpha\beta} \text{ ....(vii)} \quad \text{put (v) in (vi).}$$

$$\Rightarrow \quad P = \frac{1}{\frac{1}{2}} = 2$$

Now required quadratic equation is

$$x^2 - Sx + P = 0 \text{ ......(viii)}.$$

Put values of S and P in (viii).

$$x^2 - \frac{17}{6}x + 2 = 0$$

$$\Rightarrow 6x^2 - 17x + 12 = 0$$

Q.8: If one of the roots of quadratic equation

$2x^2 + kx + 4 = 0$ is 2, find the other root. Also find the value of k.

Solution: $2x^2 + kx + 4 = 0$ ......(i)

Let α *and* β are the roots of eq(i) .let $\alpha = 2$.

Now sum of the roots

$$\alpha + \beta = -\frac{b}{a} \quad ......(ii) \text{ and}$$

Product of the roots is

$$\alpha\beta = \frac{c}{a} \quad ....(iii)$$

Here $a = 2$, $b = k$, $c = 4$ and $\alpha = 2$

Put in (ii) and (iii)

$$2 + \beta = -\frac{k}{2} \quad ......(iv)$$

$$And \quad 2\beta = \frac{4}{2} \quad => \quad \beta = \frac{4}{4}$$

$$=> \quad \beta = 1$$

Put $\beta = 1$ in (iv) .

$$2 + 1 = -\frac{k}{2} \quad => \quad 3 = -\frac{k}{2}$$

$$=> \quad 6 = -k \quad => \quad k = -6$$

Thus $\alpha = 2$ and $\beta = 1$ and k = - 6 .

Q.9: One root of the cubic equation $x^3 + 6x^2 + 11x + 6 = 0$ is -3 . Use synthetic division to find the other roots .

Solution: $x^3 + 6x^2 + 11x + 6 = 0$ .....(i) . Since -3 is one root Then by synthetic division

-3	1	6	11	6
		-3	-9	-6
	1	3	2	0

Other factor is $Q(x) = x^2 + 3x + 2$ .Then other roots will be the roots of the equation $x^2 + 3x + 2 = 0$. Now

$$x^2 + 3x + 2 = 0 \quad => x^2 + x + 2x + 2 = 0$$

$$=> \quad x(x+1) + 2(x+1) = 0 \quad => (x+1)(x+2) = 0$$

$$=> \quad x + 1 = 0 \quad or \quad x + 2 = 0$$

$$=> \quad x = -1 \quad or \quad x = -2$$

Thus other roots are -1 and -2.

Q.10: Solve the following system of equations.

(*i*) $x+y=3$, $x^2-3xy+y^2=29$

Solution: $x+y=3......(i)$, $x^2-3xy+y^2=29.....(ii)$

From (i) $y=3-x$ ......(*iii*) put (iii) in (ii).

$x^2-3x(3-x)+(3-x)^2=29$

=> $x^2-9x+3x^2+9+x^2-6x-29=0$

=> $5x^2-15x-20=0$

=> $5(x^2-3x-4)=0$

=> $x^2-3x-4=0$

=> $x^2+x-4x-4=0$

=> $x(x+1)-4(x+1)=0$

=> $(x+1)(x-4)=0$

=> $x+1=0$ *or* $x-4=0$

=> $x=-1$ *or* $x=4$

Put x = -1 in (iii) => y = 3- (-1) = 3 + 1

=> y = 4

Put x = 4 in (iii) => y = 3 − 4 = - 1

Thus solution set = { (-1 , 4) , (4 , -1) }.

(II) $7x^2-4=5y^2$, $3x^2+2=4y^2$

Solution: $7x^2-4=5y^2....(i)$, $3x^2+2=4y^2....(ii)$

Eq(i) multiplied by 3 and eq(ii) is multiplied by 7

=> $21x^2-12=15y^2$ ....(*iii*)

$21x^2+14=28y^2$........(*iv*)

Now subtract (iii) from (iv).

$$21x^2 + 14 = 28y^2$$
$$\pm 21x^2 \mp 12 = \pm 15y^2$$
$$26 = 13y^2$$

$\Rightarrow \; y^2 = \dfrac{26}{13} \qquad \Rightarrow \; y^2 = 2$

$\Rightarrow \; \sqrt{y^2} = \pm\sqrt{2} \qquad \Rightarrow \; y = \pm\sqrt{2}$

Now put $y = \sqrt{2}$ *in(ii)*

$3x^2 + 2 = 4(\sqrt{2})^2 \qquad \Rightarrow \; 3x^2 = 4(2) - 2$

$\Rightarrow \; 3x^2 = 8 - 2 \qquad \Rightarrow \; 3x^2 = 6$

$\Rightarrow \; x^2 = \dfrac{6}{3} \qquad \Rightarrow \; x^2 = 2$

$\Rightarrow \; \sqrt{x^2} = \pm\sqrt{2} \qquad \Rightarrow \; x = \pm\sqrt{2}$

Now put $y = -\sqrt{2}$ *in(ii)*

$3x^2 + 2 = 4(-\sqrt{2})^2 \qquad \Rightarrow \; 3x^2 = 4(2) - 2$

$\Rightarrow \; 3x^2 = 8 - 2 \qquad \Rightarrow \; 3x^2 = 6$

$\Rightarrow \; x^2 = \dfrac{6}{3} \qquad \Rightarrow \; x^2 = 2$

$\Rightarrow \; \sqrt{x^2} = \pm\sqrt{2} \qquad \Rightarrow \; x = \pm\sqrt{2}$

Thus solution set

$= \{(-\sqrt{2}, \sqrt{2}), (\sqrt{2}, \sqrt{2}), (-\sqrt{2}, -\sqrt{2}), (\sqrt{2}, -\sqrt{2})\}.$

Q.11: The area of a rectangle is 48 cm^2. If length and width are each increased by 4 cm, the area of larger rectangle is 120 cm^2. Find the length and width of the original rectangle.

Solution: Let width of the original rectangle = x cm

And length of the original rectangle = y cm

Then width of the new rectangle = x + 4 cm

And length of the new rectangle = $y + 4$ cm

Now Area of original rectangle $xy = 48$......(i)

Area of new rectangle $(x+4)(y+4) = 120$

=> $xy + 4x + 4y + 16 = 120$ .....(ii)

Put (i) in (ii)

$48 + 4(x+y) + 16 = 120$ => $4(x+y) = 120 - 48 - 16$

=> $4(x+y) = 56$ => $x + y = \frac{56}{4}$

=> $x + y = 14$ ....(iii)

From (iii) $y = 14 - x$.......(iv) put (iv) in (i) .

$x(14 - x) = 48$ => $14x - x^2 = 48$

=> $0 = x^2 - 14x + 48$ => $x^2 - 14x + 48 = 0$

=> $x^2 - 6x - 8x + 48 = 0$

=> $x(x-6) - 8(x-6) = 0$

=> $(x-6)(x-8) = 0$

=> $x - 6 = 0$ *or* $x - 8 = 0$

=> $x = 6$ *or* $x = 8$

Put x = 6 in (iv)

$y = 14 - 6 = 8$

Put x = 8 in (iv)

$y = 14 - 8 = 6$

Thus width of the rectangle = 6 cm

And length of the rectangle = 8 cm .

Activity : on line Calculator.

Divided $x^5 + x^2 + 5x + 7$ by $x + 2$ and find quotient and remainder .Verify your answer b y long division .

$$
\begin{array}{r|l}
 & x^4-2x^3+4x^2-7x+19 \\
\hline
x+2 & x^5 \qquad +x^2+5x+7 \\
 & \pm x^5 \pm 2x^4 \\
\hline
 & -2x^4 \qquad +x^2+5x+7 \\
 & \mp 2x^4 \mp 4x^3 \\
\hline
 & 4x^3+x^2+5x+7 \\
 & \pm 4x^3 \pm 8x^2 \\
\hline
 & -7x^2+5x+7 \\
 & \mp 7x^2 \mp 14x \\
\hline
 & 19x+7 \\
 & \pm 19x \pm 38 \\
\hline
 & -31
\end{array}
$$

Thus quotient = $x^4-2x^3+4x^2-7x+19$

Remainder = - 31 .

<u>Activity</u> : Abuzar is trying to solve the quadratic equation $x^2+x+11=0$**. He find two real solutions to the quadratic . Habiba immediately points out that Abuzar,s answers are Wong . Habiba explain that she does not know the Solutions are right away , but she knows they are not Real numbers . whose is correct ? .**

<u>Solution</u>: $x^2+x+11=0$.

Habiba is correct . Because she use the discriminant.

Here $a=1$, $b=1$, $c=11$, **Then**

Discrimenant = $b^2-4ac=(1)^2-4(1)(11)$

$=1-44$

$=-43$

As discriminant is negative so the roots are complex . Thus The roots are not real numbers.

Unit . 3

Variation.

Ratio: If a and b are two quantities of the same unit then the ratio between a and b is a : b or $a : b = \frac{a}{b}$.

Similarly a , b , c are three numbers of the same unit then the ratio is $a:b:c$

Example: Divide Rs 5070 among three persons in the ratio 2 : 5 : 6

Solution: Given ratio 2 : 5 : 6

Sum of the ratios $= 2+5+6 = 13$

Share of the first man $= 5070 \times \frac{2}{13} = 780$ Rupees

Share of the second man $= 5070 \times \frac{5}{13} = 1950$ Rupees

Share of the third man $= 5070 \times \frac{6}{13} = 2340$ Rupees .

Proportion . If a , b , c and d are four quantities then the general form of proportion is

$$\frac{a}{b} = \frac{c}{d}$$

Then we say a , b , c , d are proportion.

Eq(i) can also be written as a : b :: c : d

$\Rightarrow a : b = c : d$ ...(ii)

From eq(i) => $a \times d = b \times c$

i.e. Product of extremities = Product of means

Example: $a^3 - b^3$, $a^2 - b^2$, $a^2 + ab + b^2$ **and** x **are in a proportion . Find the value of x .**

Solution:

$$a^3 - b^3 : a^2 - b^2 :: a^2 + ab + b^2 : x$$

Then $$\frac{a^3 - b^3}{a^2 - b^2} = \frac{a^2 + ab + b^2}{x}$$

$$\Rightarrow \quad x = \frac{(a^2 - b^2)(a^2 + ab + b^2)}{a^3 - b^3}$$

$$\Rightarrow \quad x = \frac{(a-b)(a+b)(a^2 + ab + b^2)}{(a-b)(a^2 + ab + b^2)}$$

$$\Rightarrow \quad x = a + b$$

Direct variation :Relation between two quantities x and y is called Direct variation if one increase then the Second also increase and if one decrease then the Second also decrease.

Mathematically $y \propto x$ or $y = kx$

$$\Rightarrow k = \frac{y}{x}$$

Example: There is a direct variation between x and y, and y = 27 when x = 3 . Find

(i) An equation connecting x and y .

(ii) The value of y when x = 11 .

Solution: (i) since direct variation between x and y ,then

$$y = kx \text{........}(i)$$

(ii) from (i) $\Rightarrow \quad k = \frac{y}{x} \text{........}(ii)$

Put x = 3 and y = 27 in (ii) .

$$\Rightarrow \quad k = \frac{27}{3} = 9$$

Put x = 11 and k = 9 in (i)

$$y = 9(11) \quad \Rightarrow \quad y = 99 .$$

(b)Inverse variation: If one variable is increase then the other is decreases. Then the relation is called inverse variation mathematically

$$y \propto \frac{1}{x} \quad \Rightarrow \quad y = \frac{k}{x}$$

Or $yx = k$

Exercise. No. 3.1

Q.NO.1: Which is the greater ratio 5 :7 or 151 : 208

Solution: As $5:7=\dfrac{5}{7}$

$=0.71$ ...(i)

Also $151:208=\dfrac{151}{208}$

$=0.72$ .....(ii)

Hence from (i) and (ii) it is clear that 151 : 208 is greater.

Q.2: Gold and silver are mixed in the ratio 7 : 4. If 36 grams of silver is used. How much gold is used ?.

Solution: Let gold is used = x grams. Then

$7:4::x:36$

Since Product of means = Product of extremities

Then $4x=7\times 36$

$\Rightarrow\ x=\dfrac{7\times 36}{4}=7\times 9$

$\Rightarrow\ x=63$

Hence gold is used = 63 grams.

Q.3: Divide the annual profit of Rs. 40,000 of a factory among 3 partners in the ratio of 5 : 8 : 12

Solution: Total amount = 40000

Given ratios = 5 : 8 : 12

Sum of ratios = 5 + 8 + 12 = 25

The share of Ist partner $=(40000)\left(\dfrac{5}{25}\right)$

$=(1600)(5)$

$=8{,}000$ Rupees

The share of 2nd partner $=(40000)\left(\dfrac{8}{25}\right)$

$=(1600)(8)=12800$ Rupees

The share of third partner $=(40000)\left(\dfrac{12}{25}\right)$

$= (1600)(12) = 19200$ Rupees.

Q.NO.4. **If 11 : x - 1 = 22 : 27 find the value of x**

Solution: As 11 : x - 1 = 22 : 27

Since Product of extremities = Product of means

$\Rightarrow (11)(27) = (x-1)(22)$

$\Rightarrow 297 = 22x - 22$

$\Rightarrow 297 + 22 = 22x$

$\Rightarrow x = \frac{319}{22}$

$\Rightarrow x = 14.5$

Q.NO.5. **There is direct variation between x^2 and y when x = 7 , y = 49 find (i) y when x = 9 (ii) x when y = 100**

Solution: As $y = kx^2$ ....(i) $\Rightarrow k = \frac{y}{x^2}$ ...(i)

Put x = 7 , y = 49 in eq(ii)

$\Rightarrow k = \frac{49}{(7)^2}$

$\Rightarrow k = \frac{49}{49}$ $\Rightarrow k = 1$

$\Rightarrow k = 1$

(i) Now we find y.

put x = 9 and k = 1 in eq(ii)

$\Rightarrow y = 1(9)^2$ $\Rightarrow y = 81$

(ii) we find x.

put y = 100 and k = 1 in eq(ii)

$\Rightarrow 100 = (1)x^2$ $\Rightarrow x^2 = 100$

$\Rightarrow \sqrt{x^2} = \sqrt{100}$ $\Rightarrow x = 10$

Q.NO.6. **There is an inverse variation between x and y , and when x = 4 , y = 6 , find (i) y when x = 12 (ii) x when y = 24**

Solution: (i) Since the variation is inversely

Then $y = \frac{k}{x}$ ...(i) => k = yx...(ii)

Put x = 4 and y = 6 in eq(ii)

=> k = 6(4) => k = 24

(i) Put x = 12 , k = 24 in eq(i)

=> $y = \frac{24}{12} = 2$

(ii) Put y = 24 , k = 24 in eq(i)

=> $24 = \frac{24}{x}$ => $24x = 24$

=> $x = \frac{24}{24}$ => x = 1

Note : In the book y = 24 is wrong .

Q.NO. 7: $r \propto \frac{1}{p^3}$ **and p = 9 , when r = 2**

find (i) r when p = 3 (ii) p when $r = \frac{1}{4}$

Solution: As $r \propto \frac{1}{p^3}$ => $r = \frac{k}{p^3}$ ....(i)

=> $k = rp^3$.....(ii) put p = 9 , r = 2 in eq(ii)

=> $k = 2(9)^3 = 2(729)$

=> $k = 1458$

(i) To find r put p = 3 , k = 1458 in eq (i)

$$r = \frac{1458}{(3)^3} = \frac{1458}{27}$$

=> $r = 54$

(ii) To find p put $r = \frac{1}{4}$,k = 1458 in eq(i)

$\Rightarrow \frac{1}{4} = \frac{1458}{p^3}$ $\Rightarrow p^3 = 4 \times 1458$

$\Rightarrow p^3 = (18)^3$ $\Rightarrow p = 18$

Q.NO. 8. If $y \propto x$ then complete the following Tables

x	4	6		15
y	2		3.5	

SOL: (i) As $y \alpha x$

$\Rightarrow y = kx$...(i)

$\Rightarrow k = \frac{y}{x}$ ....(ii) put $x = 4$ and $y = 2$ in eq(ii)

$\Rightarrow k = \frac{2}{4}$ $\Rightarrow k = \frac{1}{2}$

put $x = 6$, $k = \frac{1}{2}$ In eq(i)

$\Rightarrow y = (\frac{1}{2})(6) = 3$

which is corresponding value of y

put $y = 3.5$, $k = \frac{1}{2}$ in eq(i)

$\Rightarrow 3.5 = (\frac{1}{2})x$ $\Rightarrow x = 2(3.5)$

$\Rightarrow x = 7$ which is corresponding value of x

put $x = 15$, $k = \frac{1}{2}$ in eq(i)

$\Rightarrow y = (\frac{1}{2})(15) = 7.5$

which is corresponding value of y

Thus completed table is (i)

x	4	6	7	15
y	2	3	3.5	7.5

Mean , third , fourth and Continued proportion.

If a ,b ,c have direct variation

Then $\frac{a}{b}=\frac{b}{c}$ or a : b :: b :c is called Continued proportion . "b" is the mean proportion , " c" is the third proportion .

Properties of proportion .

(i) if $a:b::c:d$ => $\frac{a}{b}=\frac{c}{d}$

(ii) if $a:b::c:d$. Then $a:c::b:d$

This property is called Alternendo property .

(iii) if $\frac{a}{b}=\frac{c}{d}$ *then* $\frac{b}{a}=\frac{d}{c}$

This property is called Invertendo property .

(iv) if $\frac{a}{b}=\frac{c}{d}$ Then

(a) $\frac{a+b}{b}=\frac{c+d}{d}$ or $a+b:b=c+d:d$

(b) $\frac{a}{a+b}=\frac{c}{c+d}$ or $a:a+b=c:c+d$

This property is called Componendo property .

(v) if $\frac{a}{b}=\frac{c}{d}$ Then

(a) $\frac{a-b}{b}=\frac{c-d}{d}$ or $a-b:b=c-d:d$

(b) $\frac{a}{a-b}=\frac{c}{c-d}$ or $a:a-b=c:c-d$

This property is called dividendo property .

(vi) if $\frac{a}{b}=\frac{c}{d}$ Then

(a) $\frac{a+b}{a-b}=\frac{c+d}{c-d}$. It is Componendo – dividendo property

(b) $\frac{a-b}{a+b}=\frac{c-d}{c+d}$. It is dividendo – Componendo property .

(vii) If a, b, c are continued proportion then

$\frac{a}{b}=\frac{b}{c}$ Then (a) $ac=b^2$

(b) $\frac{a^2}{b^2}=\frac{a}{c}$

Example : If $\frac{a}{b}=\frac{c}{d}$ then prove that

$2a+3b : b = 2c+3d : d$

Solution: $\frac{a}{b}=\frac{c}{d}$ **multiplying both sides by** $\frac{2}{3}$

=> $\frac{2a}{3b}=\frac{2c}{3d}$

=> $\frac{2a+3b}{3b}=\frac{2c+3d}{3d}$ **multiplying by 3**

=> $\frac{2a+3b}{b}=\frac{2c+3d}{d}$

Or $2a+3b : b = 2c+3d : d$

EXERCISE. 3.2:

Q.NO.1. Which of the following quantities are in **continued proportion.**

(i) 4 , 12 , 36 (ii) 3 , 12 , 39 (iii) 72 , 24 , 8

Solution:(i) Since $\frac{4}{12}=\frac{1}{3}$ and $\frac{12}{36}=\frac{1}{3}$

Thus $\frac{4}{12}=\frac{12}{36}=\frac{1}{3}$

Thus 4 ,12 , 36 are continued proportion.

(ii) As $\frac{3}{12}=\frac{1}{4}$ and $\frac{12}{39}=\frac{4}{13}$

AS $\frac{3}{12}=\frac{1}{4}\neq\frac{12}{39}=\frac{4}{13}$

Hence 3 , 12 , 39 are not continued proportion

(iii) Since $\frac{72}{24} = 3$ and $\frac{24}{8} = 3$

Thus $\frac{72}{24} = \frac{24}{8} = 3$

Hence they are continued proportion

Q.2: Find the mean proportional of 12 , 3 .

Solution: Let x is the mean proportional of 12 and 3 .

$12 : x :: x : 3$

$\Rightarrow \frac{12}{x} = \frac{x}{3} \Rightarrow 12 \times 3 = x^2$

$\Rightarrow x^2 = 36 \Rightarrow x = 6$

Q. NO. 3. If 5 : 15 : x are in continued proportion find the value of x .

Solution: As 5 : 15 : x continued proportion then

$\Rightarrow \frac{5}{15} = \frac{15}{x}$

$\Rightarrow 5x = (15)(15)$

$\Rightarrow x = \frac{(15)(15)}{5} = (3)(15)$

$\Rightarrow x = 45$

Q.NO.4. 3x – 1 , 4 , 35 are continued proportion Find the value of x

Solution: 3x - 1 : 4 : 35 are continued proportion then

$\Rightarrow \frac{3x-1}{4} = \frac{4}{35}$

$\Rightarrow 35(3x-1) = 4 \times 4$

$\Rightarrow 105x - 35 = 16$

$\Rightarrow 105x = 16 + 35$

$\Rightarrow 105x = 51$

$\Rightarrow x = \frac{51}{105}$

$$=> x = \frac{17}{35}$$

Q.NO. 5. Find the mean proportional of $a^2 - b^2$ and $\frac{a+b}{a-b}$.

Solution; Let x be the mean proportion of $a^2 - b^2$ and $\frac{a+b}{a-b}$.

Then $a^2 - b^2 : x : \frac{a+b}{a-b}$ is continued proportion

$$=> \frac{a^2 - b^2}{x} = \frac{x}{(\frac{a+b}{a-b})}$$

$$=> \frac{a^2 - b^2}{x} = \frac{(a-b)x}{a+b}$$

$$=> \frac{(a-b)(a+b)(a+b)}{a-b} = x^2$$

$$=> (a+b)(a+b) = x^2$$

$$=> \sqrt{x^2} = \sqrt{(a+b)^2}$$

$$=> x = a + b$$

Q.NO. 6. If $\frac{a}{b} = \frac{c}{d}$ then prove that $\frac{ac + ad}{ac - bd} = \frac{a^2 + b^2}{a^2 - b^2}$

Solution: As $\frac{a}{b} = \frac{c}{d}$ or $\frac{c}{d} = \frac{a}{b}$

multiplied both sid s by $\frac{a}{b}$

$$=> \frac{ac}{bd} = \frac{a^2}{b^2}$$

$$=> \frac{ac+bd}{ac-bd} = \frac{a^2+b^2}{a^2-b^2} \quad (Componendo - dividendo)$$

Q.NO. 7. Solve the following equations

(i) $\frac{\sqrt{3x+2}+\sqrt{x}}{\sqrt{3x+2}-\sqrt{x}} = \frac{4}{1}$

Solution. $\frac{\sqrt{3x+2}+\sqrt{x}}{\sqrt{3x+2}-\sqrt{x}} = \frac{4}{1}$

By componendo and dividendo

$$=> \frac{\sqrt{3x+2}+\sqrt{x}+\sqrt{3x+2}-\sqrt{x}}{\sqrt{3x+2}+\sqrt{x}-(\sqrt{3x+2}-\sqrt{x})} = \frac{4+1}{4-1}$$

$$=> \frac{2\sqrt{3x+2}}{\sqrt{3x+2}+\sqrt{x}-\sqrt{3x+2}+\sqrt{x}} = \frac{5}{3}$$

$$=> \frac{2\sqrt{3x+2}}{2\sqrt{x}} = \frac{5}{3}$$

$$=> \frac{\sqrt{3x+2}}{\sqrt{x}} = \frac{5}{3}$$

$$=> 3(\sqrt{3x+2}) = 5\sqrt{x}$$

$$=> (3\sqrt{3x+2})^2 = (5\sqrt{x})^2$$

$$=> 9(3x+2) = 25x$$

$$=> 27x - 25x + 18 = 0$$

$$=> 2x = -18$$

$$=> x = -\frac{18}{2} = -9$$

(ii) $\frac{(x-1)^2+(x+2)^2}{(x-1)^2-(x+2)^2} = -\frac{17}{8}$

Solution: $\dfrac{(x-1)^2+(x+2)^2}{(x-1)^2-(x+2)^2}=-\dfrac{17}{8}$

=> $\dfrac{(x-1)^2+(x+2)^2}{(x-1)^2-(x+2)^2}=\dfrac{17}{-8}$

By componendo and dividendo

$$\frac{(x-1)^2+(x+2)^2+(x-1)^2-(x+2)^2}{(x-1)^2+(x+2)^2-\left[(x-1)^2-(x+2)^2\right]}=\frac{17+(-8)}{17-(-8)}$$

=> $\dfrac{2(x-1)^2}{(x-1)^2+(x+2)^2-(x-1)^2+(x+2)^2}=\dfrac{17-8}{17+8}$

=> $\dfrac{2(x-1)^2}{2(x+2)^2}=\dfrac{9}{25}$

=> $\dfrac{(x-1)^2}{(x+2)^2}=\dfrac{9}{25}$

=> $25(x-1)^2=9(x+2)^2$

=> $25(x^2-2x+1)=9(x^2+4x+4)$

=> $25x^2-50x+25=9x^2+36x+36$

=> $25x^2-9x^2-50x-36x+25-36=0$

=> $16x^2-86x-11=0$

Then by Quadratic formula

=> x = $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ ……(i)

Put a =16 ,b = -86 , c = -11 in eq(i)

=> $x=\dfrac{-(-86)\pm\sqrt{(-86)^2-4(16)(-11)}}{2(16)}$

=> $x=\dfrac{86\pm\sqrt{7396+704}}{32}$

$$=> \quad x=\frac{86\pm\sqrt{8100}}{32}$$

$$=> \quad x=\frac{86\pm 90}{32}$$

$$=> \quad x=\frac{86+90}{32} \quad , \quad x=\frac{86-90}{32}$$

$$=> \quad x=\frac{176}{32} \quad , \quad x=-\frac{4}{32}$$

$$=> \quad x=5.5 \quad , \quad x=-\frac{1}{8}$$

(iii) $\dfrac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}=\dfrac{1}{3}$

SOL : $\dfrac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}=\dfrac{1}{3}$

by componendo and dividendo

$$=> \frac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}+\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}-(\sqrt{x^2+a^2}-\sqrt{x^2-a^2})}=\frac{3+1}{3-1}$$

$$=> \frac{2\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}-\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}=\frac{4}{2}$$

$$=> \frac{2\sqrt{x^2+a^2}}{2\sqrt{x^2-a^2}}=2$$

$$=> \left(\frac{\sqrt{x^2+a^2}}{\sqrt{x^2-a^2}}\right)^2=(2)^2$$

$$=> \frac{x^2+a^2}{x^2-a^2}=4$$

$$=> x^2 + a^2 = 4x^2 - 4a^2$$

$$=> a^2 + 4a^2 = 4x^2 - x^2$$

$$=> 5a^2 = 3x^2$$

$$=> x^2 = \frac{5}{3}a^2$$

$$=> \sqrt{x^2} = \sqrt{\frac{5a^2}{3}}$$

$$=> x = \sqrt{\frac{5}{3}}\ a$$

Joint variation : If x is jointly proportional to y *and* z , then $x = yzk$, where k is constant.

Example: If y varies jointly as x and z , and y = 12 When x = 9 and z = 3 , find z when y = 6 and x = 15 .

Solution: As y is varies jointly as x and z , then

$$y = xzk$$

Then $k = \frac{y}{xz}$ ......(i)

Put $x = 9$, $y = 12$, $z = 3$ in (ii)

$$k = \frac{12}{9 \times 3} = \frac{4}{9}$$

As $y = xzk$ => $z = \frac{y}{xk}$ ....(ii)

Put $y = 6$, $x = 15$ in (ii)

$$z = \frac{6}{15\left(\frac{4}{9}\right)} = \frac{6 \times 9}{15 \times 4} = \frac{2 \times 9}{5 \times 4}$$

$$=> z = \frac{9}{5 \times 2} = \frac{9}{10}$$

Exercise 3.3:

Q.1: If y varies jointly as x and z, and y = 33 When x = 9 and z = 12, find y when x = 16 and z = 22.

Solution: As y is varies jointly as x and z. then

$$y = xzk$$

Then $k = \frac{y}{xz}$ ......(i)

Put $x = 9$, $y = 33$, $z = 12$ in (ii)

$$k = \frac{33}{9\times 12} = \frac{11}{9\times 4} = \frac{11}{44}$$

As $y = xzk$ ....(ii)

Put $x = 16$, $z = 22$ in (ii)

$$y = (16)(22)\left(\frac{11}{36}\right) = (4)(22)\left(\frac{11}{9}\right)$$

$$=> \quad y = \frac{968}{9}$$

Q.2: If f varies jointly as g and the cube of h, and f = 200 when g = 5 and h = 4, find f when g = 3 and h = 6.

Solution: As f varies jointly to g and h^3. Then

$$f = gh^3k \text{ ...(i)}$$

$$=> k = \frac{f}{gh^3} \text{ ....(ii)}$$

Put f = 200 , g = 5 and h = 4 in(ii)

$$k = \frac{200}{5\times(4)^3} = \frac{40}{64} = \frac{5}{8}$$

Now put $g = 3$, $h = 6$ *and* $k = \frac{5}{8}$ in(i)

$$f = 3\times(6)^3\times\frac{5}{8} = \frac{3\times 216\times 5}{8}$$

$$=> f = 3\times 27\times 5 = 405$$

Q.3: Suppose a is jointly proportional to b and c.

If $a=4$ when $b=8$ and $c=9$. then what is a when $b=2$ and $c=18$.

Solution: As a is jointly proportional to b and c, then

$$a=bck \quad ......(i)$$

$$\Rightarrow \quad k=\frac{a}{bc} \quad ....(ii).$$

Put $a=4$ when $b=8$ and $c=9$ in (ii)

$$k=\frac{4}{8\times 9}=\frac{1}{2\times 9}=\frac{1}{18}$$

Put $b=2$, $c=18$ and $k=\frac{1}{18}$ in (i)

$$\Rightarrow \quad a=2\times 18\times \frac{1}{18}=2$$

$$\Rightarrow \quad a=2$$

Q.4: If p varies jointly as q and r squared, and $p=225$, when $q=4$ and $r=3$, find p when $q=6$ and $r=8$.

Solution: p varies jointly as q and r^2. then

$$p=kqr^2 \quad ....(i)$$

$$\Rightarrow \quad k=\frac{p}{qr^2} \quad .....(ii)$$

Put $p=225$, $q=4$ and $r=3$ in(ii)

$$k=\frac{225}{4(3)^2}=\frac{225}{4\times 9}=\frac{25}{4}$$

Put $q=6$, $r=8$ and $k=\frac{25}{4}$ in (i)

$$p=\left(\frac{25}{4}\right)(6)(8)^2=\left(\frac{25}{4}\right)(6)(64)$$

$$\Rightarrow \quad p=25\times 6\times 16=2400$$

Q.5: If a is jointly proportional to b cubed and c.

and $a=36$ when $b=4$ and $c=6$. then find is a when

b = 2 and c = 14 .

Solution: As a is jointly proportional to b^3 and c , then

$$a = b^3ck \quad ......(i)$$

$$\Rightarrow \quad k = \frac{a}{b^3c} \quad ....(ii).$$

Put $a = 36$ when b = 4 and c = 6 in (ii)

$$k = \frac{36}{(4)^3 \times 6} = \frac{6}{64} = \frac{3}{32}$$

Put $b = 2$, $c = 14$ *and* $k = \frac{3}{32}$ in (i)

$$\Rightarrow \quad a = (2)^3 \times 14 \times \frac{3}{32} = 8 \times 14 \times \frac{3}{32}$$

$$\Rightarrow \quad a = 14 \times \frac{3}{4} = 7 \times \frac{3}{2}$$

$$\Rightarrow \quad a = \frac{21}{2}$$

Q.6: If z varies jointly as x and y and z = 12 , when x = 2 , y = 4 , find the constant of variation .

Solution: As z varies jointly as x and y , then

$$z = xyk$$

$$\Rightarrow \quad k = \frac{z}{xy} \quad ......(i)$$

Put $x = 2$, $y = 4$ *and* $z = 12$ in (i)

$$k = \frac{12}{2 \times 4} = \frac{6}{4}$$

$$\Rightarrow \quad k = \frac{3}{2}$$

Q.7: If y varies jointly as x^2 and z and y = 6 when x = 4 , z = 9 . Write y as a function of x and z and determine the value of y , when x = - 8 and z = 12 .

Solution: As varies jointly as x^2 and z , then

$y = kx^2z$ ......(i)

Eq(i) shows the y is a function of x and z.

From (i) => $k = \frac{y}{x^2z}$ ......(ii)

Put $x = 4$, $y = 6$ *and* $z = 9$ **in (ii)**

$$k = \frac{6}{(4)^2(9)} = \frac{2}{16\times3} = \frac{1}{8\times3}$$

=> $k = \frac{1}{24}$

Put $k = \frac{1}{24}$, $x = -8$, $z = 12$ **in (i)**

$$y = \left(\frac{1}{24}\right)(-8)^2(12) = \left(\frac{1}{2}\right)(64)(1)$$

=> $y = 32$

Q.8: If p varies jointly as q and r^2 and inversely as s and t^2 , p = 40 when q = 8 , r = 5 , s = 3 , r = 2 . Find p in term of q , r , s and t . Also find the value of p when q = - 2 , r = 4 s = 3 and t = -1 .

Solution: As p varies jointly as q and r^2 , then

$p \propto qr^2$ .....(i)

Also p is inversely as s and t^2 , then

$p \propto \frac{1}{st^2}$ .....(ii)

join (i) and (ii) , then

$$p \propto \frac{qr^2}{st^2} \quad => \quad p = k\,\frac{qr^2}{st^2} \text{ ....}(i)$$

Eq (i) shows p in terms q , r , s and t .

From (i) $k = \frac{pst^2}{qr^2}$ ......(ii)

Put p = 40 when q = 8 , r = 5 , s = 3 , r = 2 in (ii) .

$$k=\frac{40\times 3\times (2)^2}{(8)(5)^2}=\frac{5\times 3\times 4}{25}=\frac{12}{5}$$

Now put q = - 2 , r = 4 , s = 3 , t = - 1 , $k=\frac{12}{5}$ in (i)

$$p=\left(\frac{12}{5}\right)\frac{(-2)(4)^2}{(3)(-1)^2}=\left(\frac{4}{5}\right)\frac{(-2)(16)}{1}$$

$$\Rightarrow\ p=-\frac{128}{5}$$

Activity : **Example of joint variation** :

Area of a rectangle is jointly varies to the length and width of the rectangle . As area increases with the increase of width and length .

Example of direct variation: In a school if absentee fine per day is 10 rupees . Then amount of fine is increases with the number of absentee increases. Hence the absentee and fine are in direct variation .

Example of inversely variation: In retardation motion (If a car is traveling with –ve acceleration) of a car, the velocity of the car is decreases with passage (increasing) in time . Here in velocity and time has varies inversely .

K- Method: Let $a:b::c:d$ be a proportion , then

$$\frac{a}{b}=\frac{c}{d}=k$$

$$\Rightarrow\ a=bk \quad and \quad c=dk$$

Example: If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ where b , d , f are non zero zero numbers , then prove that each of these ratios is equal to the following ratios .

$$\frac{la+mc+ne}{lb+md+nf}\ .$$

Solution: Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$ ...(i)

=> $a=bk$, $c=dk$, $e=fk$

$$Now\ \frac{la+mc+ne}{lb+md+nf}=\frac{lbk+mdk+nfk}{lb+md+nf} \quad put\ values\ of\ a,c,e$$

$$=\frac{k(lb+md+nf)}{lb+md+nf}$$

$$=k\ ....(ii)$$

From (i) and (ii) it is proved that

$$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{la+mc+ne}{lb+md+nf}$$

Example: prove that

$$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{a+c+e}{b+d+f}$$

Solution: Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$ ...(i)

=> $a=bk$, $c=dk$, $e=fk$

Adding

$$a+c+e=bk+dk+fk$$

=> $a+c+e=k(b+d+f)$

=> $\frac{a+c+e}{b+d+f}=k$ .....(ii)

put (ii) and (i)

$$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{a+c+e}{b+d+f}.$$

Example . prove that $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{\sqrt{a^2+c^2+e^2}}{\sqrt{b^2+d^2+f^2}}$

Solution: Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$ ...(i)

$\Rightarrow a = bk \quad , c = dk \quad , e = fk$

$\Rightarrow a^2 = b^2k^2 \;, c^2 = d^2k^2 \quad , e^2 = f^2k^2$ **adding these**

$a^2 + c^2 + e^2 = b^2k^2 + d^2k^2 + f^2k^2$

$\Rightarrow a^2 + c^2 + e^2 = k^2(b^2 + d^2 + f^2)$

$\Rightarrow \dfrac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2} = k^2$ **take square root of both sides**

$$\sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}} = \sqrt{k^2}$$

$$\Rightarrow \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}} = k \quad ....(ii)$$

Put (ii) in (i)

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{\sqrt{a^2 + c^2 + e^2}}{\sqrt{b^2 + d^2 + f^2}}.$$

EXERCISE . NO. 3.4

Q.NO.1.if $a : b = c : d$ **then prove that**

(i) $\dfrac{2a + 3b}{2a - 3b} = \dfrac{2c + 3d}{2c - 3d}$

(ii) $pa + qb : ma - nb = pc + qd : mc - nd$

Solution. (i) As $a : b = c : d$

Then $\dfrac{a}{b} = \dfrac{c}{d}$ **multiplying both sides by** $\dfrac{2}{3}$

$\Rightarrow \dfrac{2a}{3b} = \dfrac{2c}{3d}$

$\Rightarrow \dfrac{2a + 3b}{2a - 3b} = \dfrac{2c + 3d}{2c - 3d}$ **by componendo and dividendo**

(ii) As $\dfrac{a}{b} = \dfrac{c}{d}$ **multiplying both sides by** $\dfrac{p}{q}$

$$\frac{pa}{qb}=\frac{pc}{qd}$$

=> $\frac{pa+qb}{qb}=\frac{pc+qd}{qd}$ **by componendo .**

=> $\frac{pa+qb}{b}=\frac{pc+qd}{d}$ **....(i) multiplied by q**

As $\frac{a}{b}=\frac{c}{d}$ **multiplying both sides by** $\frac{m}{-n}$

$$\frac{ma}{-nb}=\frac{mc}{-nd}$$

=> $\frac{ma-nb}{-nb}=\frac{mc-nd}{-nd}$ **by componendo .**

=> $\frac{ma-nb}{b}=\frac{mc-nd}{d}$ **....(ii) multiply by - n**

Now (i) divided by (ii)

$$\frac{\frac{pa+qb}{b}}{\frac{ma-nb}{b}}=\frac{\frac{pc+qd}{d}}{\frac{mc-nd}{d}}$$

=> $\frac{pa+qb}{b}\times\frac{b}{ma-nb}=\frac{pc+qd}{d}\times\frac{d}{mc-nd}$

=> $\frac{pa+qb}{ma-nb}=\frac{pc+qd}{mc-nd}$

Or $pa+qb:ma-nb=pc+qd:mc-nd$ **proved .**

<u>Q.NO.2</u>. Prove that $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}$

<u>Solution</u>.

Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=$ k ....(i)

From (i)

=> $\frac{a}{b}=$ k , $\frac{c}{d}=$ k and $\frac{e}{f}=$ k

$\Rightarrow a = bk \quad , c = dk \quad , e = fk$

Now $\sqrt{\dfrac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}} = \sqrt{\dfrac{p(bk)^2+q(dk)^2+(fk)^2}{pb^2+qd^2+f^2}}$

$= \sqrt{\dfrac{pb^2k^2+qd^2k^2+f^2k^2}{pb^2+qd^2+f^2}}$

$= \sqrt{\dfrac{k^2(pb^2+qd^2+f^2)}{pb^2+qd^2+f^2}}$

$= \sqrt{k^2}$

$= k$ ....(ii)

Thus from (i) and (ii)

$\Rightarrow \quad \dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = \sqrt{\dfrac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}$

Q.NO.3. $\dfrac{x-y}{z} = \dfrac{y-z}{x} = \dfrac{z-x}{y}$ **then prove that**

$x = y = z$ **where x, y, z are non zero numbers ,**

and $x+y+z \neq 0$

Solution: As $\dfrac{x-y}{z} = \dfrac{y-z}{x} = \dfrac{z-x}{y}$ ....(i)

But we now that $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = \dfrac{a+c+e}{b+d+f}$ .....(ii)

Then by (i) and (ii)

$\dfrac{x-y}{z} = \dfrac{y-z}{x} = \dfrac{z-x}{y} = \dfrac{x-y+y-z+z-x}{z+x+y}$

$= \dfrac{0}{x+y+z}$

$\Rightarrow \dfrac{x-y}{z} = \dfrac{y-z}{x} = \dfrac{z-x}{y} = 0$

$\Rightarrow \dfrac{x-y}{z}=0$, $\dfrac{y-z}{x}=0$ and $\dfrac{z-x}{y}=0$

$\Rightarrow x-y=0$, $y-z=0$, $z-x=0$

$\Rightarrow x=y$, $y=z$, $z=x$

$\Rightarrow x=y=z$

Hence proved

Q.NO.4: **If** $\dfrac{2y+2z-x}{a}=\dfrac{2z+2x-y}{b}=\dfrac{2x+2y-z}{c}$

then prove that $\dfrac{x}{2b+2c-a}=\dfrac{y}{2c+2a-b}=\dfrac{z}{2a+2b-c}$

SOL. Let $\dfrac{2y+2z-x}{a}=\dfrac{2z+2x-y}{b}=\dfrac{2x+2y-z}{c}=k$

$\Rightarrow \dfrac{2y+2z-x}{a}=k$, $\dfrac{2z+2x-y}{b}=k$,

$\dfrac{2x+2y-z}{c}=k$

$\Rightarrow 2y+2z-x=ak$ .....(i) , $2z+2x-y=bk$ ....(ii)

$\Rightarrow 2x+2y-z=ck$ ....(iii)

Eq(i) and eq(ii) multiplied by 2 and then adding

$$-2x+4y+4z=2ak$$
$$4x-2y+4z=2bk$$
$$\overline{2x+2y+8z=2ak+2bk......(iv)}$$

Eq(ii) and eq(iii) multiplied by 2 and adding

$$4x-2y+4z=2bk$$
$$4x+4y-2z=2ck$$
$$\overline{8x+2y+2z=2bk+2ck........(v)}$$

Now eq(i) and (iii) multiplied by 2 and adding

$$-2x+4y+4z=2ak$$
$$4x+4y-2z=2ck$$
$$\overline{2x+8y+2z=2ak+2ck......(vi)}$$

Subtract (iii) from (iv)

$$2x+2y+8z=2ak+2bk$$
$$\underline{\pm 2x \pm 2y \mp z = \pm ck}$$
$$9z=2ak+2bk-ck$$

$\Rightarrow \; 9z=(2a+2b-c)k$

$\Rightarrow \; \dfrac{z}{2a+2b-c}=\dfrac{k}{9}$ ....(vii)

Now (vi) – (ii)

$$2x+8y+2z=2ak+2ck$$
$$\underline{\pm 2x \mp y \pm 2z = \pm bk}$$
$$9y=2ak+2ck-bk$$

$\Rightarrow \; 9y=(2a+2c-b)k$

$\Rightarrow \; \dfrac{y}{2a+2c-b}=\dfrac{k}{9}$ ....(viii)

Now (v) - (i)

$$8x+2y+2z=2bk+2ck$$
$$\underline{\mp x \pm 2y \pm 2z = \pm ak}$$
$$9x=2bk+2ck-ak$$

$\Rightarrow \; 9x=(2b+2c-a)k$

$\Rightarrow \; \dfrac{x}{2b+2c-a}=\dfrac{k}{9}$ .....(ix)

From (vii) , (viii) and (ix) it is proved

$$\frac{x}{2b+2c-a}=\frac{y}{2c+2a-b}=\frac{z}{2a+2b-c}$$

Q.5. Prove that each of fractions in $\dfrac{x+y}{a+b}=\dfrac{y+z}{b+c}=\dfrac{z+x}{c+a}$

is equal to $\dfrac{x+y+z}{a+b+c}$

Solution: $\dfrac{x+y}{a+b}=\dfrac{y+z}{b+c}=\dfrac{z+x}{c+a} = \dfrac{x+y+z}{a+b+c}$ ...(i)

we now that $\frac{a}{b}=\frac{c}{d}=\frac{e}{f} \quad = \frac{a+c+e}{b+d+f}$ .....(ii)

Then use (ii) . Eq (i) can be written as such that

$$\Rightarrow \frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a} = \frac{x+y+y+z+z+x}{a+b+b+c+c+a}$$

$$=\frac{2x+2y+2z}{2a+2b+2c}$$

$$=\frac{2(x+y+z)}{2(a+b+c)}$$

$$=\frac{x+y+z}{a+b+c}$$

Hence proved .

Q. 6: **If** $\frac{bz+cy}{b-c}=\frac{cx+az}{c-a}=\frac{ay+bx}{a-b}$ **then**

$$(a+b+c)(x+y+z)=ax+by+cz$$

Solution: **Let** $\frac{bz+cy}{b-c}=\frac{cx+az}{c-a}=\frac{ay+bx}{a-b}=$ **k**

$$\frac{bz+cy}{b-c}=k \quad \Rightarrow \quad bz+cy=bk-ck$$

$\Rightarrow bz+cy-bk+ck=0$ **adding both sides** ax

$\Rightarrow ax+bz+cy-bk+ck=ax$ **....(i)**

Now $\frac{cx+az}{c-a}=k \quad \Rightarrow cx+az=ck-ak$

$\Rightarrow cx+az-ck+ak=0$ **adding both sides** by

$\Rightarrow by+cx+az-ck+ak=by$ **...(ii)**

Now $\frac{ay+bx}{a-b}=k \quad \Rightarrow \quad ay+bx=ak-bk$

$\Rightarrow ay+bx-ak+bk=0$ **adding both sides cz**

$\Rightarrow cz+ay+bx-ak+bk=cz$ **.....(iii)**

Adding (i) , (ii) and (iii)

$$ax+bz+cy-bk+ck+by+cx+az-ck+ak$$
$$+cz+ay+bx-ak+bk \quad =by+ax+cz$$

$=> ax+ay+az+bx+by+bz+cx+cy+cz = ax+by+cz$

$=> a(x+y+z)+b(x+y+z)+c(x+y+z) = ax+by+cz$

$=> (a+b+c)(x+y+z) = ax+by+cz$

Hence proved .

Q.7: **If** $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$

Then $(b-c)x+(c-a)y+(a-b)z = 0$

Solution: **Let** $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c} = \mathbf{k}$

Then $\frac{x}{b+c-a} = k \quad => \quad x = bk+ck-ak$

Multiply both sides by $b-c$

$(b-c)x = (b-c)(bk+ck-ak)$

$=> (b-c)x = b^2k+bck-abk-bck-c^2k+ack$

$=> (b-c)x = b^2k-abk-c^2k+ack$ **....(i)**

Now $\frac{y}{c+a-b} = k \quad => \quad y = ck+ak-bk$

Multiply both sides by $c-a$

$=> (c-a)y = (c-a)(ck+ak-bk)$

$=> (c-a)y = c^2k+ack-bck-ack-a^2k+abk$

$=> (c-a)y = c^2k-bck-a^2k+abk$ ....*(ii)*

Now $\frac{z}{a+b-c} = k \quad => \quad z = ak+bk-ck$

Multiply both sides by $a-b$

$(a-b)z = (a-b)(ak+bk-ck)$

$=> (a-b)z = a^2k+abk-ack-abk-b^2k+bck$

$=> (a-b)z = a^2k-ack-b^2k+bck$ **.....(iii)**

Adding (i) , (ii) and (iii)

$(b-c)x+(c-a)y+(a-b)z = b^2k - abk - c^2k + ack + c^2k$
$\quad - bck - a^2k + abk + a^2k - ack - b^2k + bck$

=> $(b-c)x+(c-a)y+(a-b)z = 0$ **proved .**

<u>Q. 8</u> :If $2x+3y:3y+4z:4z+5x = 4a-5b:3b-a:2b-3a$

Then $7x+6y+8z=0$

<u>Solution</u>:As $2x+3y:3y+4z:4z+5x = 4a-5b:3b-a:2b-3a$

Then $\dfrac{2x+3y}{4a-5b} = \dfrac{3y+4z}{3b-a} = \dfrac{4z+5x}{2b-3a}$

Let $\dfrac{2x+3y}{4a-5b} = \dfrac{3y+4z}{3b-a} = \dfrac{4z+5x}{2b-3a} = \mathbf{k}$

=> $\dfrac{2x+3y}{4a-5b} = k$, $\dfrac{3y+4z}{3b-a} = k$, $\dfrac{4z+5x}{2b-3a} = k$

=> $2x+3y = k(4a-5b)$....(i) , $3y+4z = k(3b-a)$.......(ii)

$4z+5x = k(2b-3a)$ ......(iii)

Adding (i) , (ii) and (iii)

=> $2x+3y+3y+4z+4z+5x = k(4a-5b)+k(3b-a)$
$\quad + k(2b-3a)$

=> $7x+6y+8z = k(4a-5b+3b-a+2b-3a)$

=> $7x+6y+8z = k(4a-4a-5b+5b)$

=> $7x+6y+8z = k(4a-4a-5b+5b)$

=> $7x+6y+8z = k(0) = 0$ **Hence proved .**

<u>Challenged</u>:

<u>Q.9</u>: If $\dfrac{a-b}{d-e} = \dfrac{b-c}{e-f}$ **, then each of them is equal to**

$\dfrac{b(f-d)+cd-af}{e(f-d)}$.

<u>Solution</u>: let $\dfrac{a-b}{d-e} = \dfrac{b-c}{e-f}$ **= k ......(i)**

=> $\dfrac{a-b}{d-e} = k$ => $a-b = dk-ek$

Multiply both sides by f.

$af - bf = dfk - efk$ .....(i)

$\frac{b-c}{e-f} = k \Rightarrow b - c = ek - fk$

Multiply both sides by d.

$bd - cd = edk - fdk$ ....(ii)

Adding (i) and (ii)

$af - bf + bd - cd = dfk - efk + edk - fdk$

$\Rightarrow -bf + af - cd + bd = -efk + edk$

$\Rightarrow -(bf - af + cd - bd) = -k(ef - ed)$

Multiplying both sides by - 1.

$(bf - af + cd - bd) = k(ef - ed)$

$\Rightarrow \frac{bf - af + cd - bd}{ef - ed} = k$

$\Rightarrow \frac{f(b-a) + cd - bd}{e(f-d)} = k$ ....(ii)

Then from (i) and (ii)

$\frac{a-b}{d-e} = \frac{b-c}{e-f} = \frac{b(f-d) + cd - af}{e(f-d)}$ **. Hence proved.**

Example: A stone is dropped from the top of a hill. The distance it falls is proportional to the square of the time of fall.

Solution: Let d is the distance fallen and t is the time of fall. Then

$d = kt^2$ ....(i) $\Rightarrow k = \frac{d}{t^2}$ .....(ii) ,

Put $d = 19.6$, $t = 2$ **in (ii)**

$k = \frac{19.6}{(2)^2} = \frac{19.6}{4} = 4.9$

Now put $t = 3$, $k = 4.9$ **in (i)**

$d = (4.9)(3)^2 = (4.9)(9)$

$\Rightarrow d = 44.1\ m$

Exercise 3.5:

Q.NO. 1. A hedge is made of wooden planks. The thickness(T) of the hedge varies directly as the number of planks. 4 planks make 12 cm thick edge. Find

(i) Thickness of the hedge when number of planks is 6

(ii) Number of planks when thickness of the hedge is 9cm.

Solution: Let the thickness of hedge = x

The number of planks = n

Since the variation is direct then $x = kn$....(i)

$$\Rightarrow k = \frac{x}{n} \quad ...(ii)$$

Put x = 12, n = 4 in eq(ii)

$$\Rightarrow k = \frac{12}{4} = 3$$

(i) Put n = 6, k = 3 in eq(i)

$\Rightarrow x = 3(6) = 18$

Thus the thickness of hedge = 18 cm

(ii) Put x = 9 and k = 3 in eq(i)

$\Rightarrow 9 = 3n$

$$\Rightarrow n = \frac{9}{3} = 3$$

Thus the number of planks = 3

Q.NO. 2 In a fountain pressure p of water at any internal point varies directly as depth d from the surface. Pressure is 51 N/cm^2 when depth is3 cm. find pressure when depth is 7 cm

Solution: Since the variation is direct

Then $P = kd$...(i)

$$\Rightarrow k = \frac{P}{d} \ ...(ii) \qquad \text{put } p = 51 \quad d = 3 \text{ in eq(ii)}$$

$$\Rightarrow k = \frac{51}{3} = 17$$

Put k = 17 d = 7 in eq(i)

=> $p = 17(7) = 119 \ N/cm^2$

Q.NO.3. Pressure of gas in a container varies directly as the temperature. When pressure is $50N/m^2$, temperature is 75^o. Find pressure when temperature is 150^o.

Solution: Let pressure of gas = P

And temperature = T as the variation is direct

Then P = kT...(i) => $k = \frac{P}{T}$ ...(ii)

Put P = 50 , T = 75 in eq(ii)

=> $k = \frac{50}{75}$ => $k = \frac{2}{3}$

Put $k = \frac{2}{3}$, T = 150 in eq (i)

$$P = \left(\frac{2}{3}\right)(150) = 2(50)$$

=> $P = 100\,N/m^2$

Q.NO. 4. If 8 persons complete a work in 10 days then how many days would 10 persons take to complete the same work

Solution: Let the number of days of work = x

And the number of persons = n

As the variation is inverse then $x \propto \frac{1}{n}$

=> $x = \frac{k}{n}$ ....(i) => k = x n ....(ii)

Put n = 8 , x = 10 in eq(ii)

=> k = 10(8) = 80

Now put n = 10 and k = 80 in eq(i)

=> $x = \frac{80}{10}$ => $x = 8$

Thus the work is completed in 8 days

Q.NO.5 Volume of gas V varies inversely as pressure p. when $P = 300N/m^2$ and $V = 4m^3$ find pressure when $V = 3m^3$

Solution: Since the variation is inverse

Then $P \propto \frac{1}{V}$ => $P = \frac{k}{V}$ .....(i) =

=> k = VP ...(ii)

Put P = 300 , V = 4 in eq(ii)

=> k = 4(300) = 1200

Now put k = 1200 and V = 3 in eq(i)

=> $P = \frac{1200}{3}$

=> $P = 400N/m^2$

Q.NO.6 Attraction force F between two magnets vary inversely as square of the distance d between them. F is 18 Newton when d is 2 cm. Find the distance when attraction force is 2 Newton.

Solution: As the variation is inversely then

=> $F \propto \frac{1}{d^2}$ => $F = \frac{k}{d^2}$ .....(i)

=> $k = Fd^2$ ....(ii) put F = 18 , d = 2 in eq(ii)

=> $k = (18)(2)^2 = (18)(4)$

=> $k = 72$

Put k = 72 , F = 2 in eq (i)

$2 = \frac{72}{d^2}$ => $2d^2 = 72$

=> $d^2 = \frac{72}{2}$ => $d^2 = 36$

=> $\sqrt{d^2} = \sqrt{36}$ => $d = 6$

Thus d = 6 cm .

Q.7: The volume of a right circular cylinder varies jointly as the height and the square of the radius . The volume of a right circular cylinder , with radius 4 cm and height 7 cm , is 352 cm^3 . Find the volume of another cylinder with radius 8cm and height 14 cm .

Solution: Let "r" be the radius and "h" be the height And "v" volume of circular cylinder . Then

$v = khr^2$ ....(i)

$\Rightarrow \quad k = \frac{v}{hr^2}$ ....(ii)

Put $v = 352$, $r = 4$, $h = 7$ in (ii) .

$k = \frac{352}{7 \times (4)^2} = \frac{352}{7 \times 16} = \frac{22}{7}$

Put $k = \frac{22}{7}$, $r = 8$, $h = 14$ in (i) .

$v = \left(\frac{22}{7}\right)(14)(8)^2 = (22)(2)(64)$

$\Rightarrow v = 2816\ cm^3$

Review Exercise 3:

Q.1: At the end of each question , four circles are given . Fill in the correct circle only .

(i): Direct variation between a and b is expressed as.

$(a)\ a = b \quad (b)\ a = \frac{1}{b} \quad (c)\ a \propto b \quad (d)\ a \propto \frac{1}{b}$

(ii) : If $m \propto \frac{1}{n}$ then

$(a)\ m = kn \quad (b)\ n = km \quad (c)\ \frac{m}{n} = k \quad (d)\ mn = k$

(iii) : Identify the item that does not have the same ratio as the other three .

$(a)\ \frac{30}{45}$ (b) 4 to 6 (c) $2:3$ (d) 3 to 2

(iv) : If $\frac{a}{b}=\frac{c}{d}$ **then by alternendo property**

$(a)\ \frac{a-b}{b}=\frac{c-d}{d}$ $(b)\ \frac{a}{a+b}=\frac{c}{c+d}$

$(c)\ \frac{a}{c}=\frac{b}{d}$ $(d)\ \frac{b}{a}=\frac{d}{c}$

(v): If $7:9::x:27$

$(a)\ x=21$ $(b)\ x=3$ $(c)\ x=7$ $(d)\ x=81$

(vi): The third proportional of x and y is

$(a)\ xy$ $(b)\ \frac{x}{y}$ $(c)\ \frac{y^2}{x}$ (d) Non of these

(vii) : If $x \propto \frac{1}{y}$ *and* $y \propto \frac{1}{z}$ *then*

$(a)\ y \propto \frac{1}{z}$ $(b)\ x \propto z$ $(c)\ xy \propto z$ $(d)\ xz \propto y$

(viii) : If $2a+1:21::4:7$ **then**

$(a)\ a=\frac{13}{2}$ $(b)\ a=\frac{11}{2}$ $(c)\ a=10$ $(d)\ a=\frac{9}{2}$

(ix) : If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$**, then each fraction is equal to**

$(a)\ \frac{la+mb+ne}{ld+mc+nf}$ $(b)\ \frac{la+mc+ne}{lb+md+nf}$

$(c)\ \frac{la+mc+ne}{lb+mc+nf}$ $(d)\ \frac{la+mb+nc}{lb+mc+nf}$

(x) Which of the following is a situation in which x directly as y ?

$(a)\ x=\frac{4}{y}$ $(b)\ xy=6$ $(c)\ x=xy$ $(d)\ x=\frac{7}{16}y$

<u>Answers</u>: (1) c (2) d (3) d (4) c (5) a (6) c (7) b

(8) b (9) b (10) d

Q.2: Find the constant of variation, when $s \propto t^2$ and t = 10 when s = 5.

Solution: Given $s \propto t^2$

$$\Rightarrow \quad s = kt^2 \qquad \Rightarrow \quad k = \frac{s}{t^2} \ ....(i)$$

Put t = 10 and s = 5 in (i)

$$\Rightarrow \quad k = \frac{5}{(10)^2} = \frac{5}{100}$$

$$\Rightarrow \quad k = \frac{1}{20}$$

Q.3: $y \propto \frac{1}{x^2}$, y = 4 when x = 3. Find the value of x when y = 9.

Solution: $y \propto \frac{1}{x^2}$ Then

$$\Rightarrow \quad y = \frac{k}{x^2} \ ....(i)$$

$$\Rightarrow \quad k = yx^2 \ ....(ii)$$

Put $x = 3$ *and* $y = 4$ in (ii)

$$\Rightarrow \quad k = (4)(3)^2 = 4(9)$$

$$\Rightarrow \quad k = 36$$

Put $y = 9$ *and* $k = 36$

$$\Rightarrow \quad 9 = \frac{36}{x^2} \qquad \Rightarrow \quad 9x^2 = 36$$

$$\Rightarrow \quad x^2 = \frac{36}{9} \qquad \Rightarrow \quad x^2 = 4$$

$$\Rightarrow \quad \sqrt{x^2} = \sqrt{4} \qquad \Rightarrow \quad x = 2$$

Q.4: Pressure of gas in a closed vessel varies directly with

temperature. If pressure is 150 unit the temperature is 70 units. What will be the pressure if temperature rises to 140 units.

Solution: Let Pressure of a gas = P

Then, temperature of gas = T.

Then $P \propto T \Rightarrow P = kT$ ......(i)

$\Rightarrow k = \dfrac{P}{T}$ ....(ii)

Put $P = 150$ and $T = 70$ in (ii)

$\Rightarrow k = \dfrac{150}{70} = \dfrac{15}{7}$

Put $T = 140$ and $k = \dfrac{15}{7}$ in (i)

$\Rightarrow P = \left(\dfrac{15}{7}\right)(140) = (15)(20)$

$\Rightarrow P = 300$ Units.

Q.5: In an electric circuit, current varies inversely as the resistance. When current is 44 amp. The resistance is 30 ohm. How much current will flow if resistance become 22 ohm.

Solution: Let Resistance = R

And Current = I

$I \propto \dfrac{1}{R}$

$\Rightarrow I = \dfrac{k}{R}$ ....(i)

$\Rightarrow k = IR$ ....(ii)

Put $R = 30$ and $I = 44$ in(ii)

$\Rightarrow k = (44)(30) = 1320$

put $k = 1320$ and $R = 22$ in(i)

$\Rightarrow I = \dfrac{1320}{22} = 60$

=> $I = 60 \ Amp.$

Note: Book answer is not correct.

Q.6: If a varies jointly as b and the square root of c, and $a = 21$ when $b = 5$ and $c = 36$, find a when $b = 9$ and $c = 225$.

Solution: a varies jointly as b and the square root of c.

Then $a = kb\sqrt{c}$ ....(i)

=> $k = \dfrac{a}{b\sqrt{c}}$ ....(ii)

Put $a = 21$, $b = 5$ **and** $c = 36$ **in (ii)**

$$k = \frac{21}{(5)(\sqrt{36})} = \frac{21}{5 \times 6} = \frac{7}{5 \times 2}$$

=> $k = \dfrac{7}{10}$

Put $b = 9$, $c = 225$, $k = \dfrac{7}{10}$ **in (i)**

$$a = \left(\frac{7}{10}\right)(9)(\sqrt{225}) = \left(\frac{7}{10}\right)(9)(15)$$

=> $a = \left(\dfrac{7}{2}\right)(9)(3) = \dfrac{189}{2}$

Q. 7 : What number should be added to each of the numbers 3 , 8 , 11 and 20 to make them in proportion.

Solution: Let x be the number to make 3 , 8 , 11 and 20 in proportion. Then

$x+3$, $x+8$, $x+11$, $x+20$ are in proportion.

=> $\dfrac{x+3}{x+8} = \dfrac{x+11}{x+20}$

=> $(x+3)(x+20) = (x+11)(x+8)$

=> $x^2 + 20x + 3x + 60 = x^2 + 8x + 11x + 88$

=> $x^2 - x^2 + 23x - 19x = 88 - 60$

=> $4x = 28$

=> $x = \frac{28}{4} = 7$

Thus required number = 7 .

Q.8: What number must be subtracted from each of 6 , 8 ,7 and 11 so that the remaining numbers are in Proportion .

Solution: Let x be the number to subtracted from each of 6 , 8 , 7 and 11 to make them in proportion .

Then $6-x$, $8-x$, $7-x$ $11-x$ *are in proportion.*

Then $\frac{6-x}{8-x} = \frac{7-x}{11-x}$

=> $(6-x)(11-x) = (7-x)(8-x)$

=> $66 - 6x - 11x + x^2 = 56 - 7x - 8x + x^2$

=> $x^2 - x^2 - 17x + 7x + 8x = 56 - 66$

=> $-2x = -10$

=> $x = \frac{-10}{-2} = 5$

Thus required number = 5

Q.9: The ratio between two numbers is 8 : 3 and their difference is 20 . Find the numbers .

Solution: Let x *and* y are two numbers .

Then according to given conditions are

=> $x : y = 8 : 3$

As Pr*oduct of extrimities* = Pr*oduct of mean*

=> $3x = 8y$

=> $3x - 8y = 0$ ......(*i*)

And $x - y = 20$ ....(*ii*)

Eq(*ii*) *is multiplied by* 8

$8x - 8y = 160$ ......(*iii*)

Subtracted (i) from (iii)

$$8x - 8y = 160$$
$$\pm 3x \mp 8y = \pm 0$$
$$5x = 160$$

=> $x = \frac{160}{5}$

=> $x = 32$

Put $x = 32$ $in(ii)$

$32 - y = 20$

=> $32 - 20 = y$

=> $y = 12$

Thus required numbers are 32 and 12 have ratio 8 : 3 and their difference is 20 .

Note: The book answer is wrong .

Q.10: Find three numbers in continued proportion such that their sum is 14 and sum of their squares is 84 .

Solution: Let x , y *and* z are three numbers which are in continued proportion .

Then according to the given conditions

$x + y + z = 14$ ....(i) *and*

$x^2 + y^2 + z^2 = 84$ ....(ii)

Since x , y *and* z are in continued ratios . Then

$\frac{x}{y} = \frac{y}{z}$ => $y^2 = xz$ ....(iii)

Put (iii) in (ii) => $x^2 + xz + z^2 = 84$

=> $x^2 + z^2 + xz + xz - xz = 84$ adding and subtract xz .

=> $x^2 + z^2 + 2xz - xz = 84$

=> $(x+z)^2 - xz = 84$ ......(iv)

From eq(i) => $x + z = 14 - y$ ......(v)

Put (v) in (iv)

$(14 - y)^2 - xz = 84$

=> $196 + y^2 - 28y - xz = 84$ .....(vi)

Subtract eq(iii) from eq(vi)

$$196 + y^2 - 28y - xz = 84$$
$$\pm y^2 \qquad \mp xz = 0$$
$$196 - 28y = 84$$

=> $196 - 84 = 28y$

=> $28y = 112$ => $y = \frac{112}{28}$

=> $y = 4$

Put $y = 4$ in eq(i)

$x + 4 + z = 14$ => $x + z = 14 - 4$

=> $x + z = 10$ ....(vii)

Put $y = 4$ in eq (3)

$(4)^2 = xz$ => $xz = 16$ ....(viii)

From eq(vii) => $z = 10 - x$ .....(ix)

Put eq(ix) in eq (viii)

$x(10 - x) = 16$

=> $10x - x^2 = 16$

=> $0 = x^2 - 10x + 16$

=> $x^2 - 10x + 16 = 0$

=> $x^2 - 2x - 8x + 16 = 0$

=> $x(x - 2) - 8(x - 2) = 0$

=> $(x - 2)(x - 8) = 0$

=> $x - 2 = 0$ *or* $x - 8 = 0$

=> $x = 2$ *or* $x = 8$

Put $x = 2$ in (ix) => $z = 10 - 2 = 8$

Put $x = 8$ in (ix) => $z = 10 - 8 = 2$

Thus required numbers are 8 , 4 , 2

Or 2 , 4 , 8 .

Q.11: The mean proportional between two numbers is 6 and their mean is 13 .Find the numbers.

Solution: Let x *and* y are two numbers whose mean is 6 .

Then $x, 6, y$ are in continued proportion.

$$\frac{x}{6} = \frac{6}{y} \quad \Rightarrow \quad xy = 36 \text{ .....}(i)$$

Also $x + y = 13$ ......(ii)

Now $(x - y)^2 = (x + y)^2 - 4xy$ ....(iii)

Put (i) and (ii) in (iii).

$$(x - y)^2 = (13)^2 - 4(36)$$

$$\Rightarrow (x - y)^2 = 169 - 144$$

$$\Rightarrow (x - y)^2 = 25 \quad \Rightarrow \quad \sqrt{(x - y)^2} = \sqrt{25}$$

$$\Rightarrow x - y = 5 \text{ ....}(iv)$$

$$x + y = 13$$

$$\underline{x - y = 5}$$

$$2x = 18$$

$$\Rightarrow x = 9$$

Put x = 9 in (ii)

$$9 + y = 13 \quad \Rightarrow \quad y = 13 - 9 = 4$$

Thus x = 9 and y = 4.

Q.12: Find the angles of a triangle which are in ratio 3 : 4 : 5.

Solution: Let A, B, C are the angles of triangle.

Since we know that sum of the angles of a triangle is 180^o.

$$\Rightarrow A + B + C = 180^o \text{ ....}(i) \quad \text{and}$$

$$A : B : C = 3 : 4 : 5$$

Now sum of the ratios = $3 + 4 + 5 = 12$

$$\text{Angle A} = \left(\frac{3}{12}\right)(180)$$

$$= (3)(15) = 45^o$$

$$Angle\ B = \left(\frac{4}{12}\right)(180)$$

$$= (4)(15) = 60^o$$

$$Angle\ C = \left(\frac{5}{12}\right)(180)$$

$$= (5)(15) = 75^o$$

Thus angles of triangle are 45^o, 60^o, 75^o.

Q.13: If $\frac{a}{b} = \frac{c}{d}$, then prove that $\frac{ac(a+c)}{bd(b+d)} = \frac{(a+c)^3}{(b+d)^3}$

Solution: As $\frac{a}{b} = \frac{c}{d}$ then we know that

$$If\ \frac{a}{b} = \frac{c}{d}\ \ then\ \ \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} \ldots\ldots(i)$$

$$Then\ \ \frac{a}{b} = \frac{a+c}{b+d} \ldots\ldots.(ii)$$ eq (ii) is multiplied by $\frac{c}{d}$

$$=> \frac{ac}{bd} = \left(\frac{c}{d}\right)\left(\frac{a+c}{b+d}\right) \ldots\ldots(iii)$$

Put in the R . H . S. in (iii) the value of $\frac{c}{d}$ from (i)

$$=> \frac{ac}{bd} = \left(\frac{a+c}{b+d}\right)\left(\frac{a+c}{b+d}\right).$$

Both sides multiplied by $\left(\frac{a+c}{b+d}\right)$

$$=> \frac{ac}{bd}\left(\frac{a+c}{b+d}\right) = \left(\frac{a+c}{b+d}\right)\left(\frac{a+c}{b+d}\right)\left(\frac{a+c}{b+d}\right)$$

$$=> \frac{ac(a+c)}{bd(b+d)} = \frac{(a+c)^3}{(b+d)^3}$$

Q.14:If a , b ,c are in continued proportion then prove

that $\frac{a}{c}=\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{a^2-b^2}{b^2-c^2}$

<u>**Solution**</u>: Since a, b, c are in continued proportion then

$$\frac{a}{b}=\frac{b}{c} \quad => \quad ac=b^2$$

$$=> \quad \frac{ac}{c^2}=\frac{b^2}{c^2} \quad => \quad \frac{a}{c}=\frac{b^2}{c^2} \;.....(i)$$

$$Let \quad \frac{a}{b}=\frac{b}{c}=k \;...(ii)$$

$$=> \quad \frac{a}{b}=k \quad , \quad \frac{b}{c}=k$$

$$=> \quad a=bk......(iii) \quad , \quad b=ck......(iv)$$

Put (iv) in (iii).

$$=> \quad a=ck.k \quad => a=ck^2 \;....(v)$$

$$Now \quad \frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{(ck^2)^2+(ck^2)(ck)+(ck)^2}{(ck)^2+(ck)c+c^2}$$

$$=\frac{c^2k^4+c^2k^3+c^2k^2}{c^2k^2+c^2k+c^2}$$

$$=\frac{c^2k^2(k^2+k+1)}{c^2(k^2+k+1)}$$

$$=> \quad \frac{a^2+ab+b^2}{b^2+bc+c^2}=k^2.....(vi)$$

$$Now \quad \frac{a^2-b^2}{b^2-c^2}=\frac{(ck^2)^2-(ck)^2}{(ck)^2-c^2}$$

$$=\frac{c^2k^4-c^2k^2}{c^2k^2-c^2}$$

$$=\frac{c^2k^2(k^2-1)}{c^2(k^2-1)}$$

$$=> \frac{a^2-b^2}{b^2-c^2}=k^2 \;.....(vii)$$

Now from (v) $\frac{a}{c}=k^2 \;.....(viii)$

From (vi) , (vii) and (viii)

$$\frac{a}{c}=\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{a^2-b^2}{b^2-c^2}$$

<u>Q.15</u>: If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ **, then prove that** $\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{ace}{bdf}$

<u>Solution</u>: *Let* $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k \;.......(i)$

$$=> a=bk \;,\; c=dk \;,\; e=fk$$

Take L. H. S.

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{(bk)^3+(dk)^3+(fk)^3}{b^3+d^3+f^3}$$

$$=\frac{k^3(b^3+d^3+f^3)}{b^3+d^3+f^3}$$

$$=> \frac{a^3+c^3+e^3}{b^3+d^3+f^3}=k^3 \;......(ii)$$

$$\frac{a}{b}=k \;,\; \frac{c}{d}=k \;,\; \frac{e}{f}=k$$

From (i) => $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)\left(\frac{e}{f}\right)=(k)(k)(k)$

$$=> \frac{ace}{bdf}=k^3 \;.......(iii)$$

From (ii) and (iii)

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{ace}{bdf}$$

Chapter 4:

Partial Fraction:

Proper Rational fraction: A rational fraction $\frac{P(x)}{Q(x)}, Q(x) \neq 0$ is a proper fraction if the degree of numerator P(x) is less than the degree of denominator Q(x). For example

$$\frac{x}{2x^2+1}, \quad \frac{1}{x+1}$$

Improper Rational fraction: A rational fraction $\frac{P(x)}{Q(x)}, Q(x) \neq 0$ is a improper fraction if the degree of numerator P(x) is greater or equal to the degree of denominator Q(x). For example

$$\frac{x^2}{3x^2+x+2}, \quad \frac{x^2}{2x+1}$$

Note: Improper rational fraction P(x) is divided by Q(x) then a polynomial and proper rational fraction are obtained.

As $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$ Where P(x) is dividend, D(x) is divisor. Q(x) is quotient and R(x) is remainder.

Partial fraction: A procedure in which splitting up a fraction into two or more fractions with only one factor of denominator is called Partial fraction.

Case 1: When denominator Q(x) consist of non- repeated linear factors. If $\frac{P(x)}{(x+a)(x+b)}$ then partial fraction is

$$\frac{P(x)}{(x+a)(x+b)} = \frac{A}{x+a} + \frac{B}{x+b}$$ where A and B are numbers to be determined.

Example 1: Resolve $\frac{1}{(x+1)(x+2)}$ into partial fraction.

Solution: Let

$$\frac{1}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2} \text{ ......}(i)$$

$$=> \frac{1}{(x+1)(x+2)}=\frac{A(x+2)+B(x+1)}{(x+1)(x+2)}$$

Both sides multiplied by $(x+1)(x+2)$

=> $1=A(x+2)+B(x+1)$ ......(ii) put x = -1 in (ii)

$1=A(-1+2)+B(-1+1)$

=> $1=A(1)+B(0)$ => $1=A$

=> $A=1$

Put x = -2 in (ii)

$1=A(-2+2)+B(-2+1)$

=> $1=A(0)+B(-1)$

=> $1=0-B$ => $1=-B$

=> $B=-1$

Put A = 1 and B = -1 in (i)

$$\frac{1}{(x+1)(x+2)}=\frac{1}{x+1}+\frac{-1}{x+2}$$

$$=> \frac{1}{(x+1)(x+2)}=\frac{1}{x+1}-\frac{1}{x+2}$$

These are the required partial fraction .

Case ii: When denominator contains repeated and linear factors . i.e. $\frac{P(x)}{(x+a)^2(x+b)}$ then partial fraction is

$$\frac{P(x)}{(x+a)^2(x+b)}=\frac{A}{x+a}+\frac{B}{(x+a)^2}+\frac{C}{x+b}$$

Example: Find partial fraction of $\frac{x}{(x+1)^2}$

Solution: Let $\frac{x}{(x+1)^2}=\frac{A}{x+1}+\frac{B}{(x+1)^2}$ ......(i)

$$\frac{x}{(x+1)^2}=\frac{A(x+1)+B}{(x+1)^2}$$

Both sides multiplied by $(x+1)^2$

$\Rightarrow \quad x=A(x+1)+B$ ......(ii)

$\Rightarrow \quad x=Ax+A+B$ ....(iii)

If we put x = - 1 in (ii) we find B

$-1=A(-1+1)+B$

$\Rightarrow \quad -1=A(0)+B \qquad \Rightarrow \quad -1=0+B$

$\Rightarrow \quad B=-1$

Now comparing the coefficients of x from (iii) we find A .

$\Rightarrow 1=A \qquad \Rightarrow A=1$

Put A = 1 and B = - 1 in (i)

$$\frac{x}{(x+1)^2}=\frac{1}{x+1}+\frac{-1}{(x+1)^2}$$

$$\Rightarrow \frac{x}{(x+1)^2}=\frac{1}{x+1}-\frac{1}{(x+1)^2}$$

Exercise 4.1:

Resolve the following rational fractions into partial fraction.

Q.1: $\dfrac{3x-2}{2x^2-x}$

Solution: $\dfrac{3x-2}{2x^2-x}=\dfrac{3x-2}{x(2x-1)}$

Let $\dfrac{3x-2}{x(2x-1)}=\dfrac{A}{x}+\dfrac{B}{2x-1}$ ....(i)

$$\Rightarrow \frac{3x-2}{x(2x-1)}=\frac{A(2x-1)+Bx}{x(2x-1)}$$

Both sides multiplied by $x(2x-1)$

$\Rightarrow 3x-2=A(2x-1)+Bx$ ....(ii)

Put x = 0 in (ii)

$$3(0)-2=A(2(0)-1)+B(0)$$

$$\Rightarrow \quad 0-2=A(0-1)+0$$

$$\Rightarrow \quad -2=-A \qquad \Rightarrow \quad A=2$$

Put $x=\frac{1}{2}$ in (ii)

$$3\left(\frac{1}{2}\right)-2=A\left(2(\frac{1}{2})-1\right)+B\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \frac{3}{2}-2=A(1-1)+\frac{1}{2}B$$

$$\Rightarrow \quad \frac{3-4}{2}=A(0)+\frac{1}{2}B$$

$$\Rightarrow \quad -\frac{1}{2}=\frac{1}{2}B \qquad \Rightarrow \quad B=-1$$

Put A = 2 and B = - 1 in (i)

$$\frac{3x-2}{2x^2-x}=\frac{2}{x}+\frac{-1}{2x-1}$$

$$\Rightarrow \quad \frac{3x-2}{2x^2-x}=\frac{2}{x}-\frac{1}{2x-1}$$

Q.2: $\frac{x-1}{x^2+6x+5}$

Solution: first we factorize denominator .

$$\frac{x-1}{x^2+6x+5}=\frac{x-1}{x^2+x+5x+5}$$

$$\Rightarrow \quad \frac{x-1}{x^2+6x+5}=\frac{x-1}{x(x+1)+5(x+1)}$$

$$=\frac{x-1}{(x+1)(x+5)}$$

Let $\frac{x-1}{(x+1)(x+5)}=\frac{A}{x+1}+\frac{B}{x+5}$ ....(i)

=> $\frac{x-1}{(x+1)(x+5)}=\frac{A(x+5)+B(x+1)}{(x+1)(x+5)}$

Both sides multiplied by $(x+1)(x+5)$

$x-1=A(x+5)+B(x+1)$ ....(ii)

Put x = -1 in (ii)

$-1-1=A(-1+5)+B(-1+1)$

=> $-2=A(4)+B(0)$

=> $-2=4A+0$ => $-2=4A$

=> $A=-\frac{2}{4}=-\frac{1}{2}$

Put x = -5 in (ii)

$-5-1=A(-5+5)+B(-5+1)$

=> $-6=A(0)+B(-4)$

=> $-6=0-4B$ => $-6=-4B$

=> $B=\frac{-6}{-4}=\frac{3}{2}$

Put $A=-\frac{1}{2}$ *and* $B=\frac{3}{2}$ in (i)

$$\frac{x-1}{(x+1)(x+5)}=\frac{-\frac{1}{2}}{x+1}+\frac{\frac{3}{2}}{x+5}$$

=> $\frac{x-1}{x^2+5x+x+5}=-\frac{1}{2(x+1)}+\frac{3}{2(x+5)}$

=> $\frac{x-1}{x^2+6x+5}=\frac{3}{2(x+5)}-\frac{1}{2(x+1)}$

Q.3: $\frac{1}{x^2-1}$

Solution: $\frac{1}{x^2-1}=\frac{1}{(x-1)(x+1)}$.

Let $\frac{1}{(x-1)(x+1)}=\frac{A}{x-1}+\frac{B}{x+1}$ ....(i)

Multiplied eq(i) both sides by $(x-1)(x+1)$.

$1= A(x+1)+B(x-1)$ ....(ii)

Put $x=1$ in (ii) => $1= A(1+1)+B(1-1)$

=> $1=2A+0$ => $A=\frac{1}{2}$

Put $x=-1$ in (ii) .

$1=A(-1+1)+B(-1-1)$

=> $1=0-2B$ => $1=-2B$

=> $B=-\frac{1}{2}$

Put values of A and B in (i).

$$\frac{1}{(x-1)(x+1)}=\frac{\frac{1}{2}}{x-1}+\frac{-\frac{1}{2}}{x+1}$$

$$=\frac{1}{2}\left(\frac{1}{x-1}-\frac{1}{x+1}\right)$$

$$=\frac{1}{2(x-1)}-\frac{1}{2(x+1)} \quad Ans.$$

Q.4: $\frac{x}{x^2+4x-5}$

Solution: $\frac{x}{x^2+4x-5}=\frac{x}{x^2-x+5x-5}$

$$=\frac{x}{x(x-1)+5(x-1)}$$

$$=\frac{x}{(x-1)(x+5)}$$

Let $\frac{x}{(x-1)(x+5)}=\frac{A}{x-1}+\frac{B}{x+5}$ ....(ii)

Eq (i) is multiplied by $(x-1)(x+5)$

$x=A(x+5)+B(x-1)$ ....(ii)

Put $x=1$ **in (ii)** => $1=A(1+5)+B(1-1)$

=> $1=6A+0$ => $A=\frac{1}{6}$

Put $x=-5$ **in (ii)** .

$-5=A(-5+5)+B(-5-1)$

=> $-5=0-6B$ => $-5=-6B$

=> $B=\frac{5}{6}$

Put values of A and B in (i) .

$$\frac{x}{(x-1)(x+5)}=\frac{\frac{1}{6}}{x-1}+\frac{\frac{5}{6}}{x+5}$$

$$=\frac{1}{6}\left(\frac{1}{x-1}+\frac{5}{x+6}\right)$$

$$=\frac{1}{6(x-1)}+\frac{5}{6(x+1)} \quad Ans.$$

<u>**Q.5**</u>: $\frac{4x+2}{(x+2)(2x-1)}$

<u>**Solution**</u> : **Let** $\frac{4x+2}{(x+2)(2x-1)}=\frac{A}{x+2}+\frac{B}{2x-1}$ .....(i)

Eq (i) is multiplied by $(x+2)(2x-1)$

$4x+2=A(2x-1)+B(x+2)$ ....(ii)

Put $x=-2$ **in (ii)** .

$4(-2)+2=A(2(-2)-1)+B(-2+2)$

=> $-8+2=A(-4-1)+0$

=> $-6=-5A$

=> $A=\frac{6}{5}$.

Put $x=\frac{1}{2}$ **in (ii)**.

$$4\left(\frac{1}{2}\right)+2=A\left(2\left(\frac{1}{2}\right)-1\right)+B\left(\frac{1}{2}+2\right)$$

=> $2+2=A(1-1)+B\left(\frac{1+4}{2}\right)$

=> $4=0+\frac{5}{2}B$ => $4=\frac{5}{2}B$

=> $B=\frac{8}{5}$

Put values of A and B in(i)

$$\frac{4x+2}{(x+2)(2x-1)}=\frac{\frac{6}{5}}{x+2}+\frac{\frac{8}{5}}{2x-1}$$

$$=\frac{6}{5(x+2)}+\frac{8}{5(2x-1)}\quad Ans.$$

<u>Q.6</u>: $\frac{x^2+5x+3}{(x^2-1)(x+1)}$

<u>Solution</u>: $\frac{x^2+5x+3}{(x^2-1)(x+1)}=\frac{x^2+5x+3}{(x-1)(x+1)(x+1)}$

=> $\frac{x^2+5x+3}{(x^2-1)(x+1)}=\frac{x^2+5x+3}{(x-1)(x+1)^2}$

Let $\frac{x^2+5x+3}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$ ....(i)

=> $\frac{x^2+5x+3}{(x-1)(x+1)^2}=\frac{A(x+1)^2+B(x-1)(x+1)+C(x-1)}{(x-1)(x+1)^2}$

Both sides multiplied by $(x-1)(x+1)^2$

$$x^2+5x+3=A(x+1)^2+B(x-1)(x+i)+C(x-1)\;....(ii)$$

$$=> x^2+5x+3=A(x^2+2x+1)+B(x^2-1)+C(x-1)$$

$$=> \quad x^2+5x+3=Ax^2+2Ax+A+Bx^2-B+Cx-C$$

$$=> \quad x^2+5x+3=(A+B)x^2+(2A+C)x+A-B-C\;....(iii)$$

Put x = 1 in (ii)

$$(1)^2+5(1)+3=A(1+1)^2+B(1-1)(1+1)+C(1-1)$$

$$=> 1+5+3=A(2)^2+B(0)(2)+C(0)$$

$$=> \; 9=A(4)+0+0 \qquad => \; 9=4A$$

$$=> \; A=\frac{9}{4}$$

Put x = - 1 in (ii)

$$(-1)^2+5(-1)+3=A(-1+1)^2+B(-1-1)(-1+1)+C(-1-1)$$

$$=> \; 1-5+3=A(0)+B(-2)(0)+C(-2)$$

$$=> \; -1=0+0-2C \qquad => \; -1=-2C$$

$$=> \quad C=\frac{-1}{-2}=\frac{1}{2}$$

Now we find B by comparing the coefficients of x^2 from (iii)

$$1=A+B\;.......(iv)$$

Put $A=\frac{9}{4}$ in (iv)

$$1=\frac{9}{4}+B \qquad => \; 1-\frac{9}{4}=B$$

$$=> \; B=\frac{4-9}{4}=-\frac{5}{4}$$

Put $A=\frac{9}{4}$ $,B=-\frac{5}{4}$ *and* $C=\frac{1}{2}$ *in*(i)

$$\frac{x^2+5x+3}{(x-1)(x+1)^2}=\frac{\frac{9}{4}}{x-1}+\frac{-\frac{5}{4}}{x+1}+\frac{\frac{1}{2}}{(x+1)^2}$$

$$\frac{x^2+5x+3}{(x-1)(x+1)(x+1)}=\frac{9}{4(x-1)}-\frac{5}{4(x+1)}+\frac{1}{2(x+1)^2}$$

$$\Rightarrow \frac{x^2+5x+3}{(x^2-1)(x+1)}=\frac{9}{4(x-1)}-\frac{5}{4(x+1)}+\frac{1}{2(x+1)^2}$$

Q.7: $\frac{x^2+2}{(x+2)(x^2+5x+6)}$

Solution: $\frac{x^2+2}{(x+2)(x^2+5x+6)}=\frac{x^2+2}{(x+2)(x^2+2x+3x+6)}$

$$\Rightarrow \frac{x^2+2}{(x+2)(x^2+5x+6)}=\frac{x^2+2}{(x+2)[x(x+2)+3(x+2)]}$$

$$\Rightarrow \frac{x^2+2}{(x+2)(x^2+5x+6)}=\frac{x^2+2}{(x+2)(x+2)(x+3)}$$

$$\Rightarrow \frac{x^2+2}{(x+2)(x^2+5x+6)}=\frac{x^2+2}{(x+2)^2(x+3)}$$

$$Let\ \frac{x^2+2}{(x+2)^2(x+3)}=\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{x+3}\ \ldots\ldots(i)$$

$$\Rightarrow \frac{x^2+2}{(x+2)^2(x+3)}=\frac{A(x+2)(x+3)+B(x+3)+C(x+2)^2}{(x+2)^2(x+3)}$$

Both sides multiplied by $(x+2)^2(x+3)$

$\Rightarrow x^2+2=A(x+2)(x+3)+B(x+3)+C(x+2)^2 \ldots(ii)$

$\Rightarrow x^2+2=A(x^2+3x+2x+6)+B(x+3)+C(x^2+4x+4)$

$\Rightarrow x^2+2=A(x^2+5x+6)+B(x+3)+C(x^2+4x+4)$

$$=> \ x^2+2=Ax^2+5Ax+6A+Bx+3B+Cx^2+4Cx+4C$$

$$x^2+0.x+2=(A+C)x^2+(5A+B+4C)x$$
$$+(6A+3B+4C)\text{.........................}(iii)$$

Put x = -2 in (ii)

$$(-2)^2+2=A(-2+2)(-2+3)+B(-2+3)+C(-2+2)^2$$

$$=> 4+2=A(0)(1)+B(1)+C(0)$$

$$=> \ 6=0+B+0 \qquad => \ B=6$$

Put x = -3 in (ii)

$$(-3)^2+2=A(-3+2)(-3+3)+B(-3+3)+C(-3+2)^2$$

$$=> 9+2=A(-1)(0)+B(0)+C(-1)^2$$

$$=> 11=0+0+C \qquad => \ C=11$$

To find A we compare the coefficients of x^2 from (iii)

$$1=A+C \ \text{.....}(iv) \quad \text{put } C=11 \text{ in (iv)}$$

$$1=A+11 \qquad => \ 1-11=A$$

$$=> \ A=-10$$

Put A = -10 , B = 6 and C = 11 in(i)

$$\frac{x^2+2}{(x+2)^2(x+3)}=\frac{-10}{x+2}+\frac{6}{(x+2)^2}+\frac{11}{x+3}$$

$$=> \ \frac{x^2+2}{(x+2)^2(x+3)}=\frac{11}{x+3}-\frac{10}{x+2}+\frac{6}{(x+2)^2}$$

Q.8: $\frac{2x-1}{x(x-3)^2}$

Solution: *Let* $\frac{2x-1}{x(x-3)^2}=\frac{A}{x}+\frac{B}{x-3}+\frac{C}{(x-3)^2}$ .....(*i*)

$$=> \frac{2x-1}{x(x-3)^2}=\frac{A(x-3)^2+Bx(x-3)+Cx}{x(x-3)^2}$$

Both sides multiplied by $x(x-3)^2$

$$\Rightarrow 2x-1 = A(x-3)^2 + Bx(x-3) + Cx \;......(ii)$$

$$\Rightarrow 2x-1 = A(x^2-6x+9) + B(x^2-3x) + Cx$$

$$\Rightarrow 2x-1 = Ax^2 - 6Ax + 9A + Bx^2 - 3Bx + Cx$$

$$\Rightarrow 0.x^2 + 2x - 1 = (A+B)x^2 + (-6A-3B+C)x + 9A \;......(iii)$$

Put x = 0 in (ii)

$$2(0)-1 = A(0-3)^2 + B(0)(0-3) + C(0)$$

$$\Rightarrow 0-1 = A(9)+0=0 \quad \Rightarrow -1 = 9A$$

$$\Rightarrow A = -\frac{1}{9}$$

Put x = 3 in (ii)

$$2(3)-1 = A(3-3)^2 + B(3)(3-3) + C(3)$$

$$\Rightarrow 6-1 = A(0) + B(3)(0) + 3C$$

$$\Rightarrow 5 = 0+0+3C \quad \Rightarrow 5 = 3C$$

$$\Rightarrow C = \frac{5}{3}$$

To find the value of B we compare the coefficients of x^2 of eq(iii)

$$0 = A + B \;.....(iv)$$

Put $A = -\frac{1}{9}$ in(iv)

$$0 = -\frac{1}{9} + B \quad \Rightarrow B = \frac{1}{9}$$

Put values of A , B and C in(i) .

$$\frac{2x-1}{x(x-3)^2} = \frac{-\frac{1}{9}}{x} + \frac{\frac{1}{9}}{x-3} + \frac{\frac{5}{3}}{(x-3)^2}$$

$$\Rightarrow \frac{2x-1}{x(x-3)^2} = -\frac{1}{9x} + \frac{1}{9(x-3)} + \frac{5}{3(x-3)^2}$$

Q.9: $\dfrac{x^2}{x^2+2x+1}$.

Solution: **It is improper fraction .**

$$x^2+2x+1 \overline{)\ x^2\ }\quad 1$$
$$\pm x^2 \pm 2x \pm 1$$
$$-2x-1$$

Then $\dfrac{x^2}{x^2+2x+1} = 1-\dfrac{2x+1}{x^2+2x+1}$

$= 1-\dfrac{2x+1}{(x+1)^2}$ ......(i)

Let $\dfrac{2x+1}{(x+1)^2} = \dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}$ ......(ii)

Both sides multiplied by $(x+1)^2$.

$2x+1 = A(x+1)+B$ ......(iii)

put $x=-1$ **in (iii) .**

$2(-1)+1 = A(-1+1)+B$

$\Rightarrow -2+1 = 0+B$

$\Rightarrow -1 = B \quad \Rightarrow B=-1$

From (iii) $\Rightarrow 2x+1 = Ax+A+B$ .....(iv)

Comparing the coefficient of x^2 **from eq(iv) .**

$A=2$

Put values of A and B in(ii)

$\dfrac{2x+1}{(x+1)^2} = \dfrac{2}{x+1}+\dfrac{-1}{(x+1)^2}$

$= \dfrac{2}{x+1}-\dfrac{1}{(x+1)^2}$ ....(v)

Put (v) in (i)

$\dfrac{x^2}{x^2+2x+1} = 1-\left[\dfrac{2}{x+1}-\dfrac{1}{(x+1)^2}\right]$

$$=1-\frac{2}{x+1}+\frac{1}{(x+1)^2} \quad \textbf{Ans.}$$

Q.10: $\dfrac{x^2}{(x-1)^2(x+1)}$

Solution: let $\dfrac{x^2}{(x-1)^2(x+1)}=\dfrac{A}{x+1}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$ ...(i)

Eq (i) is multiplied by $(x-1)^2(x+1)$.

$$x^2 = A(x-1)^2+B(x-1)(x+1)+C(x+1) \quad \text{......(ii)}$$

Put $x=1$ **in (ii)**

$$(1)^2 = A(1-1)^2+B(1-1)(1+1)+C(1+1)$$

$$\Rightarrow \quad 1=0+0+C(2)$$

$$\Rightarrow \quad 1=2C \quad \Rightarrow \quad C=\frac{1}{2}$$

Put $x=-1$ **in (ii).**

$$(-1)^2 = A(-1-1)^2+B(-1-1)(-1+1)+C(-1+1)$$

$$\Rightarrow \quad 1=(-2)^2A+0+0 \quad \Rightarrow \quad 1=4A$$

$$\Rightarrow \quad A=\frac{1}{4}$$

Now from (ii).

$$x^2 = A(x^2-2x+1)+B(x^2-1)+C(x+1)$$

$$\Rightarrow \quad x^2 = Ax^2-2Ax+A+Bx^2-B+Cx+C$$

$$\Rightarrow \quad x^2 = (A+B)x^2+(-2A+C)x+A-B+C \text{.......}(iii)$$

Comparing the coefficient of x^2 **.**

$$1=A+B \quad \textbf{Put} \quad A=\frac{1}{4}$$

$$\Rightarrow \quad 1=\frac{1}{4}+B \quad \Rightarrow \quad 1-\frac{1}{4}=B$$

$$\Rightarrow \quad B=\frac{4-1}{4} \quad \Rightarrow \quad B=\frac{3}{4}$$

Put values of A , B and C in (i) .

$$\frac{x^2}{(x-1)^2(x+1)}=\frac{\frac{1}{4}}{x+1}+\frac{\frac{3}{4}}{x-1}+\frac{\frac{1}{2}}{(x-1)^2}$$

$$=\frac{1}{4(x+1)}+\frac{3}{4(x-1)}+\frac{1}{2(x-1)^2} \quad \text{Ans.}$$

Case iii: When denominator consists of non –repeated Quadratic factors . i.e . $\frac{P(x)}{(x^2+a)(x^2+b)}$

Then partial fraction is

$$\frac{P(x)}{(x^2+a)(x^2+b)}=\frac{Ax+B}{x^2+a}+\frac{Cx+D}{x^2+b}$$

Where A , B , C , D are numbers to be determined .

Case iv: When denominator contains quadratic and repeated Factors . i.e.

$$\frac{P(x)}{(x^2+a)^2} .$$

Then partial fraction is

$$\frac{P(x)}{(x^2+a)^2}=\frac{Ax+B}{x^2+a}+\frac{Cx+D}{(x^2+a)^2}$$

Exercise 4.2:

Resolve the following fractions into partial fractions.

Q.1: $\frac{1}{x(x^2+1)}$

Solution: *Let* $\frac{1}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$ ......(*i*)

$$=> \frac{1}{x(x^2+1)}=\frac{A(x^2+1)+(Bx+C)x}{x(x^2+1)}$$

Both sides multiplied by $x(x^2+1)$

$$1 = A(x^2+1)+(Bx+C)x \quad ....(ii)$$

$$\Rightarrow 1 = Ax^2 + A + Bx^2 + Cx$$

$$\Rightarrow 0.x^2 + 0.x + 1 = (A+B)x^2 + Cx + A \quad .....(iii)$$

Put x = 0 in (ii)

$$1 = A(0+1)+(B(0)+C)(0)$$

$$\Rightarrow 1 = A + 0 \qquad \Rightarrow A = 1$$

To find B and C we compare the coefficients of x^2, x

From eq(iii).

$$0 = A + B \quad ......(iv)$$

$$0 = C \quad ....(v)$$

Put A = 1 in (iv)

$$0 = 1 + B \qquad \Rightarrow B = -1$$

Put A = 1, B = -1 and C = 0 in (i)

$$\frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-1.x+0}{x^2+1}$$

$$\Rightarrow \frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$$

Q.2: $\frac{x^2+3x+1}{(x-1)(x^2+3)}$

Solution: Let $\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+3} \quad .....(i)$

$$\Rightarrow \frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{A(x^2+3)+(Bx+C)(x-1)}{(x-1)(x^2+3)}$$

Both sides multiplied by $(x-1)(x^2+3)$

$$x^2+3x+1 = A(x^2+3)+(Bx+C)(x-1) \quad ....(ii)$$

$$\Rightarrow x^2+3x+1 = Ax^2 + 3A + Bx^2 - Bx + Cx - C$$

$$\Rightarrow x^2+3x+1 = (A+B)x^2 + (-B+C)x + 3A - C \quad ......(iii)$$

Put x = 1 in (ii)

$$(1)^2+3(1)+1=A\left((1)^2+3\right)+\left(B(1)+C\right)(1-1)$$

=> $1+3+1=A(1+3)+(B+C)(0)$

=> $5=4A+0$ => $5=4A$

=> $A=\frac{5}{4}$

To find the values of B and C we compare the coefficients of x^2, x from eq(iii).

$1=A+B$ .....(iv)

$3=-B+C$ .....(v)

Put $A=\frac{5}{4}$ in (iv)

$1=\frac{5}{4}+B$ => $1-\frac{5}{4}=B$

=> $\frac{4-5}{4}=B$ => $-\frac{1}{4}=B$

=> $B=-\frac{1}{4}$

Put $B=-\frac{1}{4}$ in (v)

$3=-\left(-\frac{1}{4}\right)+C$ => $3=\frac{1}{4}+C$

=> $3-\frac{1}{4}=C$ => $C=\frac{12-1}{4}$

=> $C=\frac{11}{4}$

Put values of A, B and C in (i).

$$\frac{x^2+3x+1}{(x-1)(x^2+3)}=\frac{\frac{5}{4}}{x-1}+\frac{-\frac{1}{4}x+\frac{11}{4}}{x^2+3}$$

$$\Rightarrow \frac{x^2+3x+1}{(x-1)(x^2+3)}=\frac{5}{4(x-1)}-\frac{x-11}{4(x^2+3)}$$

Q.3: $\frac{2x}{(x^2+1)(x-1)}$

Solution: *Let* $\frac{2x}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$ ......(*i*)

Both sides multiplied by $(x-1)(x^2+1)$

$2x=A(x^2+1)+(Bx+C)(x-1)$ ....(*ii*)

$\Rightarrow 2x=Ax^2+A+Bx^2-Bx+Cx-C$

$\Rightarrow 0.x^2+2.x=(A+B)x^2+(-B+C)x+A-C$ ....(*iii*)

Put x = 1 in (ii)

$2(1)=A(1+1)+(B(1)+C)(1-1)$

$\Rightarrow 2=2A+(B+C)(0)$

$\Rightarrow 2=2A+0 \quad \Rightarrow A=\frac{2}{2}=1$

To find B and C we compare the coefficients of x^2, x

From eq(iii).

$0=A+B$ ......(*iv*)

$2=-B+C$ ....(*v*)

Put A = 1 in (iv)

$0=1+B \quad \Rightarrow B=-1$

Put B = - 1 in(v)

$2=-(-1)+C \quad \Rightarrow 2=1+C$

$\Rightarrow C=2-1=1$

Put A = 1 , B = -1 and C = 1 in (i)

$$\frac{2x}{(x-1)(x^2+1)}=\frac{1}{x-1}+\frac{-1.x+1}{x^2+1}$$

$$=> \quad \frac{1}{(x-1)(x^2+1)}=\frac{1}{x-1}-\frac{x-1}{x^2+1} \quad Ans.$$

Q.4: $\dfrac{-3}{x^2(x^2+5)}$

Solution: Let $\dfrac{-3}{x^2(x^2+5)}=\dfrac{Ax+B}{x^2}+\dfrac{Cx+D}{x^2+5}$ ....(i)

$$=> \quad \frac{-3}{x^2(x^2+5)}=\frac{(Ax+B)(x^2+5)+(Cx+D)x^2}{x^2(x^2+5)}$$

Multiplied both sides by $x^2(x^2+5)$

$$-3=(Ax+B)(x^2+5)+(Cx+D)x^2$$

$$=> \quad -3= Ax^3+5Ax+Bx^2+5B+Cx^3+Dx^2$$

$$=> 0.x^3+0.x^2+0.x-3$$

$$=(A+C)x^3+(B+D)x^2+5Ax+5B....(ii)$$

To find the values of A and B we compare the coefficients of x^3. x^2 , x **and constant from eq(ii) .**

$0=A+C$ ......(iii)

$0= B+D$ ....(iv)

$0=5A \quad => \quad A=0$ ......(v)

$-3=5B \quad => \quad B=-\dfrac{3}{5}$

Put A = 0 in (iii) => 0 = 0 + C

=> C = 0

Put $B=-\dfrac{3}{5}$ **in (iv)**

$0=-\dfrac{3}{5}+D$

$$=> D = \frac{3}{5}$$

Put values of A , B , C , D in(i)

$$\frac{1}{x^2(x^2+5)} = \frac{0.x - \frac{3}{5}}{x^2} + \frac{0.x + \frac{3}{5}}{x^2+5}$$

$$=> \frac{1}{x^2(x^2+5)} = -\frac{3}{5x^2} + \frac{3}{5(x^2+3)}$$

Q.5: $\frac{3x-2}{(x+4)(3x^2+1)}$

Solution: *Let* $\frac{3x-2}{(x+4)(3x^2+1)} = \frac{A}{x+4} + \frac{Bx+C}{3x^2+1}$ ......(*i*)

$$=> \frac{3x-2}{(x+4)(3x^2+1)} = \frac{A(3x^2+1)+(Bx+C)(x+4)}{(x+4)(3x^2+1)}$$

Both sides multiplied by $(x+4)(3x^2+1)$

$$3x-2 = A(3x^2+1)+(Bx+C)(x+4) ......(ii)$$

$$=> 3x-2 = 3Ax^2 + A + Bx^2 + 4Bx + Cx + 4C$$

$$=> 0.x^2 + 3x - 2 = (3A+B)x^2 + (4B+C)x + A + 4C ....(iii)$$

Put x = - 4 in (ii)

$$3(-4)-2 = A(3(-4)^2+1)+(B(-4)+C)(4-4)$$

$$=> -12-2 = A(3(16)+1)+(-4B+C)(0)$$

$$=> -14 = A(48+1)+0$$

$$=> -14 = 49A$$

$$=> A = -\frac{14}{49} = -\frac{2}{7}$$

Now from (iii) we compare the coefficients of x^2 , x .

$$0 = 3A + B .....(iv)$$

$$3 = 4B + C .....(v)$$

Put $A = -\frac{2}{7}$ in (iv)

$$0=3-\left(\frac{2}{7}\right)+B \quad => \quad 0=-\frac{6}{7}=B$$

$$=> \quad B=\frac{6}{7}$$

Put $B=\frac{6}{7}$ in (v)

$$3=4\left(\frac{6}{7}\right)+C \quad => \quad 3=\frac{24}{7}+C$$

$$=> \quad C=3-\frac{24}{7}=\frac{21-24}{7}=-\frac{3}{7}$$

Put values of A , B , C in(i)

$$\frac{3x-2}{(x+4)(3x^2+1)}=\frac{-\frac{2}{7}}{x-1}+\frac{\frac{6}{7}x-\frac{3}{7}}{3x^2+1}$$

$$=> \frac{3x-2}{(x+4)(3x^2+1)}=-\frac{2}{7(x+4)}+\frac{6x-3}{7(3x^2+1)}$$

Q.6: $\dfrac{5x}{(x+1)(x^2-2)^2}$

Solution: Let $\dfrac{5x}{(x+1)(x^2-2)^2}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2-2}+\dfrac{Dx+E}{(x^2-2)^2}$ ....(i)

$$=> \frac{5x}{(x+1)(x^2-2)^2}$$

$$=\frac{A(x^2-2)^2+(Bx+C)(x+1)(x^2-2)+(Dx+E)(x+1)}{(x+1)(x^2-2)^2}$$

Both sides multiplied by $(x+1)(x^2-2)^2$.

$$\Rightarrow 5x = A(x^2-2)^2 + (Bx+C)(x+1)(x^2-2)$$
$$+ (Dx+E)(x+1) \quad ....(ii)$$

$$\Rightarrow 5x = A(x^4-4x^2+4)+(Bx+C)(x^3-2x+x^2-2)$$
$$+(Dx+E)(x+1)$$

$$\Rightarrow 5x = Ax^4 - 4Ax^2 + 4A + Bx^4 - 2Bx^2 + Bx^3 - 2Bx$$
$$+ Cx^3 - 2Cx + Cx^2 - 2C + Dx^2 + Dx + Ex + E$$

$$\Rightarrow 0.x^4 + 0.x^3 + 0.x^2 + 5x = (A+B)x^4 + (B+C)x^3$$
$$+(-4A-2B+C+D)x^2 + (-2B-2C+D+E)x$$
$$+4A-2C+E \quad ....(iii)$$

Put x = -1 in(ii).

$$5(-1) = A((-1)^2-2)^2 + (-B+C)(-1+1)((-1)^2+1)$$
$$+(-D+E)(-1+1)$$

$$\Rightarrow -5 = A(1-2)^2 + 0 + 0$$

$$\Rightarrow -5 = A(-1)^2 \qquad \Rightarrow A = -5$$

From (iii) we compare the coefficients of x^4, x^3, x^2, x

$$0 = A+B \quad ......(iv)$$
$$0 = B+C \quad ....(v)$$
$$0 = -4A-2B+C+D \quad ......(vi)$$
$$5 = -2B-2C+D+E \quad ....(vii)$$

Put A = -5 in (iv)

$$0 = -5+B \qquad \Rightarrow B = 5$$

Put B = 5 in (v)

$$0 = 5+C \qquad \Rightarrow C = -5$$

Put values of A , B , C in(vi)

$$0 = -4(-5)-2(5)-5+D$$
$$\Rightarrow 0 = 20-10-5+D$$
$$\Rightarrow 0 = 5+D \qquad \Rightarrow D = -5$$

Put values of B . C . D in(vii)

$$5=-2(5)-2(-5)-5+E$$

$$\Rightarrow 5=-10+10-5+E$$

$$\Rightarrow 5+5=E$$

$$\Rightarrow E=10$$

Put values of A , B , C , D , E in(i).

$$\frac{5x}{(x+1)(x^2-2)^2}=-\frac{5}{x+1}+\frac{5x-5}{x^2-2}+\frac{-5x+10}{(x^2-2)^2}$$

$$\Rightarrow \frac{5x}{(x+1)(x^2-2)^2}=-\frac{5}{x+1}+\frac{5x-5}{x^2-2}-\frac{5x-10}{(x^2-2)^2}$$

Q.7: $\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}$

Solution:

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+1}+\frac{Dx+E}{(x^2+1)^2}......(i)$$

$$\Rightarrow \frac{5x^2-4x+8}{(x^2+1)^2(x-2)}$$

$$=\frac{A(x^2+1)^2+(Bx+C)(x-2)(x^2+1)+(Dx+E)(x-2)}{(x-2)(x^2+1)^2}$$

Multiplied both sides by $(x-2)(x^2+1)^2$

$$5x^2-4x+8=A(x^2+1)^2+(Bx+C)(x-2)(x^2+1)$$
$$+(Dx+E)(x-2)......(ii)$$

$$5x^2-4x+8 = A\left(x^4+2x^2+1\right)+\left(Bx+C\right)\left(x^3+x-2x^2-2\right) + Dx^2-2Dx+Ex-2E$$

$$\Rightarrow\ 5x^2-4x+8 = Ax^4+2Ax^2+A+Bx^4+Bx^2-2Bx^3-2Bx + Cx^3+Cx-2Cx^2-2C+Dx^2-2Dx+Ex-2E$$

$$\Rightarrow 0.x^4+0.x^3+5x^2-4x+8 = (A+B)x^4+(-2B+C)x^3 + (2A+B-2C+D)x^2+(-2B+C-2D+E)x + A-2C-2E \;.........(iii)$$

Put x = 2 in(ii)

$$5(2)^2-4(2)+8 = A\left((2)^2+1\right)^2+(2B+C)(2-2)\left((2)^2+1\right) + (2D+E)(2-2)$$

$$\Rightarrow 5(4)-8+8 = A(4+1)^2+0+0$$

$$\Rightarrow\ 20 = (5)^2A \qquad \Rightarrow\ 20 = 25A$$

$$\Rightarrow\ A = \frac{20}{25} = \frac{4}{5}$$

From (iii) we compare the coefficients of x^4, x^3, x^2, x

$$0 = A+B \;........(iv)$$

$$0 = -2B+C \;.......(v)$$

$$5 = 2A+B-2C+D \;.......(vi)$$

$$-4 = -2B+C-2D+E \;......(vii)$$

Put value of A in (iv)

$$0 = \frac{4}{5}+B \qquad \Rightarrow\ B = -\frac{4}{5}$$

Put value of B in (v)

$$0 = -2\left(-\frac{4}{5}\right)+C \qquad \Rightarrow\ C = -\frac{8}{5}$$

Put values of A , B , C in(vi) .

$$5 = 2\left(\frac{4}{5}\right) - \frac{4}{5} - 2\left(-\frac{8}{5}\right) + D$$

$$\Rightarrow 5 - \frac{8}{5} + \frac{4}{5} - \frac{16}{5} = D$$

$$\Rightarrow \frac{25 - 8 + 4 - 16}{5} = D$$

$$\Rightarrow D = \frac{5}{5} = 1$$

Put values of B , C , D in(vii)

$$-4 = -2\left(-\frac{4}{5}\right) - \frac{8}{5} - 2(1) + E$$

$$\Rightarrow -4 = \frac{8}{5} - \frac{8}{5} - 2 + E$$

$$\Rightarrow -4 + 2 = E$$

$$\Rightarrow E = -2$$

Put values of A , B , C , D , E in(i)

$$\frac{5x^2 - 4x + 8}{(x^2+1)^2(x-2)} = \frac{\frac{4}{5}}{x-1} + \frac{-\frac{4}{5}x - \frac{8}{5}}{x^2+1} + \frac{x-2}{(x^2+1)^2}$$

$$\Rightarrow \frac{5x^2 - 4x + 8}{(x^2+1)^2(x-2)} = \frac{4}{5(x-2)} - \frac{4x+8}{5(x^2+1)} + \frac{x-2}{(x^2+1)^2}$$

Q.8: $\frac{4x-5}{(x^2+4)^2}$

Solution: *Let* $\frac{4x-5}{(x^2+4)^2} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+4)^2}$ ....(*i*)

$$\Rightarrow \frac{4x-5}{(x^2+4)^2} = \frac{(Ax+B)(x^2+4) + (Cx+D)}{(x^2+4)^2}$$

Both sides multiplied by $(x^2+4)^2$

$$4x-5=(Ax+B)(x^2+4)+(Cx+D).....(ii)$$

$$=> 4x-5=Ax^3+4Ax+Bx^2+4B+Cx+D$$

$$0.x^3+0.x^2+4x-5=Ax^3+Bx^2+(4A+C)x+4B+D....(iii)$$

From (iii) we compare the coefficients of x^3, x^2, x, **and constant**

$0=A$ ......(iv)

$0=B$ ....(v)

$4=4A+C$ ....(vi)

$-5=4B+D$ ....(vii)

Put $A=0$ in (vi)

$4=4(0)+C \quad => \quad 4=0+C$

$=> \quad C=4$

Put $B=0$ **in (vii)** $=> \quad -5=4(0)+D$

$=> \quad -5=0+D \quad => \quad D=-5$

Put values of A , B , C , D , E in(i) .

$$\frac{4x-5}{(x^2+4)^2}=\frac{0.x+0}{x^2+4}+\frac{4x-5}{(x^2+4)^2}$$

$$=\frac{4x-5}{(x^2+4)^2} \quad Ans.$$

Q.9: $\dfrac{8x^2}{(x^2+1)(1-x^4)}$

Solution: $\dfrac{8x^2}{(x^2+1)(1-x^4)}=\dfrac{8x^2}{(1+x^2)((1)^2-(x^2)^2)}$

$$=\frac{8x^2}{(1+x^2)(1+x^2)(1-x^2)}$$

$$=\frac{8x^2}{(1+x^2)^2(1+x)(1-x)}$$

Let

$$\frac{8x^2}{(1+x^2)^2(1+x)(1-x)} = \frac{Ax+B}{(1+x^2)} + \frac{Cx+D}{(1+x^2)^2} + \frac{E}{1+x} + \frac{F}{1-x} \quad ....(1)$$

$$\Rightarrow \frac{8x^2}{(1+x^2)^2(1+x)(1-x)}$$

$$= \frac{1}{(1+x^2)^2(1+x)(1-x)}\begin{bmatrix}(Ax+B)(1+x^2)(1+x)(1-x)\\ +(Cx+D)(1+x)(1-x)\\ +E(1+x^2)^2(1-x)\\ +F(1+x^2)^2(1+x)\end{bmatrix} \quad ....(2)$$

Multiplied by $(1+x^2)^2(1+x)(1-x)$

$$\Rightarrow 8x^2 = (Ax+B)(1+x^2)(1+x)(1-x) + (Cx+D)(1+x)(1-x) + E(1+x^2)^2(1-x) + F(1+x^2)^2(1+x)$$

$$\Rightarrow 8x^2 = (Ax+B)(1+x^2)(1-x^2) + (Cx+D)(1-x^2) + E(1-x)(1+x^4+2x^2) + F(1+x)(1+x^4+2x^2)$$

$$\Rightarrow 8x^2 = (Ax+B)(1-x^4) + (Cx+D)(1-x^2) + E(1+x^4+2x^2-x-x^5-2x^3) + F(1+x^4+2x^2+x+x^5+2x^3)$$

$$\Rightarrow 8x^2 = Ax - Ax^5 + B - Bx^4 + Cx - Cx^3 + D - Dx^2 + E + Ex^4 + 2Ex^2 - Ex - Ex^5 - 2Ex^3 + F + Fx^4 + 2Fx^2 + Fx + Fx^5 + 2Fx^3$$

$$\Rightarrow 0x^5 + 0.x^4 + 0.x^3 + 8.x^2 + 0.x + 0 = (-A-E+F)x^5 + (-B+E+F)x^4 + (-C-2E+2F)x^3 + (-D+2E+2F)x^2 + (A+C-E+F)x + B+D+E+F \quad ....(3)$$

From (3) we compare the coefficients of x^5, x^4, x^3, x^2, x

$$0 = -A-E+F \quad ....(4)$$

$0=-B+E+F$ ....(5)

$0=-C-2E+2F$ .....(6)

$8=-D+2E+2F$ ....(7)

$0=A+C-E+F$ ....(8)

$0=B+D+E+F$ .....(9)

From (5) => $E+F=B$ ....(10) **put eq(10) in (9)**

$0=B+D+B$ => $0=2B+D$ .....(11)

From (7) => $2E+2F=8+D$

=> $2(E+F)=8+D$ => $E+F=\frac{8+D}{2}$ .....(12)

Put eq(12) in (9)

$0=B+D+\frac{8+D}{2}$ => $0=\frac{2B+2D+8+D}{2}$

=> $0=\frac{2B+3D+8}{2}$ => $0=2B+3D+8$

=> $2B+3D=-8$ ....(13)

Subtract eq(13) from eq (11).

$$\begin{array}{r} 2B+D=0 \\ \underline{\pm 2B \pm 3D = \mp 8} \\ -2D=8 \end{array}$$

=> $D=-\frac{8}{2}=-4$.

Put $D=-4$ **in eq(11)**

$0=2B-4$ => $4=2B$

=> $B=\frac{4}{2}=2$

Now adding eq(4) and eq(8).

$$\begin{array}{l} 0=-A \quad -E+F \\ \underline{0= \quad A+C-E+F} \\ 0=C-2E+2F \end{array}$$

=> $0=C-2E+2F$ ....(14)

Subtract eq (6) from eq(14)

$$0 = C - 2E + 2F$$
$$\pm 0 = \mp C \mp 2E \pm 2F$$
$$0 = 2C$$
=> $C = 0$

Put B = 2 in eq(5) then $0 = -2 + E + F$

=> $2 = E + F$ .....(15)

Put C = 0 in eq(6) then $0 = 0 - 2E + 2F$

=> $0 = 2(-E + F)$ *Divided by 2*

=> $0 = -E + F$..........(16)

Adding eq(15) and eq(16).

$$2 = E + F$$
$$0 = -E + F$$
$$2 = 2F$$

=> $F = \frac{2}{2} = 1$. **Put** $F = 1$ **in eq(16)**

$0 = -E + 1$ => $E = 1$

Put C = 0 , E = 1 , F = 1 in eq(8)

$0 = A + 0 - 1 + 1$ => $A = 0$

Put values of A , B , C , D , E and F in (1).

$$\frac{8x^2}{(1+x^2)^2(1+x)(1-x)} = \frac{0.x+2}{(1+x^2)} + \frac{0.x-4}{(1+x^2)^2} + \frac{1}{1+x} + \frac{1}{1-x}$$
$$= \frac{2}{(1+x^2)} - \frac{4}{(1+x^2)^2} + \frac{1}{1+x} + \frac{1}{-(x-1)}$$

Or $\frac{8x^2}{(1+x^2)^2(1+x)(1-x)} = -\frac{4}{(x^2+1)^2} + \frac{2}{x^2+1} + \frac{1}{x+1} - \frac{1}{x-1}$

Q.10: $\frac{2x^2+4}{(x^2+1)^2(x-1)}$

Solution: *Let* $\frac{2x^2+4}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$........$(i)$

$$\Rightarrow \frac{2x^2+4}{(x-1)(x^2+1)^2} = \frac{1}{(x-1)(x^2+1)^2}\begin{bmatrix} A(x^2+1)^2 \\ +(Bx+C)(x-1)(x^2+1) \\ +(Dx+E)(x-1) \end{bmatrix}$$

Both sides multiplied by $(x-1)(x^2+1)^2$

$$2x^2+4 = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1) \ldots\ldots(ii)$$

$$\Rightarrow 2x^2+4 = A(x^4+2x^2+1) + (Bx+C)(x^3+x-x^2-1) + (Dx+E)(x-1)$$

$$\Rightarrow 2x^2+4 = A(x^4+2x^2+1) + Bx^4+Bx^2-Bx^3-Bx + Cx^3+Cx-Cx^2-C+Dx^2-Dx+Ex-E$$

$$\Rightarrow 2x^2+4 = Ax^4+2Ax^2+A+Bx^4+Bx^2-Bx^3-Bx + Cx^3+Cx-Cx^2-C+Dx^2-Dx+Ex-E$$

$$\Rightarrow 2x^2+4 = (A+B)x^4 + (-B+C)x^3 + (2A+B-C+D)x^2 + (-B+C-D+E)x + A-C+E \ \ldots\ldots.(iii)$$

Put x = -1 in (ii)

$$2(1)^2+4 = A((1)^2+1)^2 + (B(1)+C)(1-1)(1+1) + (D(1)+E)(1-1)$$

$$\Rightarrow 2+4 = A(2)^2 + (B+C)(0) + (D+E)(0)$$

$$\Rightarrow 6 = 4A$$

$$\Rightarrow A = \frac{6}{4} = \frac{3}{2}$$

From (iii) we compare the coefficients of x^4, x^3, x^2, x.

$$0 = A+B \ \ldots\ldots(iv)$$

$$0 = -B+C \ \ldots\ldots..(v)$$

$$2 = 2A+B-C+D \ \ldots\ldots.(vi)$$

$$0 = -B+C-D+E \ \ldots\ldots..(vii)$$

Put $A=\frac{3}{2}$ in (iv)

$$0=\frac{3}{2}+B \quad => \quad B=-\frac{3}{2}$$

Put $B=-\frac{3}{2}$ in eq(v) .

$$0=-\left(-\frac{3}{2}\right)+C \quad => \quad 0=\frac{3}{2}+C$$

$$=> C=-\frac{3}{2}$$

Put values of A and B and C in (vi)

$$2=2\left(\frac{3}{2}\right)-\frac{3}{2}-\left(-\frac{3}{2}\right)+D$$

$$=> \quad 2=3-\frac{3}{2}+\frac{3}{2}+D$$

$$=> \quad 2-3=D \quad => \quad D=-1$$

Put values of B , C , D in(vii)

$$0=-\left(-\frac{3}{2}\right)-\frac{3}{2}-(-1)+E$$

$$=> \quad 0=\frac{3}{2}-\frac{3}{2}+1+E \quad => \quad 0=1+E$$

$$=> E=-1$$

Put values of A , B , C , D , E in(i) .

$$\frac{2x^2+4}{(x-1)(x^2+1)^2}=\frac{\frac{3}{2}}{x-1}+\frac{-\frac{3}{2}x-\frac{3}{2}}{x^2+1}+\frac{-x-1}{(x^2+1)^2}$$

$$=> \frac{2x^2+4}{(x-1)(x^2+1)^2}=\frac{3}{2(x-1)}+\frac{-3x-3}{2(x^2+1)}+\frac{-x-1}{(x^2+1)^2}$$

$$\text{Or} => \frac{2x^2+4}{(x-1)(x^2+1)^2}=\frac{-x-1}{(x^2+1)^2}+\frac{-3x-3}{2(x^2+1)}+\frac{3}{2(x-1)}$$

Review Exercise 4:

Q.1: At the end of each question , four circles are given . Fill in the correct circle only .

(i) $\frac{1}{x^2-1} =$

(a) $\frac{1}{x+1}-\frac{1}{x-1}$ (b) $\frac{1}{2(x+1)}-\frac{1}{2(x-1)}$

(c) $\frac{1}{2(x-1)}-\frac{1}{2(x+1)}$ (d) $\frac{2}{x-1}-\frac{1}{2(x+1)}$

(ii) If P(x) and Q(x) are two polynomials then $\frac{P(x)}{Q(x)}$,

$Q(x)\neq 0$ is a

(a) Rational fraction (b) Irrational fraction

(c) Proper fraction (d) Improper fraction

(iii) $\frac{x^2+2}{x^2+2x+2}$

(a) Proper fraction (b) Improper fraction

(c) Irrational fraction (d) Non of these

(iv) What is the quotient when $x^3-8x^2+16x-5$ is divided by $x-5$?.

(a) x^2-x+5 (b) x^2-3x+2

(c) x^2-3x+1 (d) $x^2+13x-49+\frac{240}{x+5}$

Ans : (i) c (ii) a (iii) b (iv) a (v) c

Q.2: Resolve the following fractions into partial fractions.

(i): $\frac{2x^2+1}{(x+1)(x-1)}$

Solution: $\frac{2x^2+1}{(x+1)(x-1)}=\frac{2x^2+1}{x^2-1}$

it is improper fraction then

$$\begin{array}{r|l} & \quad 2 \\ x^2-1 & 2x^2+1 \\ & \underline{\pm 2x^2 \mp 2} \\ & \quad 3 \end{array}$$

Then $\dfrac{2x^2+1}{x^2-1} = 2 + \dfrac{3}{x^2-1}$

$\Rightarrow \dfrac{2x^2+1}{(x-1)(x+1)} = 2 + \dfrac{3}{(x-1)(x+1)}$ ....(i)

Now we solve $\dfrac{3}{(x-1)(x+1)}$ by partial fraction .

$\dfrac{3}{(x-1)(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$ ....(ii)

$\Rightarrow \dfrac{3}{(x-1)(x+1)} = \dfrac{A(x+1)+B(x-1)}{(x-1)(x+1)}$

Multiplied both sides by $(x-1)(x+1)$

$3 = A(x+1) + B(x-1)$ ....(iii)

Put $x = 1$ in (iii)

$3 = A(1+1) + B(1-1)$

$\Rightarrow 3 = A(2) + 0$

$\Rightarrow A = \dfrac{3}{2}$

Put $x = -1$ in (iii)

$3 = A(-1+1) + B(-1-1)$

$\Rightarrow 3 = A(0) + B(-2)$

$\Rightarrow 3 = -2B$

$\Rightarrow B = -\dfrac{3}{2}$

Put values of A and B in(ii)

$$\frac{3}{(x-1)(x+1)}=\frac{\frac{3}{2}}{x-1}+\frac{-\frac{3}{2}}{x+1}$$

$$=> \frac{3}{(x-1)(x+1)}=\frac{3}{2(x-1)}-\frac{3}{2(x+1)} \quad ......(iv)$$

Put (iv) in (i)

$$\frac{2x^2+1}{(x-1)(x+1)}+2+\frac{3}{2(x-1)}-\frac{3}{2(x+1)}$$

(ii): $\frac{2x^3-3x^2+9x+8}{x^2-3x+2}$

Solution: $\frac{2x^3-3x^2+9x+8}{x^2-3x+2}$ It is improper fraction . Then

$$\begin{array}{r|l} & 2x+3 \\ x^2-3x+2 & 2x^3-3x^2+9x+8 \\ & \pm 2x^3 \mp 6x^2 \pm 4x \\ \hline & 3x^2+5x+8 \\ & \pm 3x^2 \mp 9x \pm 6 \\ \hline & 14x+2 \end{array}$$

$$Then \quad \frac{2x^3-3x^2+9x+8}{x^2-3x+2}=2x+3+\frac{14x+2}{x^2-3x+2} \quad ....(i)$$

Now we solve $\frac{14x+2}{x^2-3x+2}$ *by partial fraction.*

$$\frac{14x+2}{x^2-3x+2}=\frac{14x+2}{x^2-x-2x+2}$$

$$=\frac{14x+2}{x(x-1)-2(x-1)}$$

$$=\frac{14x+2}{(x-1)(x-2)}$$

$$Let\ \frac{14x+2}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} \ ....(ii)$$

$$=> \frac{14x+2}{(x-1)(x-2)} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$$

Multiplied both sides by $(x-1)(x-2)$

$$14x+2 = A(x-2)+B(x-1) \ ....(iii)$$

Put x = 1 in (iii)

$$14(1)+2 = A(1-2)+B(1-1)$$

$$=> 14+2 = A(-1)+B(0)$$

$$=> 16 = -A \qquad => A = -16$$

Put x = 2 in (iii) .

$$14(2)+2 = A(2-2)+B(2-1)$$

$$=> 28+2 = A(0)+B(1)$$

$$=> 30 = 0+B \qquad => B = 30$$

Put values of A and B in(ii)

$$\frac{14x+2}{(x-1)(x-2)} = -\frac{16}{x-1} + \frac{30}{x-2} \ ....(iv)$$

Put (iv) in (i)

$$\frac{2x^3-3x^2+9x+8}{x^2-3x+2} = 2x+3-\frac{16}{x-1}+\frac{30}{x-2}$$

<u>**(iii)**</u>: $\frac{3x-1}{x^3-2x^2+x}$

<u>**Solution**</u>: $\frac{3x-1}{x^3-2x^2+x} = \frac{3x-1}{x(x^2-2x+1)}$

$$=> \frac{3x-1}{x^3-2x^2+x} = \frac{3x-1}{x(x-1)^2}$$

Let $\frac{3x-1}{x(x-1)^2}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$ ....(i)

=> $\frac{3x-1}{x(x-1)^2}=\frac{A(x-1)^2+Bx(x-1)+Cx}{x(x-1)^2}$

Both sides multiplied by $x(x-1)^2$

$3x-1=A(x-1)^2+Bx(x-1)+Cx$ ......(ii)

=> $3x-1=A(x^2-2x+1)+Bx(x-1)+Cx$

=> $3x-1=Ax^2-2Ax+A+Bx^2-Bx+Cx$

=> $0.x^2+3x-1=(A+B)x^2+(-2A-B+C)x+A$

Put x = 0 in (ii)

$3(0)-1=A(0-1)^2+B(0)(0-1)+C(0)$

=> $0-1=A(-1)^2+0+0$

=> $-1=A$

Now to find the values of B and C we compare the coefficients of x^2 , x .

$0=A+B$ ....(iii)

$3=-2A-B+C$ ....(iv)

Put A = -1 in (iii)

$0=-1+B$ => $B=1$

Put A = -1 and B = 1 in(iv)

$3=-2(-1)-1+C$

=> $3=2-1+C$

=> $3=1+C$

=> $3-1=C$ => $C=2$

Put values of A , B and C in(i) .

$\frac{3x-1}{x(x-1)^2}=-\frac{1}{x}+\frac{1}{x-1}+\frac{2}{(x-1)^2}$

(iv): $\frac{x+1}{(x-1)^2}$

Solution: $\frac{x+1}{(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}$ ......(i)

=> $\frac{x+1}{(x-1)^2}=\frac{A(x-1)+B}{(x-1)^2}$

Both sides multiplied by $(x-1)^2$

$x+1=A(x-1)+B$ ......(ii)

=> $x+1=Ax-A+B$ ....(iii)

Comparing the coefficients of x and constants

$A=1$ .....(iv)

$-A+B=1$ .....(v)

Put A = 1 in (v)

$-1+B=1$ => $B=1+1$

=> $B=2$

Put A = 1 and B = 2 in(i)

$\frac{x+1}{(x-1)^2}=\frac{1}{x-1}+\frac{2}{(x-1)^2}$

(v): $\frac{2x^2}{x^4-4}$

Solution: $\frac{2x^2}{x^4-4}=\frac{2x^2}{(x^2-2)(x^2+2)}$

Let $\frac{2x^2}{(x^2-2)(x^2+2)}=\frac{Ax+B}{x^2-2}+\frac{Cx+D}{x^2+2}$ .....(i)

=> $\frac{2x^2}{(x^2-2)(x^2+2)}=\frac{(Ax+B)(x^2+2)+(Cx+D)(x^2-2)}{(x^2-2)(x^2+2)}$

Multiplied by $(x^2-2)(x^2+2)$

$2x^2=(Ax+B)(x^2+2)+(Cx+D)(x^2-2)$

$$\Rightarrow 2x^2 = Ax^3 + 2Ax + Bx^2 + 2B + Cx^3 - 2Cx + Dx^2 - 2D$$

$$\Rightarrow 0.x^3 + 2x^2 + 0.x + 0 = (A+C)x^3 + (B+D)x^2 + (2A-2C)x + 2B - 2D \;....(ii)$$

From (ii) we compare the coefficients of x^3, x^2, x and constant

$0 = A + C$ ......(iii)

$2 = B + D$ ......(iv)

$0 = 2A - 2C$.......(v)

$0 = 2B - 2D$ ....(vi)

Eq(iii) multiplied by 2 => $0 = 2A + 2B$ ......(vii)

Adding (v) and (vii)

$$\begin{aligned} 0 &= 2A - 2C \\ 0 &= 2A + 2C \\ \hline 0 &= 4A \end{aligned}$$

$\Rightarrow A = 0$

Put A= 0 in(iii)

$0 = 0 + C \quad \Rightarrow C = 0$

Eq(iv) multiplied by 2 => $4 = 2B + 2D$ .....$(viii)$

Adding (vi) and (viii)

$$\begin{aligned} 0 &= 2B - 2D \\ 4 &= 2B + 2D \\ \hline 4 &= 4B \end{aligned}$$

$\Rightarrow B = 1$

Put B = 1 in (iv)

$2 = 1 + D \quad \Rightarrow 2 - 1 = D$

$\Rightarrow D = 1$

Put values of A , B , C , D in(i)

$$\frac{2x^2}{(x^2-2)(x^2+2)} = \frac{0.x+1}{x^2-2} + \frac{0.x+1}{x^2+2}$$

$$=> \frac{2x^2}{x^4-4}=\frac{1}{x^2-2}+\frac{1}{x^2+2}$$

(vi): $\frac{3x^2+3x+2}{x^4-1}$

Solution: $\frac{3x^2+3x+2}{x^4-1}=\frac{3x^2+3x+2}{(x^2-1)(x^2+1)}$

$$=\frac{3x^2+3x+2}{(x-1)(x+1)(x^2+1)}$$

$$Let \quad \frac{3x^2+3x+2}{(x-1)(x+1)(x^2+1)}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{Cx+D}{x^2+1} \ldots.(i)$$

$$=> \frac{3x^2+3x+2}{(x-1)(x+1)(x^2+1)}$$

$$=\frac{A(x+1)(x^2+1)+B(x-1)(x^2+1)+(Cx+D)(x-1)(x+1)}{(x-1)(x+1)(x^2+1)}$$

Both sides multiplied by $(x-1)(x+1)(x^2+1)$

$$=> 3x^2+3x+2=A(x+1)(x^2+1)+B(x-1)(x^2+1)$$
$$+(Cx+D)(x-1)(x+1) \ldots\ldots(ii)$$

$$=> 3x^2+3x+2=A(x^3+x+x^2+1)+B(x^3+x-x^2-1)$$
$$+(Cx+D)(x^2-1)$$

$$=> 3x^2+3x+2=Ax^3+Ax+Ax^2+A+Bx^3+Bx-Bx^2-B$$
$$+Cx^3-Cx+Dx^2-D$$

$$=> 0.x^3+3x^2+3x+2=(A+B+C)x^3+(A-B+D)x^2$$
$$+(A+B-C)x+A-B-D \ldots\ldots\ldots.(iii)$$

Put x = 1 in (ii)

$$3(1)^2+3(1)+2=A(1+1)((1)^2+1)+B(1-1)((1)^2+1)$$
$$+(C(1)+D)(1-1)(1+1)$$

$$3+3+2=A(2)(1+1)+B(0)(1+1)+(C+D)(0)(2)$$

$=> 8=4A+0+0$

$=> 4A=8 \quad => A=2$

Put x = -1 in (ii)

$$3(-1)^2+3(-1)+2=A(-1+1)((-1)^2+1)+B(-1-1)((-1)^2+1)$$
$$+(-C+D)(-1-1)(-1+1)$$

$=> 3-3+2=A(0)(1+1)+B(-2)(1+1)+(-C+D)(-2)(0)$

$=> 2=0-4B+0 \quad => \quad 4B=-2$

$=> \quad B=-\frac{1}{2}$

Now we compare the coefficients of x^3, x^2, x from (iii) .

$0=A+B+C$ ......(iv)

$3=A-B+D$ ....(v)

$3=A+B-C$........(vi)

Put values of A , B in(iv)

$$3=2-\frac{1}{2}+C$$

$=> 3-2+\frac{1}{2}=C$

$=> \frac{6-4+1}{2}=C \quad => \quad C=\frac{3}{2}$

Put values of A , B in(v)

$$3=2-\left(-\frac{1}{2}\right)+D$$

$=> 3-2=\frac{1}{2}+D \quad => 1-\frac{1}{2}=D$

$=> \quad D=\frac{2-1}{2}=\frac{1}{2}$

Put values of A , B , C , D in(v)

$$\frac{3x^2+3x+2}{(x-1)(x+1)(x^2+1)}=\frac{2}{x-1}+\frac{-\frac{1}{2}}{x+1}+\frac{\frac{3}{2}x+\frac{1}{2}}{x^2+1}$$

$$=> \frac{3x^2+3x+2}{(x-1)(x+1)(x^2+1)}=\frac{2}{x-1}-\frac{1}{2(x+1)}+\frac{3x+1}{2(x^2+1)}$$

(vii): $\frac{x^3+3x^2+1}{(x^2+1)^2}$

Solution: *Let* $\frac{x^3+3x^2+1}{(x^2+1)^2}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}$ ......(*i*)

$$=> \frac{x^3+3x^2+1}{(x^2+1)^2}=\frac{(Ax+B)(x^2+1)+Cx+D}{(x^2+1)^2}$$

Both sides multiplied by $(x^2+1)^2$

$=> x^3+3x^2+1=(Ax+B)(x^2+1)+Cx+D$

$=> x^3+3x^2+1=Ax^3+Ax+Bx^2+B+Cx+D$

$=> x^3+3x^2+1=Ax^3+Bx^2+(A+C)x+B+D$ ...(*ii*)

Now we compare the coefficients of x^3, x^2, x and constant from (ii) .

$1=A$ ....(*iii*)

$3=B$.....(*iv*)

$0=A+C$ ......(*v*)

$1=B+D$ ....(*vi*)

Put A = 1 in(v)

$0=1+C$

$=> C=-1$

Put B = 3 in(vi)

$1 = 3 + D$

$\Rightarrow D = 1 - 3 = -2$

Put values of A , B , C , D in(i) .

$$\frac{x^3+3x^2+1}{(x^2+1)^2} = \frac{1.x+3}{x^2+1} + \frac{-x-2}{(x^2+1)^2}$$

$$\Rightarrow \frac{x^3+3x^2+1}{(x^2+1)^2} = \frac{x+3}{x^2+1} - \frac{x+2}{(x^2+1)^2}$$

Q.(viii) : $\dfrac{2x^3-1}{x^3+x^2}$

Solution: It is improper fraction .

$$\begin{array}{r|l} & 2 \\ x^3+x^2 & 2x^3 \qquad -1 \\ & \pm 2x^3 \pm 2x^2 \\ \hline & -2x^2 - 1 \end{array}$$

$$\frac{2x^3-1}{x^3+x^2} = 2 - \frac{2x^2+1}{x^3+x^2}$$

$$= 2 - \frac{2x^2+1}{x^2(x+1)} \quad ....(i)$$

$$Let \quad \frac{2x^2+1}{x^2(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2} \quad ......(ii)$$

Both sides multiplied by $x^2(x+1)$

$2x^2+1 = Ax^2 + (Bx+C)(x+1)$

$\Rightarrow 2x^2+1 = Ax^2 + Bx^2 + Bx + Cx + C$

$\Rightarrow 2x^2+1 = (A+B)x^2 + (B+C)x + C$

To find A, B and C we compare the coefficients of x^2, x

From eq(iii) .

$2 = A ......(iv)$

$0 = B + C ....(v)$

$1 = C$ ......(vi)

Put $C = 1$ **in (v)**

$0 = B + 1 \quad => \quad B = -1$

Put A = 2 , B = -1 and C = 1 in (i)

$$\frac{2x^2+1}{x^2(x+1)} = \frac{2}{x+1} + \frac{-x+1}{x^2}$$

$$= \frac{2}{x+1} - \frac{x-1}{x^2} \text{.......(vii)}$$

Put (vii) in (i).

$$\frac{2x^3-1}{x^3+x^2} = 2 - \frac{2}{x+1} + \frac{x-1}{x^2}$$

$$= 2 - \frac{2}{x+1} + \frac{x}{x^2} - \frac{1}{x^2}$$

$$= 2 - \frac{2}{x+1} + \frac{1}{x} - \frac{1}{x^2} \quad Ans.$$

Q.(ix): $\frac{4x^2+3x+14}{x^3-8}$

Solution: $\frac{4x^2+3x+14}{x^3-8} = \frac{4x^2+3x+14}{(x)^3-(2)^3}$

$$= \frac{4x^2+3x+14}{(x-2)(x^2+2x+4)}$$

Let $\frac{4x^2+3x+14}{(x-2)(x^2+2x+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4}$ .....(i)

$$\frac{4x^2+3x+14}{(x-2)(x^2+2x+4)} = \frac{A(x^2+2x+4)+(Bx+C)(x-2)}{(x-2)(x^2+2x+4)}$$

Multiplied by $(x-2)(x^2+2x+4)$

$$4x^2+3x+14 = A(x^2+2x+4)+(Bx+C)(x-2) \text{....(ii)}$$

$=> 4x^2+3x+14 = Ax^2+2Ax+4A+Bx^2-2Bx+Cx-2C$

$=> 4x^2+3x+14 = (A+B)x^2+(2A-2B+C)x+4A-2C$

Comparing the coefficients of x^2 , x *and constants* .

$4 = A + B \quad ....(iii)$

$3 = 2A - 2B + C \,......(iv)$

$14 = 4A - 2C \quad ....(v)$

Put $x = 2$ in (ii)

$4(2)^2 + 3(2) + 14 = A((2)^2 + 2(2) + 4) + (B(2) + C)(2 - 2)$

$\Rightarrow 4(4) + 6 + 14 = A(4 + 4 + 4) + (2B + C)(0)$

$\Rightarrow 16 + 20 = 12A + 0$

$\Rightarrow 36 = 12A \quad \Rightarrow A = \frac{36}{12} = 3$

Put A = 3 in(iii)

$4 = 3 + B \quad \Rightarrow 4 - 3 = B$

$\Rightarrow B = 1$

Put A = 3 in eq(v).

$14 = 4(3) - 2C \quad \Rightarrow 14 = 12 - 2C$

$\Rightarrow 2C = 12 - 14 \quad \Rightarrow 2C = -2$

$\Rightarrow C = -\frac{2}{2} = -1$

Put values of A , B and C in (i)

$$\frac{4x^2 + 3x + 14}{(x-2)(x^2 + 2x + 4)} = \frac{3}{x-2} + \frac{x-1}{x^2 + 2x + 4} \quad \text{Ans.}$$

<u>Challenge</u> :

<u>Q .3</u>: Resolve the following fraction into partial fraction.

$$\frac{x^4 + 3x^2 + x + 1}{(x+1)(x^2+1)^2}$$

<u>Solution</u>: *Let* $\frac{x^4 + 3x^2 + x + 1}{(x+1)(x^2+1)^2} = \frac{A}{x+1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2+1)^2} \,........(i)$

$$\Rightarrow \frac{x^4 + 3x^2 + x + 1}{(x+1)(x^2+1)^2} = \frac{1}{(x+1)(x^2+1)^2}\begin{bmatrix} A(x^2+1)^2 \\ +(Bx + C)(x+1)(x^2+1) \\ +(Dx + E)(x+1) \end{bmatrix}$$

Both sides multiplied by $(x+1)(x^2+1)^2$

$$x^4+3x^2+x+1=A(x^2+1)^2+(Bx+C)(x+1)(x^2+1)$$
$$+(Dx+E)(x+1).....(ii)$$

$$\Rightarrow x^4+3x^2+x+1=A(x^4+2x^2+1)+(Bx+C)(x^3+x+x^2+1)$$
$$+(Dx+E)(x+1)$$

$$\Rightarrow x^4+3x^2+x+1=A(x^4+2x^2+1)+Bx^4+Bx^2+Bx^3+Bx$$
$$+Cx^3+Cx+Cx^2+C+Dx^2+Dx+Ex+E$$

$$\Rightarrow x^4+3x^2+x+1=Ax^4+2Ax^2+A+Bx^4+Bx^2+Bx^3+Bx$$
$$+Cx^3+Cx+Cx^2+C+Dx^2+Dx+Ex+E$$

$$\Rightarrow x^4+3x^2+x+1=(A+B)x^4+(B+C)x^3$$
$$+(2A+B+C+D)x^2$$
$$+(B+C+D+E)x+A+C+E \quad ....(iii)$$

Put x = - 1 in (ii)

$$(-1)^4+3(-1)^2-1+1=A((-1)^2+1)^2+(B(-1)+C)(-1+1)(1+1)$$
$$+(D(1)+E)(-1+1)$$

$$\Rightarrow 1+3=A(1+1)^2+(-B+C)(0)(2)+(D+E)(0)$$

$$\Rightarrow 4=A(2)^2+0+0 \quad \Rightarrow 4=4A$$

$$\Rightarrow A=\frac{4}{4}=1$$

From (iii) we compare the coefficients of x^4, x^3, x^2, x.

$1 = A+B$ ......(iv)

$0 = B+C$ ....(v)

$3 = 2A+B+C+D$ ....(vi)

$1 = B+C+D+E$ ....(vii)

Put $A=1$ in (iv)

$1 = 1 + B \quad => \quad B = 1 - 1$

$=> B = 0$

Put $B = 0$ **in eq(v) .**

$0 = 0 + C \quad => \quad 0 = C$

$=> C = 0$

Put values of A and B and C in (vi)

$3 = 2(1) + 0 + 0 + D$

$=> \quad 3 = 2 + D$

$=> \quad 3 - 2 = D \quad => \quad D = 1$

Put values of B , C , D in(vii)

$1 = 0 + 0 + 1 + E$

$=> \quad 1 - 1 = E \quad => \quad E = 0$

Put values of A , B , C , D , E in(i) .

$$\frac{x^4 + 3x^2 + x + 1}{(x+1)(x^2+1)^2} = \frac{1}{x+1} + \frac{0.x + 0}{x^2 + 1} + \frac{1.x + 0}{(x^2+1)^2}$$

$$=> \quad \frac{x^4 + 3x^2 + x + 1}{(x+1)(x^2+1)^2} = \frac{1}{x+1} + \frac{x}{(x^2+1)^2}$$

Unit 5:

Sets and Functions:

Set: A collection of well defined and distinct objects is called a set. A set is generally denoted by capital letters of English alphabets A, B, C, ..., Z. The objects of a set are called elements or members and denoted by small letters of English alphabets a, b, c.....z. If "x" is a member of a set A then it is written as $x \in A$. If x is not a member of A then we write $x \notin A$.

Methods of a set writing: A set is writing by three methods

(i) Descriptive Method : In this method the elements of a set is writing by a sentence. For example

The set of first ten natural numbers.

(ii) Tabular Method: In this method the elements of a set is Written one by one. For example

A = { 1 , 2 , 3, 4 , 5 . 6 , 7 , 8 , 9 , 10 }

(iii) Set builder method : In this method the elements of a Set is represented by x. i.e.

$A = \{x / x \in N , x \leq 10\}$

Some important set:

(i) Set of natural numbers = N = { 1 , 2 , 3 , ...}

(ii) Set whole numbers = W = { 0 , 1 . 2 . ...}

(iii) Set of integers = Z = {0 ,±1 , ± 2, ± 3, ...}

(iv) Set of even numbers = E = {0 , ± 2 , ± 4, ...}

(v) Set of odd numbers = O = {±1 , ± 3. ±5,.........}

(vi) Set of prime numbers = P = { 2 , 3 , 5 , 7 , 11 , ...}

(vii) Set of rational numbers =

$$Q = \{x / x \in \frac{p}{q} , p, q \in Z , q \neq 0\}$$

Union of two sets: The union of two sets A and B is a set Which contains the elements of set A or set B or both sets A and B. It is denoted by A U B.

In set builder form $AUB = \{x / x \in A \vee x \in B\}$.

Intersection of two sets: Intersection of two sets A and B is a

set which contains the common elements of set A and set B. It is denoted by $A \cap B$. In set builder form

$A \cap B = \{x / x \in A \wedge x \in B\}$

Difference of two sets: The difference of two sets A and B is denoted by A − B or A / B and contains those elements of set A which does not belong to B. In set builder form

$A - B = \{x / x \in A \wedge x \notin B\}$

Complement of a set: Complement of a set A is a set which Contains those elements of universal set which does not belong to set A. It is denoted by A' or A^c

Exercise 5.1:

Q.1: If A = { 1 , 2 , 3} , B = { 0 , 1} and C = { 1 , 3 ,4 } then find

(i) AUB (ii) $A \cap B$ (iii) AUC

(iv) $A \cap C$ (v) BUC (vi) $A \cap A$

Solution: (i) A UB

$AUB = \{1, 2, 3\} U \{0, 1\}$

$= \{0, 1, 2, 3\}$

(ii) $A \cap B = \{1, 2, 3\} \cap \{0, 1\} = \{1\}$

(iii) $AUC = \{1, 2, 3\} U \{1, 3, 4\} = \{1, 2, 3, 4\}$

(iv) $A \cap C = \{1, 2, 3\} \cap \{1, 3, 4\} = \{1, 3\}$

(v) $BUC = \{0, 1\} U \{1, 3, 4\} = \{0, 1, 3, 4\}$

(vi) $A \cap A = \{1, 2, 3\} \cap \{1, 2, 3\} = \{1, 2, 3\}$

Q.2: Find A / B and B / A when

(i) $A = \{1, 3, 5, 7\}$, $B = \{3, 4, 5, 6, 7, 8\}$

(ii) $A = \{0, \pm 1, \pm 2, \pm 3\}$, $B = \{-1, -2, -3\}$

(iii) $A = \{1, 2, 3, 4, ...\}$, $B = \{1, 3, 5, 7, ...\}$

Solution: (i) $A / B = \{1, 3, 5, 7\} / \{3, 4, 5, 6, 7, 8\}$

=> $A / B = \{1\}$

$B / A = \{3, 4, 5, 6, 7, 8\} / \{1, 3, 5, 7\}$

=> $B / A = \{4, 6, 8\}$

(ii) $A=\{0, \pm1, \pm2, \pm3\}$, $B=\{-1,-2,-3\}$

$A/B=\{0, \pm1, \pm2, \pm3\}/\{-1,-2,-3\}$

$\Rightarrow A/B=\{0, 1, 2, 3\}$

$B/A=\{-1,-2,-3\}/\{0, \pm1, \pm2, \pm3\}$

$\Rightarrow B/A=\{\,\}$

(iii) $A=\{1, 2, 3, 4, \ldots\}$, $B=\{1, 3, 5, 7, \ldots\}$

$A/B=\{1, 2, 3, 4, \ldots\}/\{1, 3, 5, 7, \ldots\}$

$\Rightarrow A/B=\{2, 4, 6, \ldots\}$

B/A = $\{1, 3, 5, 7, \ldots\}/\{1, 2, 3, 4, \ldots\}$

$\Rightarrow B/A=\{\,\}$

Q.3: If U = { 1 , 2, 3, ...,20} , A = { 2 , 4 , 6, ..., 20} , B = { 1 , 3 , 5, ..., 19} and $C=\phi$ then find

(i) A' (ii) B' (iii) C' (iv) $A' \cup B'$ (v) $A' \cap B'$

(vi) $A' \cap B$ (vii) $A' \cup C'$ (viii) $A \cap C'$ (ix) $C' \cap C$

(x) $B' \cup C'$

Solution: (i) A'

$A' = U/A = \{1, 2, 3, \ldots, 20\}/\{2, 4, 6, \ldots, 20\}$

$\Rightarrow A' = \{1, 3, 5, \ldots, 19\} \ldots\ldots (i)$

(ii) B'

$B' = U/B = \{1, 2, 3, \ldots, 20\}/\{1, 3, 5, \ldots, 19\}$

$\Rightarrow B' = \{2, 4, 6, \ldots, 20\} \ldots\ldots (ii)$

(iii) C'

$C' = U/C = \{1, 2, 3, \ldots, 20\}/\phi$

$\Rightarrow C' = \{1, 2, 3, \ldots, 20\} \ldots\ldots (iii)$

(iv) $A' \cup B'$

From (i) $A' = \{1, 3, 5, \ldots, 19\}$

From (ii) $B' = \{2, 4, 6, ...,20\}$

$A'UB' = \{1, 3, 5, ...,19\} U \{2, 4, 6, ...20\}$

$= \{1,2,3,4,.......,20\}$

(v) $A' \cap B'$

From (i) $A' = \{1, 3, 5, ...,19\}$

From (ii) $B' = \{2, 4, 6, ...,20\}$

$A' \cap B' = \{1,3,5,...,19\} \cap \{2, 4, 6,....,20\}$

$= \{\ \}$

(vi) $A' \cap B$

From (i) $A' = \{1, 3, 5, ...,19\}$

$A' \cap B = \{1,3,5, ...,19\} \cap \{1,3,5,......,19\}$

$= \{1,3,5,......,19\}$

(vii) $A'UC'$

From (i) $A' = \{1, 3, 5, ...,19\}$

From (iii) $C' = \{1, 2, 3,.......,20\}$

$A'UC' = \{1, 3, 5,.......,19\} U \{1, 2, 3.......,20\}$

$= \{1, 2, 3,.......,20\}$

(viii) $A \cap C'$

From (iii) $C' = \{1, 2, 3,.......,20\}$

$A \cap C' = \{2, 4, 6,.......,20\} \cap \{1, 2, 3, ...,20\}$

$= \{2, 4, 6,.......,20\}$

(ix) $C' \cap C$

From (iii) $C' = \{1, 2, 3,.......,20\}$

$C' \cap C = \{1, 2, 3,.......,20\} \cap \phi$

$= \phi$

(x) $B'UC'$

From (ii) $B' = \{2\ ,\ 4,\ 6,\ .....,20\}$

From (iii) $C' = \{1,\ 2,3,.......,20\}$

$B'U\ C' = \{2\ ,\ 4\ ,6\ ,......,20\}U\{1,\ 2,\ 3,......,20\}$

$= \{1,\ 2,\ 3,......,20\}$

<u>Q.4</u>: If U = set of natural numbers up to 15
and A = set of even numbers up to 15
and B = set of odd numbers up to 15 , then find

$(i)\ A'UB'$ $(ii)\ A'\cap B'$ $(iii)\ U'$ $(iv)\ \phi'$

$(v)\ B\cap A'$ $(vi)\ BUB'$ $(vii)\ A\cap A'$ $(viii)\ AUB'$

<u>Solution</u>: U = { 1 , 2 , 3 , ……., 15}
A = { 2 , 4 , 6 , ….., 14}
B = { 1 , 3 , 5 , …., 15 }

(i) $A'U\ B'$

$A' = U/A = \{1,2\ ,3,........,15\}/\{2\ ,4,6\ ,\ ......,14\}$

$\Rightarrow\ A' = \{1\ ,\ 3,5,.......,15\}\ .....(i)$

$B' = U/B = \{1,2\ ,3,........,15\}/\{1\ ,3,5\ ,\ ......,15\}$

$\Rightarrow B' = \{\ 2\ ,\ 4,\ 6,\ ......,14\}\ ......(ii)$

$A'UB' = \{1,\ 3,\ 5,......,15\}U\{2,\ 4\ ,6,......,14\}$

$= \{1,\ 2\ ,3,4,.......,15\}$

(ii) $A'\cap B'$

From (i) $A' = \{1\ ,\ 3,5,.......,15\}$

From (ii) $B' = \{\ 2\ ,\ 4,\ 6,\ ......,14\}$

$A'\cap B' = \{1,3,5,....,15\}\cap\{\ 2\ ,\ 4,\ 6,\ ......,14\}$

$= \{\ \}$

(iv) U'

$U' = U/U = \{1\ ,\ 2\ ,3,.......,15\}/\{1\ ,\ 2\ ,3,.......,15\}$

$=> U' = \phi$

(iv) ϕ'

$\phi' = U / \phi = \{1, 2, 3, ..., 15\} / \phi$

$= \{1, 2, 3, ..., 15\}$

(v) $B \cap A'$

From (i) $A' = \{1, 3, 5, ..., 15\}$

$B \cap A' = \{1, 3, 5, ..., 15\} \cap \{1, 3, 5, ..., 15\}$

$= \{1, 3.5, ..., 15\}$

(vi) BUB'

From (ii) $B' = \{2, 4, 6, ..., 14\}$

$BUB' = \{1, 3, 5, ..., 15\} U \{2, 4, 6, ..., 14\}$

$= \{1, 2, 3, 4, ..., 15\}$

(vii) $A \cap A'$

From (i) $A' = \{1, 3, 5, ..., 15\}$

$A \cap A' = \{2, 4, 6, ..., 14\} \cap \{1, 3, 5, ..., 15\}$

$= \phi$

(viii) AUB'

From (ii) $B' = \{2, 4, 6, ..., 14\}$

$AUB' = \{2, 4, 6, ..., 14\} U \{2, 4, 6, ..., 14\}$

$= \{2, 4, 6, ..., 14\}$

Properties of union and intersection:

(a) Commutative property of union:

$AUB = BUA$

Prof: $Let \ x \in AUB$

$=> x \in A \ or \ x \in B$

$=> x \in B \ or \ x \in A$

$=> x \in BUA$

$AUB \subseteq BUA .....(i)$

$Let\ x \in BUA$

$\Rightarrow\ x \in B\ \ or\ \ x \in A$

$\Rightarrow\ x \in A\ \ or\ \ x \in B$

$\Rightarrow\ x \in AUB$

$\Rightarrow\ BUA \subseteq AUB$ .....(ii)

(b) Commutative property of intersection:

$A \cap B = B \cap A$

$Let\ \ x \in A \cap B$

$\Rightarrow\ x \in A\ \wedge\ x \in B$

$\Rightarrow\ x \in B\ \wedge\ x \in A$

$\Rightarrow x \in B \cap A$

$A \cap B \subseteq B \cap A$ .....(i)

$Let\ x \in B \cap A$

$\Rightarrow\ x \in B\ \wedge\ x \in A$

$\Rightarrow\ x \in A\ \wedge\ x \in B$

$\Rightarrow\ x \in A \cap B$

$\Rightarrow\ B \cap A \subseteq A \cap B$ .....(ii)

From (i) and (ii) $\quad A \cap B = B \cap A$

(c) Associative property of union:

$AU(BUC) = (AUB)UC$

(d) Associative property of intersection:

$A \cap (B \cap C) = (A \cap B) \cap C$

(e) Distributive property of union over intersection:

$AU(B \cap C) = (AUB) \cap (AUC)$

(f) Distributive property of intersection over union:

$A \cap (BUC) = (A \cap B)U(A \cap C)$

<u>De Morgan,s Laws</u>: (i) $(AUB)' = A' \cap B'$

(ii) $(A \cap B)' = A'UB'$

Proof: $(AUB)' = A' \cap B'$

$Let \quad x \in (AUB)'$

$\Rightarrow x \notin AUB$

$\Rightarrow x \notin A \quad and \quad x \notin B$

$\Rightarrow x \in A' \quad and \quad x \in B'$

$\Rightarrow x \in A' \cap B'$

$(AUB)' \subseteq A' \cap B' \;.......(i)$

$Let \quad x \in A' \cap B'$

$\Rightarrow x \in A' \quad and \quad x \in B'$

$\Rightarrow x \notin A \quad and \quad x \notin B$

$\Rightarrow x \notin AUB$

$\Rightarrow x \in (AUB)'$

$A' \cap B \subseteq (AUB)' \;......(ii)$

From (i) and (ii) $(AUB)' = A' \cap B'$.

(ii) $(A \cap B)' = A'UB'$

$Let \quad x \in (A \cap B)'$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A \quad or \quad x \notin B$

$\Rightarrow x \in A' \quad or \quad x \in B'$

$\Rightarrow x \in A'U\, B'$

$(A \cap B)' \subseteq A'U\, B' \;.....(i)$

$Let \quad x \in A'U\, B'$

$\Rightarrow x \in A' \quad or \quad x \in B'$

$\Rightarrow x \notin A \quad or \quad x \notin B$

$\Rightarrow x \notin A \cap B$

$x \in (A \cap B)'$

$(AUB)' \subseteq A' \cap B'$ .....(ii)

From (i) and (ii) $(A \cap B)' = A'UB'$.

Exercise 5.2:

Q.1: Verify commutative property of union and intersection for the following sets:

(i) $A = \{1, 2, 3, ..., 12\}$, $B = \{2, 4, 5, 8, 10, 12\}$

(ii) $A = N$, $B = \{x / x \in N$, x *is an even integer*$\}$

(iii) A = Set of first ten prime numbers.
B = Set of first ten composite numbers.

Solution: (i) $A = \{1, 2, 3, ..., 12\}$, $B = \{2, 4, 5, 8, 10, 12\}$

(a) Commutative property of Union : A U B = B U A .

$AUB = \{1, 2, 3, ..., 12\} U \{2, 4, 5, 8, 10, 12\}$

$= \{1, 2, 3, ..., 12\}$ ....(i)

$BUA = \{2, 4, 5, 8, 10, 12\} U \{1, 2, 3, ..., 12\}$

$\Rightarrow BUA = \{1, 2, 3, ..., 12\}$........(ii)

From (i) and (ii) it is proved A U B = B U A .

(b) Commutative property of intersection: $A \cap B = B \cap A$

$A \cap B = \{1, 2, 3, ..., 12\} \cap \{2, 4, 5, 8, 10, 12\}$

$= \{2, 4, 5, 8, 10, 12\}$ ....(i)

Now $B \cap A = \{2, 4, 5, 8, 10, 12\} \cap \{1, 2, 3, ..., 12\}$

$\Rightarrow B \cap A = \{2, 4, 5, 8, 10, 12\}$........(ii)

From (i) and (ii) it is proved $A \cap B = B \cap A$

(ii) $A = \{1, 2, 3, ...\}$, $B = \{2, 4, 6, ...\}$

(a) Commutative property of Union : A U B = B U A

$AUB = \{1, 2, 3, ...\} U \{2, 4, 6, ...\}$

$= \{1, 2, 3, ...\}$ ....(i)

$Now \quad B \cup A = \{2, 4, 6, ...\} \cup \{1, 2, 3, ...\}$

$= \{1, 2, 3, ...\} ....(ii)$

From (i) and (ii) it is proved A U B = B U A .

(b) Commutative property of intersection: $A \cap B = B \cap A$

$A \cap B = \{1, 2, 3, ...\} \cap \{2, 4, 6, ...\}$

$= \{2, 4, 6, ...\} ....(i)$

$Now \quad B \cap A = \{2, 4, 6, ...\} \cap \{1, 2, 3, ...\}$

$= \{2, 4, 6, ...\} ....(ii)$

From (i) and (ii) it is proved $A \cap B = B \cap A$.

(iii) $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

$B = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

(a) Commutative property of Union : A U B = B U A

$A \cup B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

$\cup \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$= \{2, 3, 4, 5, ..., 19, 23, 29\} ....(i)$

$Now \quad B \cup A = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$\cup \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

$= \{2, 3, 4, 5, ..., 19, 23, 29\} ....(ii)$

From (i) and (ii) it is proved A U B = B U A .

(b) Commutative property of intersection: $A \cap B = B \cap A$

$A \cap B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

$\cap \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$= \{\ \} ....(i)$

$Now \quad B \cap A = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$\cap \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

$= \{\ \} ....(ii)$

From (i) and (ii) it is proved $A \cap B = B \cap A$.

<u>Q.2</u>: Verify associative properties of union and intersection for the following sets:

(i) $A = \{a, b, c, ..., z\}$, $B = \{a, e, i, o, u\}$

$C=\{a, d, i, l, m, n, o\}$

(ii) $A=\{1,2,\ldots,100\}$, $B=\{2,4,\ldots,100\}$

$C=\{1,3,5,\ldots,99\}$

Solution: (i) $A=\{a,b,c,\ldots,z\}$, $B=\{a,e,i,o,u\}$

$C=\{a,d,i,l,m,n,o\}$

(a) Associative property of union: AU(BUC) = (AUB) UC

$BUC=\{a,e,i,o,u\}U\{a,d,i,l,m,n,o\}$

$=\{a,d,e,i,l,m,n,o,u\}$

$AU(BUC)=\{a,b,c,\ldots,z\}U\{a,d,e,i,l,m,n,o,u\}$

$=\{a,b,c,\ldots,z\}$ ....(i)

Now $AUB=\{a,b,c,\ldots,z\}U\{a,e,i,o,u\}$

$=\{a,b,c,\ldots,z\}$

$(AUB)UC=\{a,b,c,\ldots,z\}U\{a,d,i,l,m,n,o\}$

$=\{a,b,c,\ldots,z\}$..................(ii)

From (i) and (ii) it is proved $AU(BUC)=(AUB)UC$

(b) Associative property of intersection:

$A\cap(B\cap C)=(A\cap B)\cap C$

$B\cap C=\{a,e,i,o,u\}\cap\{a,d,i,l,m,n,o\}$

$=\{a,i,o\}$

$A\cap(B\cap C)=\{a,b,c,\ldots,z\}\cap\{a,i,o\}$

$=\{a,i,o\}$ ....(i)

Now $A\cap B=\{a,b,c,\ldots,z\}\cap\{a,e,i,o,u\}$

$=\{a,e,i,o,u\}$

$(A\cap B)\cap C=\{a,e,i,o,u\}\cap\{a,d,i,l,m,n,o\}$

$=\{a,i,o\}$..................(ii)

From (i) and (ii) it is proved $A\cap(B\cap C)=(A\cap B)\cap C$.

(ii) $A=\{1,2,\ldots,100\}$, $B=\{2,4,\ldots,100\}$

$C=\{1,3,5,\ldots,99\}$

Associative property of union: AU(BUC) = (AUB) UC

$BUC = \{2, 4, ..., 100\} U \{1, 3, 5, ..., 99\}$

$= \{1, 2, 3, ..., 100\}$

$AU(BUC) = \{1, 2, 3, ..., 100\} U \{1, 2, 3, ..., 100\}$

$= \{1, 2, 3, ..., 100\}$ ....(i)

Now $AUB = \{1, 2, 3, ..., 100\} U \{2, 4, ..., 100\}$

$= \{1, 2, 3, ..., 100\}$

$(AUB)UC = \{1, 2, 3, ..., 100\} U \{1, 3, 5, ..., 99\}$

$= \{1, 2, 3, ..., 100\}$ ....(ii)

From (i) and (ii) it is proved $AU(BUC) = (AUB)UC$

(b) Associative property of intersection:

$A \cap (B \cap C) = (A \cap B) \cap C$

$B \cap C = \{2, 4, ..., 100\} \cap \{1, 3, 5, ..., 99\}$

$= \{\ \}$

$A \cap (B \cap C) = \{1, 2, 3, ..., 100\} \cap \{\ \}$

$= \{\ \}$ ....(i)

Now $A \cap B = \{1, 2, 3, ..., 100\} \cap \{2, 4, ..., 100\}$

$= \{2, 4, ..., 100\}$

$(A \cap B) \cap C = \{2, 4, ..., 100\} \cap \{1, 3, 5, ..., 99\}$

$= \{\ \}$ ....(ii)

From (i) and (ii) it is proved $A \cap (B \cap C) = (A \cap B) \cap C$.

<u>Q.3</u>: Verify distributive properties of union over intersection and intersection over union.

(i) $A = \{0, 1, 2\}$, $B = \{0\}$, $C = \phi$

(ii) $A = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$

$B = \{-1, -2, -3, -4, -5\}$, $C = \{-1, -2, 3, 4\}$

<u>Solution</u>: (i) $A = \{0, 1, 2\}$, $B = \{0\}$, $C = \phi$

(a) Distributive property of union over intersection:

$AU(B \cap C) = (AUB) \cap (AUC)$

$B \cap C = \{0\} \cap \phi = \phi$

$AU(B\cap C)=\{0,1,2\}U\phi$

$=\{0,1,2\}$ ......(i)

$Now\quad AUB=\{0,1,2\}U\{0\}$

$=\{0,1,2\}$

$AUC=\{0,1,2\}U\phi$

$=\{0,1,2\}$

$(AUB)\cap(AUC)=\{0,1,2\}\cap\{0,1,2\}$

$=\{0,1,2\}$ ....(ii)

From (i) and (ii) it is proved $AU(B\cap C)=(AUB)\cap(AUC)$

(b) Distributive property of intersection over union:

$A\cap(BUC)=(A\cap B)U(A\cap C)$

$BUC=\{0\}U\phi=\{0\}$

$A\cap(BUC)=\{0,1,2\}\cap\{0\}$

$=\{0\}$ ....(i)

$Now\quad A\cap B=\{0,1,2\}\cap\{0\}$

$=\{0\}$

$A\cap C=\{0,1,2\}\cap\phi=\phi$

$(A\cap B)U(A\cap C)=\{0\}U\phi=\{0\}$......$(ii)$

From (i) and (ii) it is proved $A\cap(BUC)=(A\cap B)U(A\cap C)$

(ii) $A=\{0,\pm1,\pm2,\pm3,\pm4,\pm5\}$

$B=\{-1,-2,-3,-4,-5\}$, $C=\{-1,-2,3,4\}$

(a) Distributive property of union over intersection:

$AU(B\cap C)=(AUB)\cap(AUC)$

$B\cap C=\{-1,-2,-3,-4,-5\}\cap\{-1,-2,3,4\}$

$=\{-1,-2\}$

$AU(B\cap C)=\{0,\pm1,\pm2,\pm3,\pm4,\pm5\}U\{-1,-2\}$

$=\{0,\pm1,\pm2,\pm3,\pm4,\pm5\}$ ....(i)

$Now \quad AUB = \{0,\pm1, \pm2, \pm3, \pm4, \pm5\}U\{-1,-2.-3,-4,-5\}$

$= \{0,\pm1, \pm2, \pm3, \pm4, \pm5\}$

$AUC = \{0,\pm1, \pm2, \pm3, \pm4, \pm5\}U\{-1,-2\}$

$= \{0,\pm1, \pm2, \pm3, \pm4, \pm5\}$

$(AUB)\cap(AUC) = \{0,\pm1, \pm2, \pm3, \pm4, \pm5\}$

$\cap\{0,\pm1, \pm2, \pm3, \pm4, \pm5\}$

$= \{0,\pm1, \pm2, \pm3, \pm4, \pm5\}$ ....(*ii*)

From (i) and (ii) it is proved $AU(B\cap C) = (AUB)\cap(AUC)$

(b) Distributive property of intersection over union:

$A\cap(BUC) = (A\cap B)U(A\cap C)$

$BUC = \{-1,-2,-3,-4,-5\}U\{-1,-2, 3, 4\}$

$= \{-1,-2,-3,-4,-5, 3, 4\}$

$A\cap(BUC) = \{0,\pm1, \pm2, \pm3, \pm4, \pm5\}$

$\cap\{-1,-2,-3,-4,-5, 3, 4\}$

$= \{-1,-2,-3,-4,-5, 3, 4\}$ ....(*i*)

$Now \quad A\cap B = \{0,\pm1, \pm2, \pm3, \pm4, \pm5\}\cap\{-1,-2,-3,-4,-5\}$

$= \{-1,-2,-3,-4,-5\}$

$A\cap C = \{0,\pm1, \pm2, \pm3, \pm4, \pm5\}\cap\{-1,-2, 3, 4\}$

$= \{-1,-2, 3, 4\}$

$(A\cap B)U(A\cap C) = \{-1,-2,-3,-4,-5\}U\{-1,-2, 3, 4\}$

$= \{-1,-2, -3, -4, -5, 3, 4\}$........(*ii*)

From (i) and (ii) it is proved $A\cap(BUC) = (A\cap B)U(A\cap C)$

<u>**Q.4**</u>**: Verify De, Morgan's laws for the following sets.**

(*i*) $U = \{x/x\in N \wedge 1\le x\le 20\}$

$A = \{2, 3, 5, 7, 11, 12, 13, 17\}$

$B = \{1, 4, 6, 8, 10, 14, 17, 18\}$

(ii) $U = \{1,2,....,10\}$, $A = \{2,4,6,8,10\}$, $B = \{1,3,5,7,9\}$

<u>**Solution**</u>: **(i)** U = { 1 , 2 , 3, 4 , ..., 20 }

$A=\{2,3,5,7,11,12,13,17\}$

$B=\{1,4,6,8,10,14,17,18\}$

(a) $(AUB)'=A'\cap B'$

Take L . H. S.

$AUB=\{2,3,5,7,11,12,13,17\}$

$U\{1,4,6,8,10,14,17,18\}$

$=\{1,2,3,4,5,6,7,8,10,11,12,13,14,17,18\}$

$(AUB)'=U/AUB=\{1,2,3,........,20\}$

$/\{1,2,3,4,5,6,7,8,10,11,12,13,14,17,18\}$

$=\{9,15,16,19,20\}$ ....(i)

Take R . H . S.

$A'=U/A=\{1,2,3.....,20\}/\{2,3,5,7,11,12,13,17\}$

$=\{1,4,6,8,9,10,14,15,16,18,19,20\}$

$B'=U/B=\{1,2,3.....,20\}/\{1,4,6,8,10,14,17,18\}$

$=\{2,3,5,7,9,11,12,13,15,16,19,20\}$

$\Rightarrow A'\cap B'=\{1,4,6,8,9,10,14,15,16,18,19,20\}$

$\cap\{2,3,5,7,9,11,12,13,15,16,19,20\}$

$=\{9,15,16,19,20\}$ ....(ii)

From (i) and (ii) it is proved $(AUB)'=A'\cap B'$

(b) $(A\cap B)'=A'UB'$

Take L . H. S.

$A\cap B=\{2,3,5,7,11,12,13,17\}$

$\cap\{1,4,6,8,10,14,17,18\}$

$=\{17\}$

$(A\cap B)'=U/A\cap B=\{1,2,3,........,20\}/\{17\}$

$=\{1,2,3,......,15,16,18,20\}$ ....(i)

Take R . H . S.

$A' = U / A = \{1, 2, 3 ....., 20\} / \{2, 3, 5, 7, 11, 12, 13, 17\}$

$= \{1, 4, 6, 8, 9, 10, 14, 15, 16, 18, 19, 20\}$

$B' = U / B = \{1, 2, 3 ....., 20\} / \{1, 4, 6, 8, 10, 14, 17, 18\}$

$= \{2, 3, 5, 7, 9, 11, 12, 13, 15, 16, 19, 20\}$

$\Rightarrow A'UB' = \{1, 4, 6, 8, 9, 10, 14, 15, 16, 18, 19, 20\}$

$U\{2, 3, 5, 7, 9, 11, 12, 13, 15, 16, 19, 20\}$

$= \{1, 2, 3, ... 15, 16, 18, 19, 20\}$ ....(ii)

From (i) and (ii) it is proved $(A \cap B)' = A'UB'$

(ii) $U = \{1, 2, ..., 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 3, 5, 7, 9\}$

(a) $(AUB)' = A' \cap B'$

Take L . H. S.

$AUB = \{2, 4, 6, 8, 10\} U \{1, 3, 5, 7, 9\}$

$= \{1, 2, ..., 10\}$

$(AUB)' = U / AUB = \{1, 2, ..., 10\} / \{1, 2, ..., 10\}$

$= \{\ \}$ ....(i)

Take R.H.S.

$A' = U / A = \{1, 2, ..., 10\} / \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

$B' = U / B = \{1, 2, ..., 10\} / \{1, 3, 5, 7, 9\}$

$= \{2, 4, 6, 8, 10\}$

$A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8, 10\}$

$= \{\ \}$............(ii)

From (i) and (ii) it is proved $(AUB)' = A' \cap B'$

(b) $(A \cap B)' = A'UB'$

Take L . H. S.

$A \cap B = \{2, 4, 6, 8, 10\} \cap \{1, 3, 5, 7, 9\}$

$= \{ \}$

$(A \cap B)' = U / AUB' = \{1, 2, ..., 10\} / \{ \}$

$= \{1, 2, ..., 10\}$ ....(*i*)

Take R.H.S.

$A' = U / A = \{1, 2, ..., 10\} / \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

$B' = U / B = \{1, 2, ..., 10\} / \{1, 3, 5, 7, 9\}$

$= \{2, 4, 6, 8, 10\}$

$A'UB' = \{1, 3, 5, 7, 9\} U \{2, 4, 6, 8, 10\}$

$= \{1, 2, ..., 10\}$ ....(*ii*)

From (i) and (ii) it is proved $(AUB)' = A' \cap B'$

Venn diagram: In Venn diagram the universal set U is represented by a rectangle and other sets are represented by small circles inside the rectangle.

Exercise 5.3:

Q.1: If A = { 1 , 2 , 3 , 4 , 5 } and B = { 2 , 3 , 6 , 7} .Then Draw Venn diagram for the following .

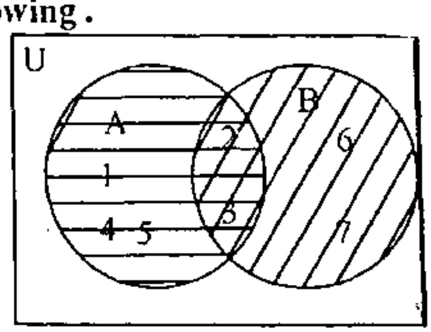
(*i*) AUB (*ii*) $A \cap B$

Solution: (i) AUB

$AUB = \{1, 2, 3, 4, 5\} U \{2, 3, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

The shaded part show AUB.



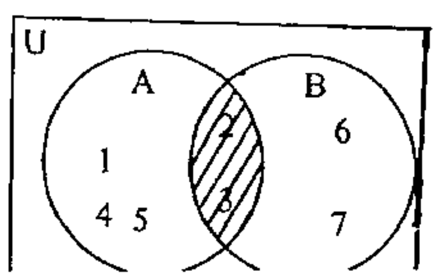
Venn diagram A UB

(ii) $A \cap B$

$A \cap B = \{1, 2, 3, 4, 5\} \cap \{2, 3, 6, 7\}$

$= \{3\}$

The shaded part show $A \cap B$



Venn diagram $A \cap B$

Q.2: If $A=\{1,2,3,4,5,6\}$, $B=\{3,4,5,6,7,8\}$ *and* $C=\{5,6,9,10\}$ **then with the help of Venn diagram.**

(i) $AU(BUC)=(AUB)UC$

(ii) $A\cap(B\cap C)=(A\cap B)\cap C$

(iii) $AU(B\cap C)=(AUB)\cap(AUC)$

(iv) $A\cap(BUC)=(A\cap B)U(A\cap C)$

Solution: $A=\{1,2,3,4,5,6\}$, $B=\{3,4,5,6,7,8\}$ *and* $C=\{5,6,9,10\}$

(i) $AU(BUC)=(AUB)UC$

Take L . H . S.

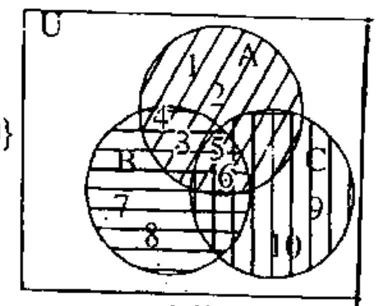
$BUC=\{3,4,5,6,7,8\}U\{5,6,9,10\}$

$=\{3,4,5,6,7,8,9,10\}$

$AU(BUC)=\{1,2,3,4,5,6\}$

$U\{3,4,5,6,7,8,9,10\}$

$=\{1,2,3,4,5,6,7,8,9,10\}$.....(*i*)



(*i*)

The shaded part show A U (B U C)

Venn diagram of A U (B U C)

Take R . H . S.

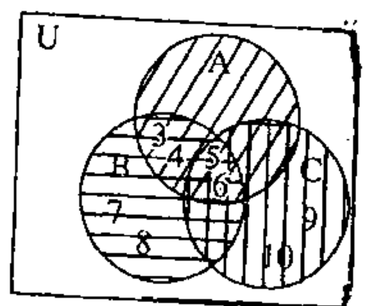
$AUB=\{1,2,3,4,5,6\}U\{3,4,5,6,7,8\}$

$=\{1,2,3,4,5,6,7,8\}$

$(AUB)UC=\{1,2,3,4,5,6,7,8\}$

$U\{5,6,9,10\}$

$=\{1,2,3,4,5,6,7,8,9,10\}$.....(*i*)



(*ii*)

The shaded part show (AUB)UC .

Venn diagram (AUB)UC.

It is clear from figure (i) and (ii) the parts which shows AU(BUC) and (AUB)UC are equal .

Thus $AU(BUC)=(AUB)UC$

(*ii*) $A\cap(B\cap C)=(A\cap B)\cap C$

Take L . H . S.

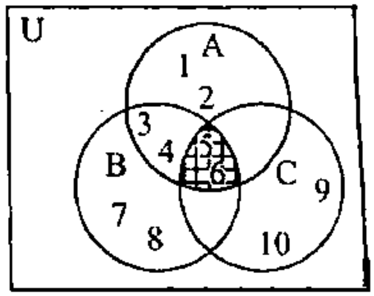
$B\cap C=\{3,4,5,6,7,8\}\cap\{5,6,9,10\}$

$=\{5, 6\}$

$A\cap(B\cap C)=\{1,2,3,4,5,6\}\cap\{5, 6\}$

$=\{5,6\}.....(i)$

The shaded part show $A\cap(B\cap C)$



Venn diagram of $A\cap(B\cap C)$ (i)

Take R . H . S.

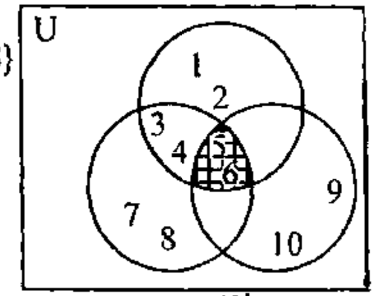
$A\cap B=\{1,2,3,4,5,6\}\cap\{3,4,5,6,7,8\}$

$=\{3,4,5,6\}$

$(A\cap B)\cap C=\{3,4,5,6\}\cap\{5,6,9,10\}$

$=\{5,6\}.....(ii)$

The shaded part show $(A\cap B)\cap C$.



Venn diagram $(A\cap B)\cap C$. (ii)

It is clear from figure (i) and (ii) the parts which shows $A\cap(B\cap C)$ and $(A\cap B)\cap C$ are equal .

Thus $A\cap(B\cap C)=(A\cap B)\cap C$

(iii) $AU(B\cap C)=(AUB)\cap(AUC)$

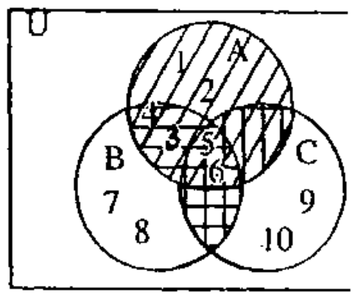
Take L . H . S.

$B\cap C=\{3,4,5,6,7,8\}\cap\{5,6,9,10\}$

$=\{5, 6\}$

$AU(B\cap C)=\{1,2,3,4,5,6\}U\{5, 6\}$

$=\{1,2,3,4,5,6\}.....(i)$



(i)

The shaded part show $AU(B\cap C)$

Venn diagram of $AU(B\cap C)$

Take R . H . S.

$AUB=\{1,2,3,4,5,6\}U\{3,4,5,6,7,8\}$

$=\{1,2,3,4,5,6,7,8\}$

$AUC=\{1,2,3,4,5,6\}U\{5,6,9,10\}$

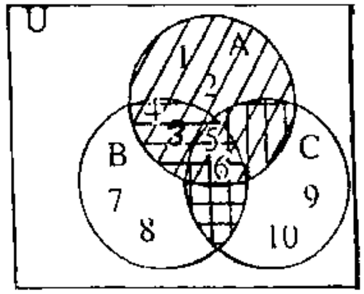
$=\{1,2,3,4,5,6,9,10\}$

$(AUB)\cap(AUC)=\{1,2,3,4,5,6,7,8\}$

$\cap\{1,2,3,4,5,6,9,10\}$

$=\{1,2,3,4,5,6\}.....(ii)$

The shaded part show $(AUB)\cap(AUC)$.



Venn diagram $(AUB)\cap(AUC)$ (ii)

It is clear from figure (i) and (ii) the parts which shows $AU(B\cap C)$ and $(AUB)\cap(AUC)$ are equal .

Thus $AU(B\cap C) = (AUB)\cap(AUC)$

(iv) $A\cap(BUC)=(A\cap B)U(A\cap C)$

Take L . H . S.

$BUC=\{3,4,5,6,7,8\}U\{5,6,9,10\}$

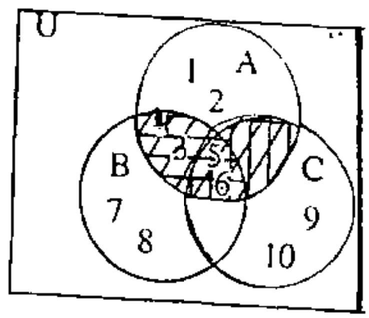
$=\{3,4,5,6,7,8,9,10\}$

$A\cap(BUC)=\{1,2,3,4,5,6\}$

$\cap\{3,4,5,6,7,8,9,10\}$

$=\{3,4,5,6\}.....(i)$

The shaded part show $A\cap(BUC)$



Venn diagram of $A\cap(BUC)$ (i)

Take R . H . S.

$A\cap B=\{1,2,3,4,5,6\}\cap\{3,4,5,6,7,8\}$

$=\{3,4,5,6\}$

$A\cap C=\{1,2,3,4,5,6\}\cap\{5,6,9,10\}$

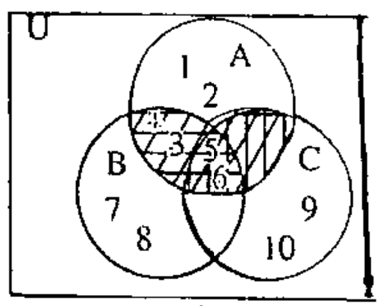
$=\{5,6\}$

$(A\cap B)U(A\cap C)=\{3,4,5,6\}$

$U\{5,6\}$

$=\{3,4,5,6\}.....(ii)$

The shaded part show $(A\cap B)U(A\cap C)$.



Venn diagram $(A\cap B)U(A\cap C)$ (ii)

It is clear from figure (i) and (ii) the parts which shows

$A \cap (BUC)$ and $(A \cap B)U(A \cap C)$ are equal.

Thus $A \cap (BUC) = (A \cap B)U(A \cap C)$

Q.3:If $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3, 4\}$

And $B = \{3, 4, 5\}$ **then draw Venn diagrams for**

A', B', $A'UB'$ *and* $A' \cap B'$ **and also verify that**

(i) $(AUB)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

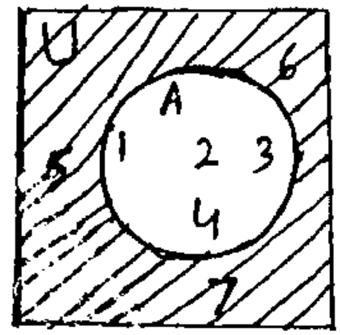
Solution: A'

$A' = U / A = \{1, 2, 3, 4, 5, 6, 7\}$

$/\{1, 2, 3, 4\}$

$= \{5, 6, 7\} \ldots\ldots (i)$

The shaded part is show A'.



Venn diagram of A'

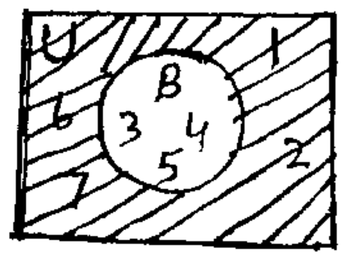
B':

$B' = U / B = \{1, 2, 3, 4, 5, 6, 7\}$

$/\{3, 4, 5\}$

$= \{1, 2, 6, 7\} \ldots\ldots (ii)$

The shaded part show B'.



Venn diagram of B'.

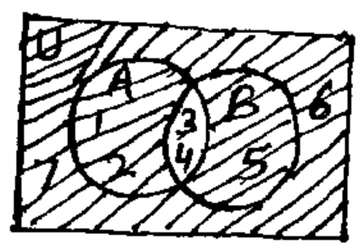
$A'UB'$: From (i) $A' = \{5, 6, 7\}$

From (ii) $B' = \{1, 2, 6, 7\}$

$A'UB' = \{5, 6, 7\}U\{1, 2, 6, 7\}$

$= \{1, 2, 5, 6, 7\}$

The shaded part show $A'UB'$.



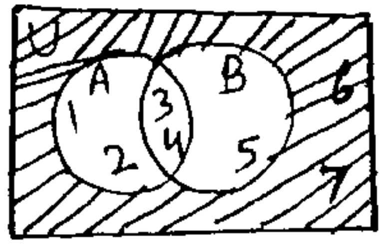
Venn diagram of $A'UB'$.

$A' \cap B'$:

$A' \cap B' = \{5, 6, 7\} \cap \{1, 2, 6, 7\}$

$= \{6, 7\}$

The shaded part show $A' \cap B'$



Venn diagram of $A' \cap B'$

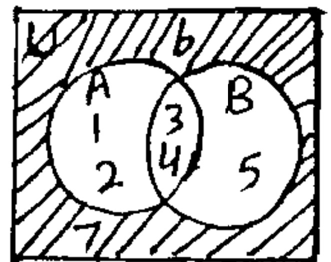
(i) $(AUB)' = A' \cap B'$

$AUB = \{1, 2, 3, 4\} U \{3, 4, 5\}$

$= \{1, 2, 3, 4, 5\}$

$(A \cup B)' = U / (A \cup B) = \{6, 7\}$

The shaded part show $(A \cup B)'$.



Venn diagram of $(A \cup B)'$.

fig(i)

$A' \cap B'$:

$A' = U / A = \{5, 6, 7\}$

$B' = U / B = \{1, 2, 6, 7\}$

$A' \cap B' = \{5, 6, 7\} \cap \{1, 2, 6, 7\}$

$= \{6, 7\}$

The shaded part show $A' \cap B'$:.

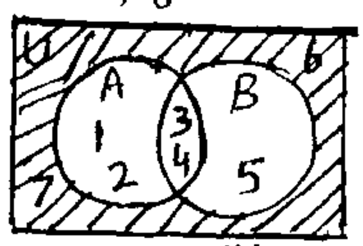


Fig (ii)

Venn diagram of B U A .

.It is clear from figure (i) and (ii) the parts which shows

$(A \cup B)'$ and $A' \cap B'$ which are equal .

(ii) $(A \cap B)' = A' \cup B'$

$A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 5\}$

$= \{3, 4\}$......(i)

$(A \cap B)' = U / (A \cap B) = \{1, 2, 5, 6, 7\}$



FIg(i)

The shaded part show $(A \cap B)'$.

Venn diagram of $(A \cap B)'$.

$A' \cup B' = \{5, 6, 7\} \cup \{1, 2, 6, 7\}$

$= \{1, 2, 5, 6, 7\}$



Fig (ii)

The shaded part show $A' \cup B'$.

Venn diagram of $A' \cup B'$.

It is clear from figure (i) and (ii) the parts which shows $(A \cap B)'$ and $A' \cup B'$ which are equal.

Q.4: **If** $U = \{a, b, c, 1, 2, 3, 4\}$, $A = \{c, 3\}$ *and* $B = \{a, 3, 4\}$.

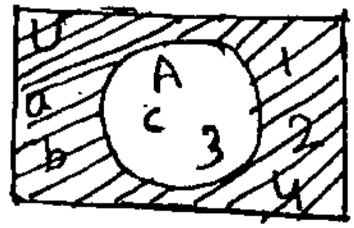
Then draw Venn diagrams of A' , B' , A/B *and* B/A .

Solution: A'

$A' = U/A = \{a, b, c, 1, 2, 3, 4\}/\{c, 3\}$

$= \{a, b, 1, 2, 4\}$

The shaded part is show A'.



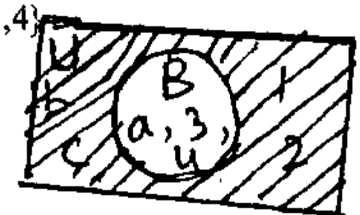
Venn diagram of A'

B':

$B' = U - B = \{a, b, c, 1, 2, 3, 4\}/\{a, 3, 4\}$

$= \{b, c, 1, 2\}$

The shaded part show B'.

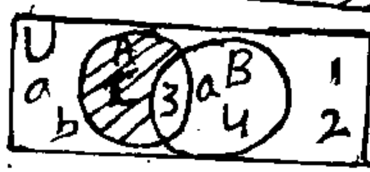


Venn diagram of B'.

A/B:

$A/B = \{c, 3\}/\{a, 3, 4\} = \{c\}$

The shaded part show A / B.

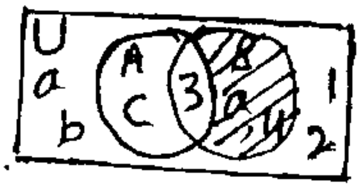


Venn diagram of A / B.

B/A:

$B/A = \{a, 3, 4\}/\{c, 3 = \{a\}$

The shaded part show B/ A.



Venn diagram of B / A.

Q.5: **If** $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, b, c\}$ **and** $B = \{c, d, e\}$ **then verify De- Morgan's laws with the help of Venn diagrams.**

Solution: $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, b, c\}$

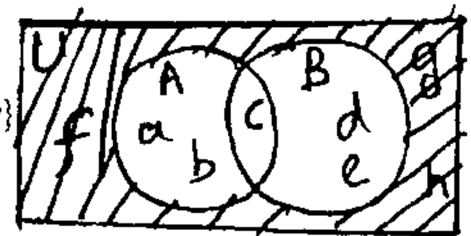
$B = \{c, d, e\}$

(a) $(AUB)' = A' \cap B'$

Take L .H . S.

$AUB = \{a, b, c\} U \{c, d, e\}$

$= \{a, b, c, d, e\}$



Fig(i)

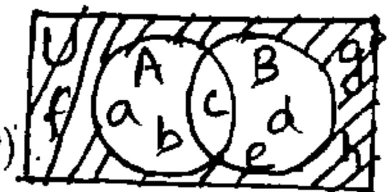
$(AUB)' = U / AUB$

$= \{a, b, c, d, e, f, g, h\} / \{a, b, c, d, e\}$

$= \{f, g, h\}$........(i)

The shaded part show $(AUB)'$

Venn diagram of $(AUB)'$.



Fig(ii)

Take R . H . S.

$A' = U / A = \{a, b, c, d, e, f, g, h\} / \{a, b, c\}$

$= \{d, e, f, g, h\}$

$B' = U / B = \{a, b, c, d, e, f, g, h\} / \{c, d, e\}$

$= \{a, b, f, g, h\}$

$A' \cap B' = \{d, e, f, g, h\} \cap \{a, b, f, g, h\}$

$= \{f, g, h\}$......(ii)

The shaded part show $A' \cap B'$.

Venn diagram of $A' \cap B'$.

It is clear from figure (i) and (ii) the parts which shows $(AUB)'$ and $A' \cap B'$. which are equal .

(ii) $(A \cap B)' = A' U B'$ .Take L .H . S.

$A \cap B = \{a, b, c\} \cap \{c, d, e = \{c\}$

$(A \cap B)' = U / A \cap B$

$= \{a, b, c, d, e, f, g, h\} / \{c\}$

$= \{a, b, d, f, e, g, h\}$........(i)



Fig(i)

The shaded part show $(A \cap B)'$

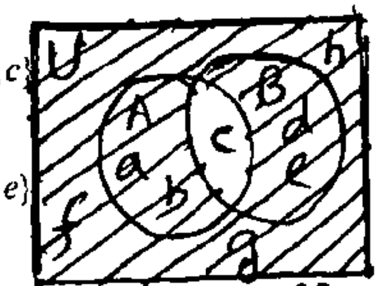
Venn diagram of $(A \cap B)'$.

Take R . H . S.

$A' = U / A = \{a,b,c,d,e,f,g,h\} / \{a,b,c\}$
$= \{d, e, f, g, h\}$

$B' = U / B = \{a,b,c,d,e,f,g,h\} / \{c,d,e\}$
$= \{a, b, f, g, h\}$

$A'UB' = \{d, e, f, g, h\} U \{a, b, f, g, h\}$
$= \{a, b, d, e, f, g, h\}......(ii)$



Fig(ii)

The shaded part show $A'UB'$.

Venn diagram of $A'UB'$

It is clear from figure (i) and (ii) the parts which shows $(A \cap B)'$ and $A'UB'$ which are equal.

Thus $(A \cap B)' = A'UB'$.

Ordered pair: (a, b) is called ordered pair in which "a" is the first element and "b" is the second element.

The two ordered pairs (a, b) *and* (c, d) are only equal if $a = c$ *and* $b = d$.

Cartesian product: Let "A" and "B" are non empty sets then $A \times B$ is called Cartesian product and the elements of $A \times B$ are in ordered pairs in which the first element of each ordered pair belong to A and second element belong to set B. symbolically we write

$$A \times B = \{(a, b) / a \in A \wedge b \in B\}$$

Binary relation: If A and B are any two non empty sets then any subset of $A \times B$ is called Binary relation.

Note: If set A has m elements and set B has n elements then The number of elements of $A \times B$ is mn and the number Subsets of $A \times B$ are 2^{mn}.

Domain and range of binary relation: If R is a binary relation from set A to set B then the set which contains the first elements of all ordered pairs in R is called Domain. While the set which contains the second elements of all ordered pairs in R is called

Range .

Exercise 5.4:

Q.1: **If $A=\{1,2,3\}$, $B=\{4,5\}$ .Then**

(i) Write 3 binary relations from A to B .

(ii) Write 4 binary relations from B to A .

(iii) Write 4 binary relations in A .

(iv) Write 2 binary relations in B.

Solution: $A=\{1,2,3\}$, $B=\{4,5\}$

(i) $A\times B=\{(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)\}$

Since any subset of $A\times B$ is called binary relation .

Then 3 binary relations from A to B are

$R_1=\{(1,4)\}$, $R_2=\{(1,4),(2,5)\}$

$R_3=\{(3,4)\}$

(ii) $B\times A=\{(4,1),(4,2),(4,3),(5,1),(5,2),(5,3)\}$

Then any 4 binary relations from B to A are

$R_1=\{(4,1)\}$, $R_2=\{(4,2)\}$

$R_3=\{(4,1),(4,2)\}$, $R_4=\{(5,1)\}$

(iii) $A\times A=\left\{\begin{matrix}(1,1),(1,2),(1,3),(2,1),(2,2),(2,3)\\ (3,1),(3,2),(3,3)\end{matrix}\right\}$

Then any 4 binary relations from A to A are

$R_1=\{(1,1),(1,2)\}$, $R_2=\{(3,1)\}$

$R_3=\{(1,2)\}$, $R_4=\{(3,2),(2,1)\}$

(iv) $B\times B=\{(4,4),(4,5),(5,4),(5,5)\}$

Then any 2 binary relations from B to B are

$R_1=\{(4,4)\}$, $R_2=\{(4,5),(5,5)\}$

Q.2: If $A=\{1,2,3,4\}$, $B=\{1,3,5\}$ and

$R=\{(x,y)/\ y<x\}$ is a binary relation from A to B , then write in tabular form .

Solution: $A=\{1,2,3,4\}$, $B=\{1,3,5\}$

$R=\{(x,y)/\ y<x\}$ ......(i)

$$A \times B = \begin{Bmatrix} (1,1), (1,3), (1,5), (2,1), (2,3), (2,5) \\ (3,1), (3,3), (3,5), (4,1), (4,3), (4,5) \end{Bmatrix}$$

Then relation (i) in tabular form is

$R = \{(2,1), (3,1), (4,1), (4,3)\}$

Q.3: Domain of a binary relation $R = \{(x, y) / y = 2x\}$ in set W is { 0 , 4 , 8 }. Find range .

Solution: $R = \{(x, y) / y = 2x\}$

$y = 2x$ ....(i)

Put x = 0 in (i) $y = 2(0) = 0$

Put x = 4 in (i) $y = 2(4) = 8$

Put x = 8 in (i) $y = 2(8) = 16$

Thus range is { 0 , 8 ,16}.

Q.4: Domain of a binary relation $R = \{(x, y) / y + 1 = 2x^2\}$ is set N .Find its range.

Solution: $R = \{(x, y) / y + 1 = 2x^2\}$ ....(i)

$y + 1 = 2x^2$

=> $y = 2x^2 - 1$......(i)

Since N is domain of R . Then

Put x = 1 in (i)

$y = 2(1)^2 - 1 = 2 - 1 = 1$

Put x = 2 in(i)

$y = 2(2)^2 - 1 = 2(4) - 1$

$= 8 - 1 = 7$

Put x = 3 in(i)

$y = 2(3)^2 - 1 = 2(9) - 1$

=> $y = 18 - 1 = 17$

And so on .

Thus range of R is {1 , 7 , 17 ,........} .

Function: Let A and B are two non empty sets then a binary relation f from A to B is called function if f contains those ordered pairs whose first elements are all distinct and domain set is equal to set A

OR

Let A and B are two non empty sets then a binary relation f from A to B is called function if

(i) Domain f = A

(ii) There should be no repetition in the first element of all ordered pairs contained in f. Symbolically we write

$$f : A \longrightarrow B$$

Kinds of function: **(i) Into function**: Let f be a function from A to B, then f is into function if $Range\ of\ \ f \neq B$

(ii) One –one or (1 – 1) function: A function f from A to B is called one –one if there is no repetition in the second element of all ordered pairs contained in f. i.e. Distinct elements of set A mapped to distinct elements of set B.

(iii) Into and one – one function (Injective): Let f be a function from A to B, then f is called into and one –one function if

(i) $Range\ f \neq B$

(iv) Onto function: Let f be a function from A to B, then f is called onto function if every element of set B is the image of some elements of set A. or Range f = B.

(v) one –one and onto function: A function f from set A to set B which is one –one and onto is called bijective function.

One – one correspondence: If A and B are two non empty Sets then a rule for which each element of a set A is Paired with one and only one element of set B and each element of a set B is Paired with one and only one element of set A. Then it is called one –one correspondence

Exercise 5.5:

Q.1: $A = \{1, 2, 3, 4\}$ and $B = \{6, 7\}$ and the following are relations from A to B, then state whether these are functions or not?. If these are functions then state which kind of functions are they ?.

$R_1 = \{(1,5), (2,7), (3,6)\}$
$R_2 = \{(1,6), (2,6), (3,7), (4,7)\}$
$R_3 = \{(1,6), (2,6), (3,6), (4,6)\}$

Solution: (i) $R_1 = \{(1,5), (2,7), (3,6)\}$ it is not a function

Because $Domain\ R_1 = \{1, 2, 3\}$ .Since

$$Domain\ R_1 = \{1, 2, 3\} \neq A$$

(ii) $R_2 = \{(1,6), (2,6), (3,7), (4,7)\}$

It is a function . it is onto. Because

$$Range\ R_2 = \{6, 7\} = B$$

(iii) $R_3 = \{(1,6), (2,6), (3,6), (4,6)\}$

It is into function because $Range\ R_3 = \{6\} \neq B$

Q.2: Which of the following relations on set $A = \{a, b, c, d\}$ are functions ? State the kind of functions as well ?

(i) $\{(a,b), (c,d), (b,d), (d,b)\}$
(ii) $\{(b,a), (a,b), (a,b), (d,d)\}$
(iii) $\{(d,c), (c,b), (a,b), (d,d)\}$
(iv) $\{(a,d), (b,c), (c,d), (d,a)\}$

Solution: (i) $\{(a,b), (c,d), (b,d), (d,b)\}$

It is into function . Because domain set { a , c , b , d} is equal to set A and range set { b , d} is not equal to set A .

(ii) $\{(b,a), (a,b), (a,b), (d,d)\}$

It is not a function because Domain set { b , a , d } is not equal to set A .

(iii) $\{(d,c), (c,b), (a,b), (d,d)\}$

It is not a function because Domain set { d , c , a } is not equal to set A .

(iv) $\{(a,d),(b,c),(c,d),(d,a)\}$

It is into function . Because domain set { a . b , c , d} is equal to set A and range set { d ,c ,a} is not equal to set A .

Q.3: If $A=\{0,1,2,3\}$ and $B=\{x,y,z,p\}$ then state Whether following relations show that there exist one –one Correspondence between the elements of set A and B . If not give reason .

(i) $\{(0,x),(2,z),(3,y),(1,p)\}$

(ii) $\{(0,x),(1,z),(2,y),(3,z)\}$

Solution: (i) $\{(0,x),(2,z),(3,y),(1,p)\}$

There is one – one correspondence because each element of a set A is Paired with one and only one element of set B and each element of a set B is Paired with one and only one element of set A.

(ii) $\{(0,x),(1,z),(2,y),(3,z)\}$

There is not one – one correspondence because member p of set B is unpaired .

Q.4: If $C=\{a,b,c\}$ and $D=\{2,3,4,5\}$ then , state whether the following relations shows that there exist one- one correspondence between the elements of set C and D if not what kind of these relations.

(i): $\{(a,2),(b,3),(c,4)\}$

(ii) $\{(a,3),(b,4),(c,3)\}$

Solution: (i): $\{(a,2),(b,3),(c,4)\}$

It is not one – one correspondence because 5 is the member of set D is not paired with element of C .
it into and one –one function because domain set { a , b , c} is equal to set C and range set { 2 , 3 , 4} is not equal to set D.

(ii) $\{(a,3),(b,4),(c,3)\}$

It is not one – one correspondence because 2 and 5 are the members of set D is not paired with element of C .
it into and one –one function because domain set { a , b , c} is equal to set C and range set { 3 , 4} is not equal to set D.

Q.5: If $X=\{1, 2, 3, 4\}$ and $Y=\{5, 6, 7, 8\}$ then write,

(i) A function from X to Y.

(ii) A one – one function from X to Y.

(iii) A relation which shows that there exist one – one Correspondence between X and Y.

(iv) A function which is onto from Y to X.

(v) A bijective function from Y to X.

(vi) A function from X to Y which is neither one –one nor onto.

Solution: $X=\{1, 2, 3, 4\}$ and $Y=\{5, 6, 7, 8\}$

(i) $R_1=\{(1, 5), (2, 6), (3, 6), (4, 7)\}$
is a function from X to Y.

(ii) $R_2=\{(1, 5), (2, 6), (3, 7), (4, 8)\}$
is a one –one function.

(iii) $R_3=\{(1, 6), (2, 5), (3, 7), (4, 8)\}$
It is one – one correspondence.

(iv) $R_4=\{(6, 1), (5, 2), (7, 3), (8, 4)\}$
It is a function from Y to X.

(v) $R_5=\{(5, 1), (6, 2), (7, 3), (8, 4)\}$
It is bijective function from Y to X.

(vi) $R_6=\{(1, 5), (2, 5), (3, 6), (4, 6)\}$
It is a function from X to Y which is neither one – one nor onto.

Q.6: Let $A=\{1,2,3,4,5\}$. Check whether the following sets are functions on A. In case these are functions indicate their ranges. Which functions is onto.

(i) $\{(1,5), (2,3), (3, 3), (4, 2), (5, 1)\}$

(ii) $\{(1, 1), (2, 4), (3, 2), (4, 1), (5, 3)\}$

(iii) $\{(1, 2), (2, 1), (3, 1), (4, 4), (5, 5)\}$

Solution: (i) $R_1=\{(1,5), (2,3), (3, 3), (4, 2), (5, 1)\}$

It is a function.

Range = $\{1, 2, 3,, 5\}$. **not onto.**

(ii) $R_2 = \{(1,1),(2,4),(3,2)(4,1),(5,3)\}$

It is a function . not onto .

Range = {1 ,4, 2,,3}

(iii) $R_3 = \{(1,2),(2,1),(3,1),(4,4),(5,5)\}$

It is a function .

Range = {2 ,1 ,4, 5}. not onto .

Review exercise 5:

Q.1: At the end of each question , four circles are given . Fill in the c orrect circle only .

(i) If $A = \{1, 2, 3\}$, $B = \{4, 5\}$ **and** $R = \{(1,4),(2,5),(3,4)\}$ **then R is ....**

(a) one –one function from A to B.

(b) A function from A to A .

(c) not a function (d) A onto function from A to B.

(ii) If A has two elements and B has 3 elements , then number of binary relations in $A \times B$ **.**

$(a) 2\times 3$ $(b)\ 2^3$ $(c)\ 2^6$ $(d)\ 2^2$.

(iii) Which one of the following is an example of disjoint Sets ? .

(a) $\{0,1,2,3\}$ *and* $\{3,2,1,0\}$ **(b)** $\{0,2,4,6\}$ *and* $\{2,4,6,8\}$

(c) $\{0,3,6,9\}$ *and* $\{9,16,25,36\}$ **(d)** $\{0,4,8,12\}$ *and* $\{6,10,14,18\}$

(iv) If the universal set

$U = \{x | x \text{ is positive odd integer less than } 30\}$ **and**

$R = \{1, 5, 7\}$ **and** $S = \{1, 3, 7, 11, 13\}$ **, how many elements are in** $(R \cap S)'$ **? .**

(a) 15 (b) 13 (c) 7 (d) 2

(v)If f is a function from A to B , then f is onto function if

(a) *Range* $f = B$ (b) *Range* $f \neq A$ (c) *Dom* $f = A$

(d) second element of all ordered pairs in f is not repeated.

(vi) *If* $R = \{(0,0),(8,2),(10,3),(14,12)\}$, **then Dom R = ...**

(a) { 0,8 ,10, 14} (b) {0 , 2 ,3 ,12}

(c) {8 ,10 ,4} (d) {0 ,10}

Answers: (i) d (ii) c (iii) d (iv) b (v) a (vi) a

Q.2: If U = { Natural numbers up to 100 }

A ={ Positive even numbers up to 100}

B = { Positive odd numbers up to 100} then find

(i) $A'UB'$ (ii) $A'\cap B'$ (iii) $A\cap B'$ (iv) $A'\cap B$

Solution: $U=\{1,2,3,.......,100\}$

$A=\{2,4,6,....,100\}$, $B=\{1,3,5,........,99\}$

(i) $A'UB'$:

$A'=U/A=\{1,2,3,......,100\}/\{2,4,6,....,100\}$

$\Rightarrow A'=\{1,3,5,......,99\}$

$B'=U/B=\{1,2,3,......,100\}/\{1,3,5,......,99\}$

$\Rightarrow B'=\{2,4,6,.......,100\}$

$A'UB'=\{1,3,5,......,99\}U\{2,4,6,......,100\}$

$=\{1,2,3,4,............,100\}$

(ii) $A'\cap B'$:

As $A'=\{1,3,5,......,99\}$ **and** $B'=\{2,4,6,.......,100\}$

$A'\cap B'=\{1,3,5,......,99\}\cap\{2,4,6,.......,100\}$

$=\{\ \}$

(iii) $A\cap B'$:

As $B'=\{2,4,6,.......,100\}$

$A\cap B'=\{2,4,6,.......,100\}\cap\{2,4,6,.......,100\}$

$=\{2,4,6,.......,100\}$

(iv) $A'\cap B$:

As $A'=\{1,3,5,......,99\}$

$A' \cap B = \{1, 3, 5, ..., 99\} \cap \{1, 3, 5, ..., 99\}$

$= \{1, 3, 5, ..., 99\}$

Q.3: If $A = \{1, 2, 3, 5, 7\}$, $B = \{2, 4, 6\}$, $C = \{2, 5, 9\}$

Then verify (i) Associative property of union.

(ii) Associative property of intersection .

(iii) Distributive property of union over intersection.

(iv) Distributive property of intersection over union .

Solution: $A = \{1, 2, 3, 5, 7\}$, $B = \{2, 4, 6\}$, $C = \{2, 5, 9\}$

(i) Associative property of union. $AU(BUC) = (AUB)UC$

$BUC = \{2, 4, 6\}U\{2, 5, 9\}$

$=> BUC = \{2, 4, 5, 6, 9\}$

$AU(BUC) = \{1, 2, 3, 5, 7\}U\{2, 4, 5, 6, 9\}$

$AU(BUC) = \{1, 2, 3, 4, 5, 6, 7, 9\}......(i)$

Take R . H .S.

$AUB = \{1, 2, 3, 5, 7\}U\{2, 4, 6\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

$(AUB)UC = \{1, 2, 3, 4, 5, 6, 7\}U\{2, 5, 9\}$

$= \{1, 2, 3, 4, 5, 6, 7, 9\}.......(ii)$

From (i) and (ii) it is proved $AU(BUC) = (AUB)UC$

(ii) Associative property of intersection. $A \cap (B \cap C) = (A \cap B) \cap C$

Take L . H . S.

$B \cap C = \{2, 4, 6\} \cap \{2, 5, 9\}$

$=> B \cap C = \{2\}$

$A \cap (B \cap C) = \{2\} \cap \{1, 2, 3, 5, 7\}$

$= \{2\} ....(i)$

Take R . H .S.

$A \cap B = \{1, 2, 3, 5, 7\} \cap \{2, 4, 6\}$

$= \{2\}$

$(A \cap B) \cap C = \{2\} \cap \{2, 5, 9\}$

$= \{2\} \ldots\ldots.(ii)$

From (i) and (ii) it is proved $A \cap (B \cap C) = (A \cap B) \cap C$

(iii) Distributive property of union over intersection.

$AU(B \cap C) = (AUB) \cap (AUC)$

$B \cap C = \{2, 4, 6\} \cap \{2, 5, 9\}$

$\Rightarrow B \cap C = \{2\}$

$AU(B \cap C) = \{1, 2, 3, 5, 7\} U \{2\}$

$= \{1, 2, 3, 5, 7\} \ldots\ldots\ldots.(i)$

Take R . H .S.

$AUB = \{1, 2, 3, 5, 7\} U \{2, 4, 6\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

$AUC = \{1, 2, 3, 5, 7\} U \{2, 5, 9\}$

$= \{1, 2, 3, 5, 7, 9\}$

$(AUB) \cap (AUC) = \{1, 2, 3, 4, 5, 6, 7\} \cap \{1, 2, 3, 5, 7, 9\}$

$= \{1, 2, 3, 5, 7\} \ldots\ldots.(ii)$

$AU(B \cap C) = (AUB) \cap (AUC)$

(iv) Distributive property of intersection over union .

$A \cap (BUC) = (A \cap B) U (A \cap C)$

Take L . H .S.

$BUC = \{2, 4, 6\} U \{2, 5, 9\}$

$\Rightarrow BUC = \{2, 4, 5, 6, 9\}$

$A \cap (BUC) = \{1, 2, 3, 5, 7\} \cap \{2, 4, 5, 6, 9\}$

$= \{2, 5\} \ldots\ldots\ldots.(i)$

Take R . H . S.

$A \cap B = \{1, 2, 3, 5, 7\} \cap \{2, 4, 6\}$

$= \{2\}$

$A \cap C = \{1, 2, 3, 5, 7\} \cap \{2, 5, 9\}$

$= \{2, 5\}$

$(A \cap B) U (A \cap C) = \{2, 5\} \cap \{2, 5\}$

$= \{, 2, 5\} \ldots\ldots (ii)$

From (i) and (ii) it is proved $A \cap (BUC) = (A \cap B) U (A \cap C)$

Q.4: **If** $U = \{x / x \in N \wedge 1 \le x \le 40\}$, $A = \{1, 6, 11, 16, 21, 26, 31\}$

$B = \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

Then verify De- Morgan's laws .

Solution: $U = \{1, 2, 3, \ldots\ldots, 40\}$, $A = \{1, 6, 11, 16, 21, 26, 31\}$

$B = \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

(a) $(AUB)' = A' \cap B'$

Take L .H . S.

$AUB = \{1, 6, 11, 16, 21, 26, 31\}$

$U\{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

$= \{1, 2, 5, 8, 11, 14, 16, 17, 20, 21, 23, 26, 29, 31, 32\}$

$(AUB)' = U / AUB = \{1, 2, 3, \ldots\ldots, 40\}$

$/\{1, 2, 5, 8, 11, 14, 16, 17, 20, 21, 23, 26, 29, 31, 32\}$

$$= \left\{ \begin{matrix} 3, 4, 6, 7, 9, 10, 12, 13, 15, 18, 19, 22, 24, 25, \\ 27, 28, 30, 33, 34, \ldots\ldots\ldots\ldots, 40 \end{matrix} \right\} \ldots\ldots (i)$$

Take R . H . S.

$A' = U / A = \{1, 2, 3, \ldots\ldots, 40\} / \{1, 6, 11, 16, 21, 26, 31\}$

$$= \left\{ \begin{matrix} 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15, 17, 18, 19 \\ 20, 22, 23, 24, 25, 27, 28, 29, 30, 32, 33, \ldots, 40 \end{matrix} \right\}$$

$B' = U / B = \{1, 2, 3, \ldots\ldots, 40\}$

$/\{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32\}$

$$=\begin{Bmatrix}1,3,4,6,7,9,10,12,13,15,16,18,19,21,22,24,\\25,27,28,30,31,33,34,\ldots\ldots,40\end{Bmatrix}$$

$$A'\cap B'=\begin{Bmatrix}2,3,4,5,7,8,9,10,12,13,14,15,17,18,19\\20,22,23,24,25,27,28,29,30,32,33,\ldots,40\end{Bmatrix}$$

$$\cap\begin{Bmatrix}1,3,4,6,7,9,10,12,13,15,16,18,19,21,22,24,\\25,27,28,30,31,33,34,\ldots\ldots,40\end{Bmatrix},$$

$$=\begin{Bmatrix}3,4,6,7,9,10,12,13,15,18,19,22,24,25,\\27,28,30,33,34,\ldots\ldots,40\end{Bmatrix}\ldots\ldots(ii)$$

From (i) and (ii) it is proved that $(AUB)'=A'\cap B'$.

(b) $(A\cap B)'=A'UB'$

Take L . H . S.

$A\cap B=\{1,6,11,16,21,26,31\}$

$\cap\{2,5,8,11,14,17,20,23,26,29,32\}$

$=\{11,26\}$

$(A\cap B)'=U/A\cap B=\{1,2,3,\ldots\ldots,40\}/\{11,26\}$

$=\{1,2,\ldots\ldots,10,12,13,\ldots\ldots,25,27,28,\ldots,40\}\ldots\ldots(i)$

Take R . H . S.

$A'=U/A=\{1,2,3,\ldots\ldots,40\}/\{1,6,11,16,21,26,31\}$

$$=\begin{Bmatrix}2,3,4,5,7,8,9,10,12,13,14,15,17,18,19\\20,22,23,24,25,27,28,29,30,32,33,\ldots,40\end{Bmatrix}$$

$B'=U/B=\{1,2,3,\ldots\ldots,40\}$

$/\{2,5,8,11,14,17,20,23,26,29,32\}$

$$=\begin{Bmatrix}1,3,4,6,7,9,10,12,13,15,16,18,19,21,22,24,\\25,27,28,30,31,33,34,\ldots\ldots,40\end{Bmatrix}$$

$$A'UB' = \begin{Bmatrix} 2,3,4,5,7,8,9,10,12,13,14,15,17,18,19 \\ 20,22,23,24,25,27,28,29,30,32,33,.....40 \end{Bmatrix}$$

$$U' \begin{Bmatrix} 1,3,4,6,7,9,10,12,13,15,16,18,19,21,22,24, \\ 25,27,28,30,31,33,34.......................40 \end{Bmatrix}$$

$$= \{1,2,.........10,12,13.........25,\ 27,28......40\} .....(ii)$$

From (i) and (ii) it is proved $(A \cap B)' = A'UB'$.

<u>Q.5</u>: If $U = \{1,2,3,5,6,7\}$, $A = \{2,5,6\}$, $B = \{1,2,3\}$
Then verify De-Morgan's laws with the help of Venn diagrams
<u>Solution</u>: $U = \{1,2,3,5,6,7\}$, $A = \{2,5,6\}$, $B = \{1,2,3\}$

(a) $(AUB)' = A' \cap B'$ **Take L.H.S.**

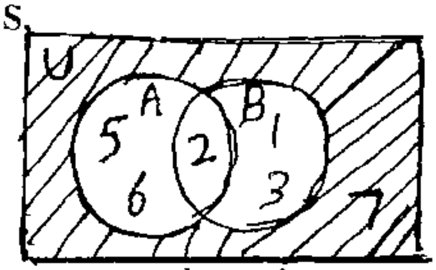
$AUB = \{2,5,6\}U\{1,2,3\}$

$= \{1,2,3,5,6\}$

$(AUB)' = U / AUB$

$= \{1,2,3,5,6,7\}/\{1,2,3,5,6\}$

$= \{7\}.......(i)$



Fig(i)

The shaded part show $(AUB)'$.

Venn diagram of $(AUB)'$

Take L.H.S.

$A' = U / A = \{1,2,3,5,6,7\}/\{2,5,6\}$

$= \{1,3,7\}$

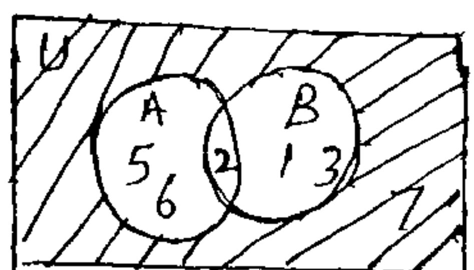
$B' = U / B$

$= \{1,2,3,5,6,7\}/\{1,2,3\}$

$= \{5,6,7\}$

$A' \cap B' = \{1,3,7\} \cap \{5,6,7\}$

$= \{7\}........(ii)$



Fig(ii)

The shaded part show $A' \cap B'$.

Venn diagram $A' \cap B'$.

It is clear from figure (i) and (ii) the parts which shows $(AUB)'$ **and** $A'\cap B'$**, which are equal.**

Thus $(AUB)' = A'\cap B'$.

(ii) $(A\cap B)' = A'UB'$

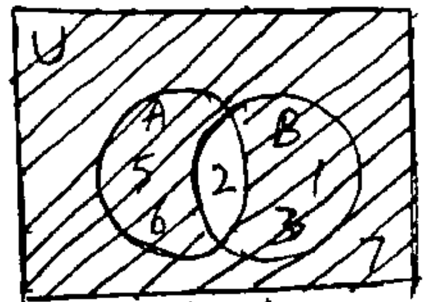
$A\cap B = \{2,5,6\}\cap\{1,2,3\}$

$= \{2\}$

$(A\cap B)' = U / A\cap B$

$= \{1,2,3,5,6,7\}/\{2\}$

$= \{1,3,5,6,7\}$.......(i)



Fig(i)

Take L . H . S.

$A' = U / A$

$= \{1,2,3,5,6,7\}/\{2,5,6\}$

$= \{1,3,7\}$

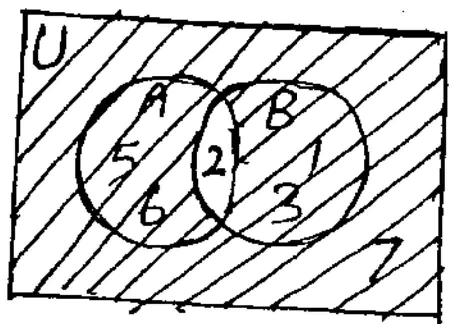
$B' = U / B$

$= \{1,2,3,5,6,7\}/\{1,2,3\}$

$= \{5,6,7\}$

$A'UB' = \{1,3,7\}U\{5,6,7\}$

$= \{1,3,5,6,7\}$........(ii)



Fig(ii)

The shaded part show $A'UB'$.

It is clear from figure (i) and (ii) the parts which shows $(A\cap B)'$ **and** $A'UB'$ **which are equal.**

Thus $(A\cap B)' = A'UB'$.

<u>Q.6</u>: If $U = \{1,2,3,.....,10\}$,

$A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$, $C = \{3,4,7,8\}$ **then verify distributive laws by Venn diagrams.**

<u>Solution</u>: (i) $AU(B\cap C) = (AUB)\cap(AUC)$

Take L . H. S.

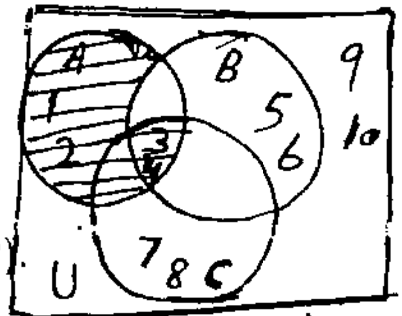
$B \cap C = \{3, 4, 5, 6\} \cap \{3, 4, 7, 8\}$

$= \{3, 4\}$

$AU(B \cap C) = \{1, 2, 3, 4\} U \{3, 4\}$

$= \{1, 2, 3, 4\}$........(i)

The shaded part show $AU(B \cap C)$.



Venn diagram of $AU(B \cap C)$. Fig(i)

Take R . H . S.

$AUB = \{1, 2, 3, 4\} U \{3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6\}$

$AUC = \{1, 2, 3, 4\} U \{3, 4, 7, 8\}$

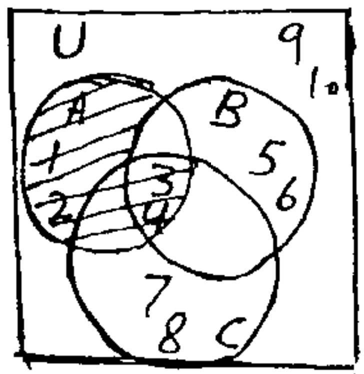
$= \{1, 2, 3, 4, 7, 8\}$

$(AUB) \cap (AUC) = \{1, 2, 3, 4, 5, 6\}$

$\cap \{1, 2, 3, 4, 7, 8\}$

$= \{1, 2, 3, 4\}$........(ii)

The shaded part show $(AUB) \cap (AUC)$.



Fig(ii)

Venn diagram of $(AUB) \cap (AUC)$.

It is clear from figure (i) and (ii) the shaded parts which shows $AU(B \cap C)$ **and** $(AUB) \cap (AUC)$ **are equal.**

Thus $AU(B \cap C) = (AUB) \cap (AUC)$

(b) $A \cap (BUC) = (A \cap B) U (A \cap C)$

Take L . H. S.

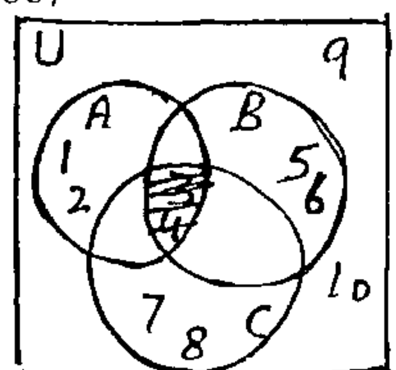
$BUC = \{3, 4, 5, 6\} U \{3, 4, 7, 8\}$

$= \{3, 4, 5, 6, 7, 8\}$

$A \cap (BUC)$

$= \{1, 2, 3, 4\} \cap \{3, 4, 5, 6, 7, 8\}$

$= \{3, 4\}$........(i)



The shaded part show $A \cap (BUC)$.

Venn diagram of $A \cap (BUC)$. Fig(i)

Take R.H.S.

$A\cap B=\{1,2,3,4\}\cap\{3,4,5,6\}$

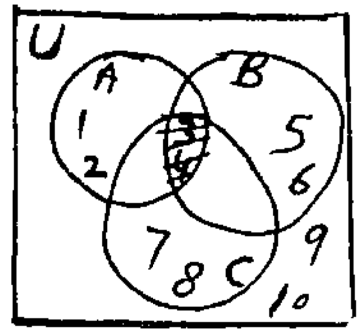
$=\{3,4\}$

$A\cap C=\{1,2,3,4\}\cap\{3,4,7,8\}$

$=\{3,4\}$

$(A\cap B)U(A\cap C)=\{3,4\}U\{3,4\}$

$=\{3,4\}$........(ii)



Fig(ii)

The shaded part show $(A\cap B)U(A\cap C)$

Venn diagram of $(A\cap B)U(A\cap C)$.

The shaded parts which show $A\cap(BUC)$ and $(A\cap B)U(A\cap C)$ are equal.

Thus $A\cap(BUC)=(A\cap B)U(A\cap C)$.

Q.7: Let $A=\{-2,-1,0,1,2\}$, $B=\{a, b, c, d, e\}$.

Determine which sets of ordered pairs represent a function. In case of a function, mention one –one function, onto function and bijective function.

(i) $\{(-2,a),(-1,a),(0,b),(1,c),(2,d)\}$

(ii) $\{(-1,a),(1,e),(-2,d),(0,c),(2,b)\}$

(iii) $\{(2,d),(0,a),(-2,b),(-1,c),(1,e)\}$

(iv) $\{(-2,b),(-1,c),(0,a),(1,d),(-2,e)\}$

Solution: (i) $\{(-2,a),(-1,a),(0,b),(1,c),(2,d)\}$

It is a function.

Range = $\{a, b, c, d\}$

(ii) $\{(-1,a),(1,e),(-2,d),(0,c),(2,b)\}$

It is a function. it is bijective function.

Range = $\{a, b, c, d, e\}$

(iii) $\{(2,d),(0,a),(-2,b),(-1,c),(1,e)\}$

It is a function. it is bijective function.

Range = $\{a, b, c, d, e\}$

(iv) $\{(-2,b),(-1,c),(0,a),(1,d),(-2,e)\}$

It is not a function.

Q.8: Let $A = \{1, 2, 3, 4, 5\}$, check whether the following sets are function on A. In case these are function, indicate their ranges. which function are onto.

(i) $\{(1, 5), (2, 3), (3, 3), (4, 2), (5, 1)\}$

(ii) $\{(1, 1), (2, 4), (3, 2), (4, 1), (5, 3)\}$

(iii) $\{(1, 2), (2, 1), (3, 1), (4, 4), (5, 5)\}$

(iv) $\{(1, 2), (2, 3), (1, 4), (3, 5)\}$

Solution: (i) $\{(1, 5), (2, 3), (3, 3), (4, 2), (5, 1)\}$

It is a function.

Range $= \{1, 2, 3, 5\}$

(ii) $\{(1, 1), (2, 4), (3, 2), (4, 1), (5, 3)\}$

It is a function.

Range $= \{1, 2, 3, 4\}$

(iii) $\{(1, 2), (2, 1), (3, 1), (4, 4), (5, 5)\}$

It is a function.

Range $= \{1, 2, 4, 5\}$

(iv) $\{(1, 2), (2, 3), (1, 4), (3, 5)\}$

It is not a function.

Q.9: If $X = \{-5, -6, -4, -3\}$, $Y = \{1, 2, 3, 4\}$

(i) A one – one function from X to Y.

(ii) Onto function from X to Y.

(iii) A function which is one –one and onto from X to Y.

(iv) A function which is neither one –one and nor onto from X to Y.

Solution: $X = \{-5, -6, -4, -3\}$, $Y = \{1, 2, 3, 4\}$

(i) $R_1 = \{(-5, 1), (-6, 2), (-4, 3), (-3, 4)\}$

It is one –one function from X to Y.

(ii) $R_2 = \{(-6, 2), (-5, 1), (-4, 3), (-3, 4)\}$

It is onto function from X to Y.

(iii) $R_3 = \{(-4, 1), (-6, 2), (-5, 3), (-3, 4)\}$

It is one –one and onto function from X to Y .

(iv) $R_4 = \{(-5,1),(-6,1),(-4,2),(-3,2)\}$

It is a function which is neither one – one nor onto .

Project: $(-2,5)$, $(-1,0)$, $(0,1)$ *and* $(2,5)$ is a function.

If the x – values and y – values of the function are switched , the relation that results is not a function . Provide an example of a function that will still be a function if the x – values and y- values are switched .

Solution: Given $R = \{(-2,5),(-1,0),(0,1),(2,5)\}$

It is a function .

Now the x – values and y – values of the function R are switched ,then

$\{(5,-2),(0,-1),(1,0),(5,2)\}$ it is not a function .

Example of a function when the x – values and y – values of the function are switched is also a function . Let $A = \{a, b, c\}$, $B = \{x, y, z\}$

$f = \{(a,x),(b,y),(c,z)\}$ it is a function .

Now the x – values and y – values of the function f are switched ,then

$\{(x,a),(y,b),(z,c)\}$ it is also a function .

Chapter No .6:

Basic Statics:

Frequency distribution: The group frequency table is a statistical method to organize and simplify a large set of data into smaller groups .

Group Frequencies: The number of appearing a number is called frequency . For example in an examination if 10 students scored 80 numbers in Maths paper . Then frequency is 10.

Class limit: The selected number which shows the start and end of a class is called class limit.
Every class has two limits ,the upper class limit and lower class limit . for each class the two limits may be fixed such that the mid point of each class falls on an integer . mid point of each class is calculated as

$$mid\ point = \frac{lower\ class\ limit + upper\ class\ limit}{2}$$

Class width: The difference between the upper and lower class boundaries is called class width .
For example
5 – 1 = 4 is the class width .

Histogram: A Histogram is a graph in which the data is represented in the forms of adjacent rectangles . The base of the rectangle shows the class boundaries and the height shows the frequency of that class.
In unequal class intervals the area of rectangle shows the frequency of that class.

Frequency polygon: A frequency polygon is a graphical representation of frequency distribution. A frequency polygon is drawn by joining all the mid points of rectangles of a histogram .

Exercise 6.1:

Q.1: Construct a frequency distribution of the marks of 30 students during a quiz with 100 points by taking 10 as the class – interval .Indicate the class – boundaries and class – marks .

40 ,60 ,65 ,70 ,35 , 50 ,56, 74 , 72 , 49 , 85 ,76 ,82 , 83 ,
68 , 90 , 67 , 66 , 58 , 46 , 74 , 88 , 76 , 69 , 57 , 63 , 66,
47 , 82 , 90 .

Solution: Here the lowest number is 35 and highest number is 90 .

Class limits	Tallymarks	Frequency (f)
35 – 45	||	2
46 – 55	||||	4
56 – 65	卌	6
66 – 75	卌 ||||	9
76 – 85	卌 |	6
86 – 90	|||	3

Q.2: Following are the mistakes made by a group of students of class 10th in a test of essay writing . Using an appropriate size of class interval , make a frequency distribution and also indicate the number of class intervals.

4 ,7,12 ,9 ,21 ,16 ,3,19 ,17,24 ,14 ,15,8,13 ,11 ,16 ,15 ,6,
5 ,8 ,11,20,18,22 ,6.

Solution: Largest number = 24 , smallest number = 3
we take size of class - interval 3 .
Number of groups are 8 .
The frequency distribution is

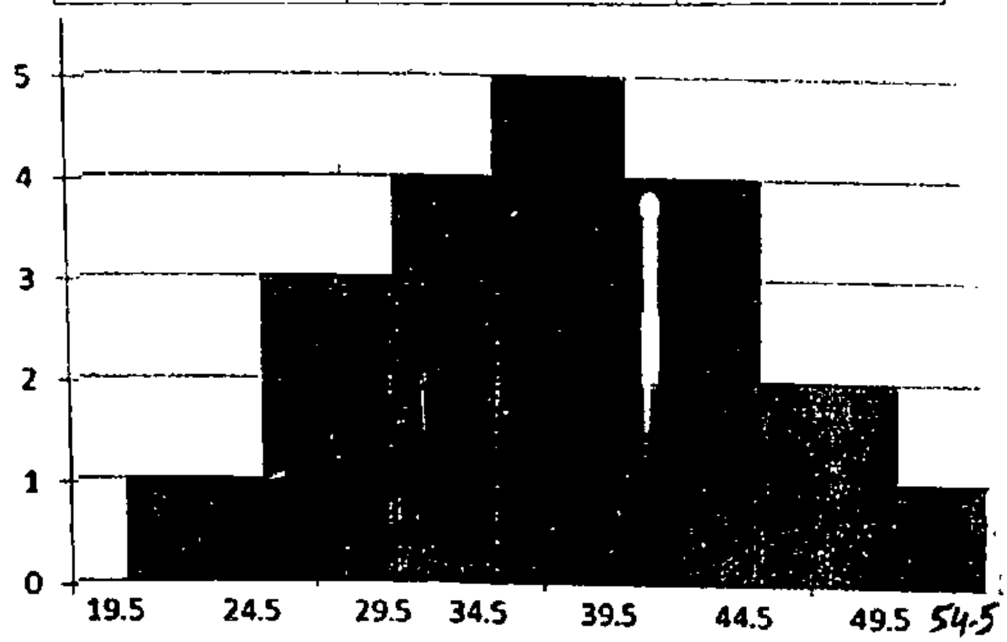
S.No.	Class interval	Frequency
1	1 – 3	1
2	4 – 6	4
3	7 – 9	4
4	10 – 12	3
5	13 – 15	4
6	16 – 18	4
7	19 – 21	3
8	22 – 24	2
		$\sum f = 25$

Q.3: Draw a histogram for the following data

Class limits	20-24	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49	50 – 54
Frequency	1	3	4	5	4	2	1

Solution: To draw a histogram class boundaries are marked along x – axis and frequencies of each class are marked along Y – axis as shown in the figure .

Class – limits	Frequency	Class boundaries
20 – 24	1	19.5 – 24.5
25 – 29	3	24.5 – 29.5
30 – 34	4	29.5 – 34.5
35 – 39	5	34.5 – 39.5
40 – 44	4	39.5 – 44.5
45 – 49	2	44.5 – 49 .5
50 – 54	1	49.5 – 54 .5



Q.4: **The following data give the weights in (kg) of the students in the 10^{th} class .**

25 , 30 ,32 , 29 , 24 , 40 , 36 , 37 , 28 , 27 , 41 , 42 , 35 , 39 , 31 , 32 , 34 , 42 , 40 , 43 , 36 , 26 , 22 , 23 , 42 , 39, 35 , 41 , 39 , 29 .

(i) Prepare a frequency distribution using a suitable Class interval .

(ii) Draw histogram and frequency – polygon.

Solution: (i) Here minimum weight = 22

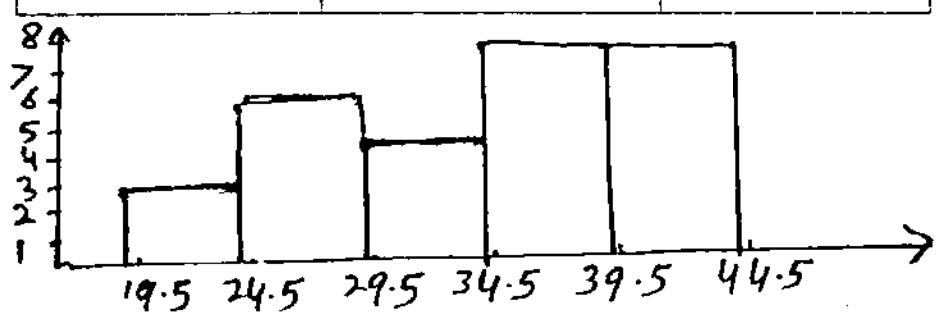
And maximum weight = 43 .

We take width of class interval 4 .

Class limits	Tally marks	frequency
20 – 24	|||	3
25 – 29	~~||||~~ |	6
30 – 34	~~||||~~	5
35 – 39	~~||||~~ |||	8
40 – 44	~~||||~~ |||	8

(ii) Histogram:

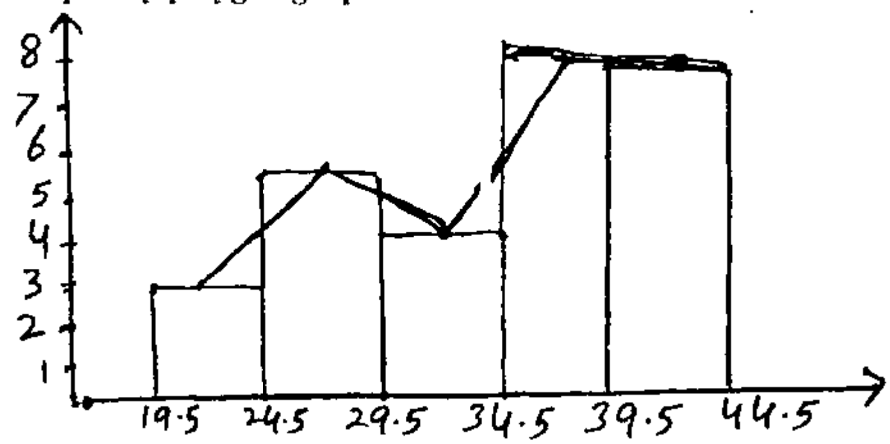
Class – limits	Frequency	Class boundaries
20 – 24	3	19.5 – 24.5
25 – 29	6	24.5 – 29.5
30 – 34	5	29.5 – 34.5
35 – 39	8	34.5 – 39.5
40 – 44	8	39.5 – 44.5



Frequency polygon:

Class – limits	Class boundaries	Class Marks (mid points)	Frequency
20 – 24	19.5 – 24.5	19.5 – 24.5	3
25 – 29	24.5 – 29.5	24.5 – 29.5	6
30 – 34	29.5 – 34.5	29.5 – 34.5	5
35 – 39	34.5 – 39.5	34.5 – 39.5	8
40 – 44	39.5 – 44.5	39.5 – 44.5	8

Frequency polygon graph



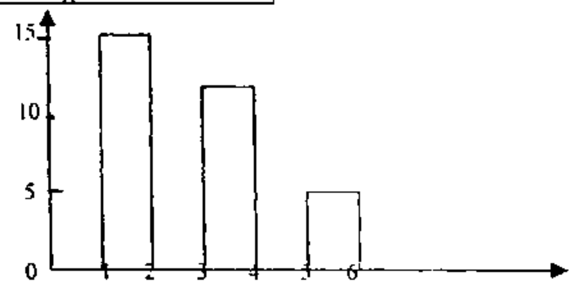
Q.5: A teacher asked the students about their time spent on homework completion. The following set of data was obtained (Time in hours).

4	4	6	3	1	2	2	3	1	4
1	2	5	3	4	5	2	2	3	1
3	1	2	2	3	1	4	2	6	2

Construct a frequency table and draw a histogram Showing the results.

Solution: Here largest number = 6 , Smallest number = 1 ,
We take class intervals of length 1 .

Class interval	Frequency
1 – 2	15
3 – 4	11
5 – 6	4
	$\sum f = 30$



Cumulative frequency table: A cumulative frequency table is obtained by adding successive class frequencies .

Cumulative Frequency polygon: A polygon in which cumulative .Frequency .is used for plotting the curve is called cumulative frequency polygon. The curve is also called ogive.

Exercise 6.2:

Q.1: The following data give the wages (in Rs) of workers. 60 , 75 , 80 , 85 , 90 ,84 , 95 , 100 , 150 , 66 , 58 ,90 , 98 , 120 , 77 , 90 .

By Taking 10 as the class interval , prepare .

(i) Cumulative frequency distribution

(ii) Cumulative frequency polygon .

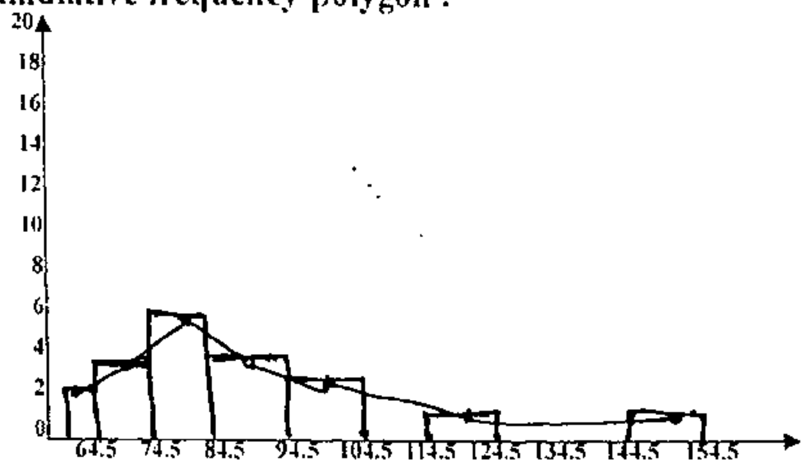
Solution: (i) Cumulative frequency distribution

As class interval = 10 .

Class - limits	Class boundaries	Frequency (f)	Cumulative Frequency (c .f)
55 – 64	54.5 – 64.5	2	2
65 – 74	64.5 – 74 .5	3	2 + 3 = 5
75 – 84	74.5 – 84.5	6	5 + 6 = 11
85 – 94	84.5 – 94.5	4	11 + 4 = 15

95 – 104	94.5 – 104 .5	3	15 + 3 = 18
105 – 114	104.5 – 114.5	0	18 + 0 = 18
115 – 124	114.5 – 124.5	1	18 + 1 = 19
125 – 134	124.5 – 134.5	0	19 + 0 = 19
135 – 144	134.5 – 144.5	0	19 + 0 =19
145 – 154	144.5 – 154.5	1	19 + 1 = 20

(ii) Cumulative frequency polygon .



Take upper class boundaries.

Q.2: Make Cumulative frequency table for the following data.

Age in years	20-24	25-29	30-34	34 – 39	40-44	45-49	50-54	55-59
Number of persons	1	2	16	10	22	20	15	14

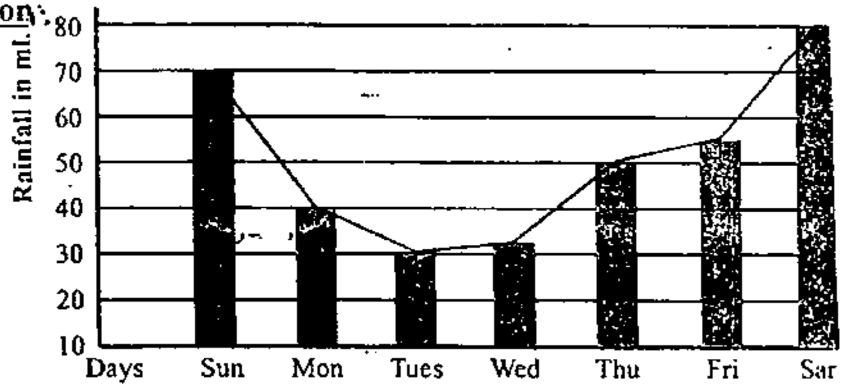
Solution: Cumulative frequency table

Age in years	Upper class Boundaries	Frequency(f)	Cumulative Frequency
20 – 24	24.5	1	1
25 – 29	29.5	2	1 +2 = 3
30 – 34	34.5	16	3 +16 = 19
35 – 39	39.5	10	19 +10 =29
40 – 44	44.5	22	29+22 = 51
45 – 49	49.5	20	51+20=71
50 – 54	54.5	15	71+15=86
55 – 59	59.5	14	86+14=100

Q. 3: In a city during the first week of august rainfall recorded is as follows . Construct a cumulative frequency graph.

Day	sun	Mon	Tue	Wed	Thu	Fri	Sat
Rain fall	70	40	30	35	50	55	80

Solution:

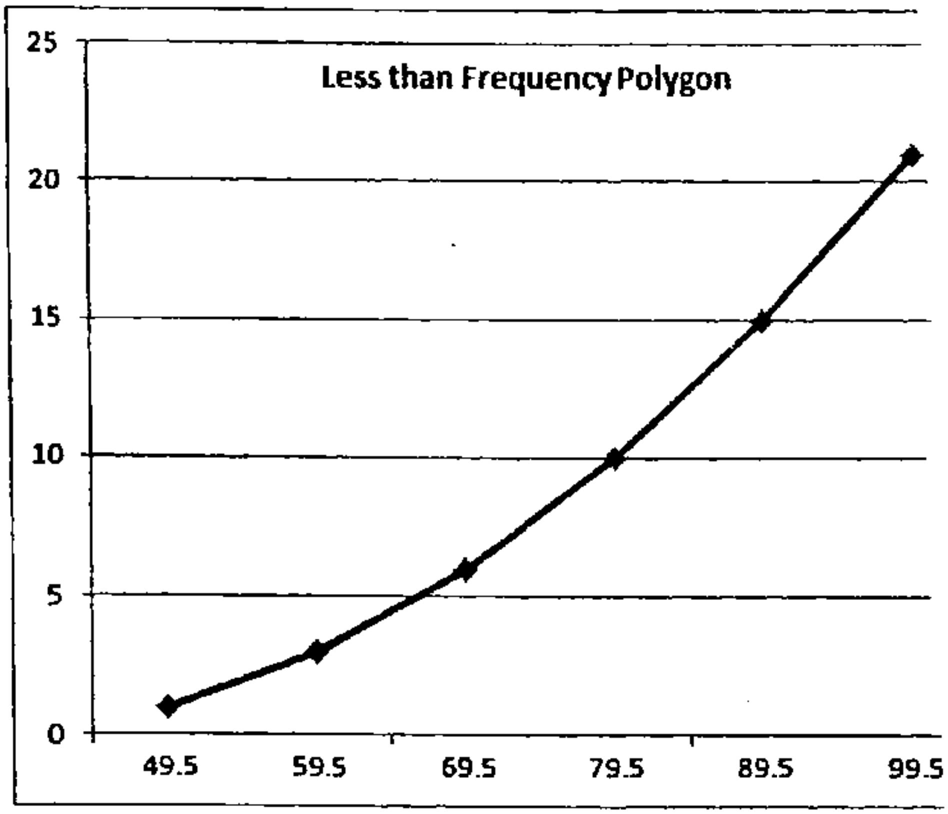


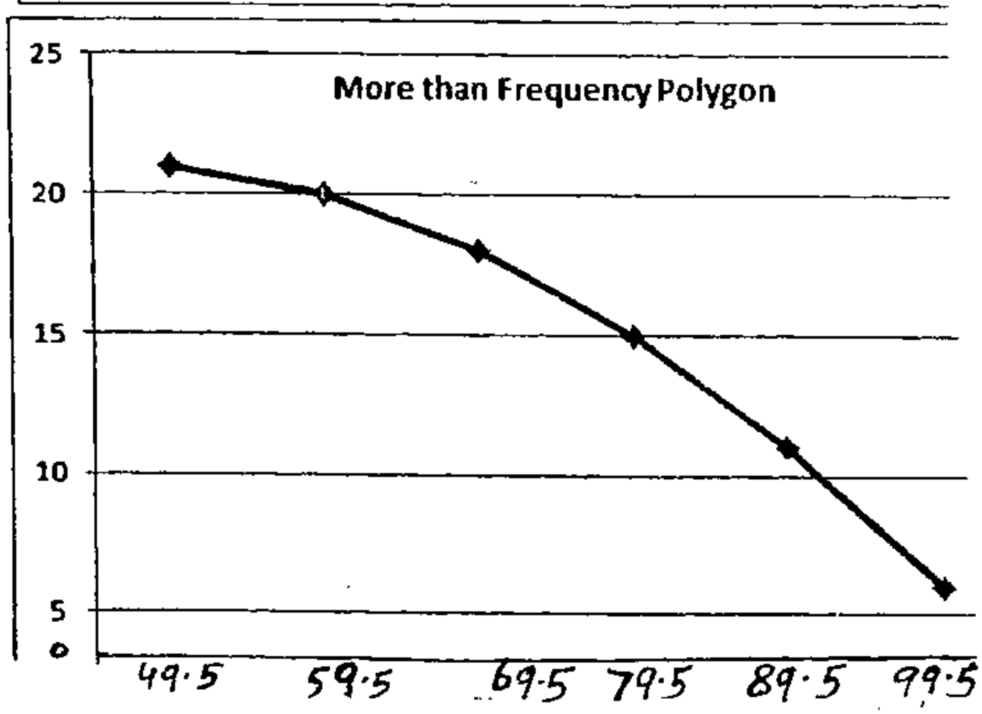
Q.4: Draw less than and more than cumulative frequency polygon for the data given .

Marks	Number of students
40 – 49	1
50 – 59	2
60 – 69	3
70 – 79	4
80 – 89	5
90 – 99	6

Solution: we construct the cumulative frequency table .

Class - limits	Class boundaries	Frequency	Cumulative requency
40 – 49	40.5 – 49.5	1	1
50 – 59	49.5 – 59.5	2	1+ 2 = 3
60 – 69	59.5 – 69.5	3	3 + 3 = 6
70 – 79	69.5 – 79.5	4	6 + 4 = 10
80 – 89	79.5 - 89.5	5	10 + 5 = 15
90 - 99	89.5 - 99.5	6	15 + 6 = 21


Less than Frequency Polygon
25
20
15
10
5
0
49.5
59.5
69.5
79.5
89.5
99.5


More than Frequency Polygon
25
20
15
10
5
0
49.5
59.5
69.5
79.5
89.5
99.5

Q.5: Determine from the data of Q .4 , the following .
(i) Number of students who obtained more than 50 marks.
(ii) Number of students who obtained less than 70 marks.
(iii) Number of students who secured marks between 50 and 70 .
(iv) class interval of all classes .
(v) lower class boundary of 5^{th} class .

Solution: (i) Number of students who obtained more than 50 marks are 20 .
(ii) Number of students who obtained less than 70 marks are 6 .
(iii) Number of students who secured marks between 50 and 70 are 5 .
(iv) class interval of all classes is 10 .
(v) lower class boundary of 5^{th} class is 79.5 .

Q.6: Construct an ogive for the following table .

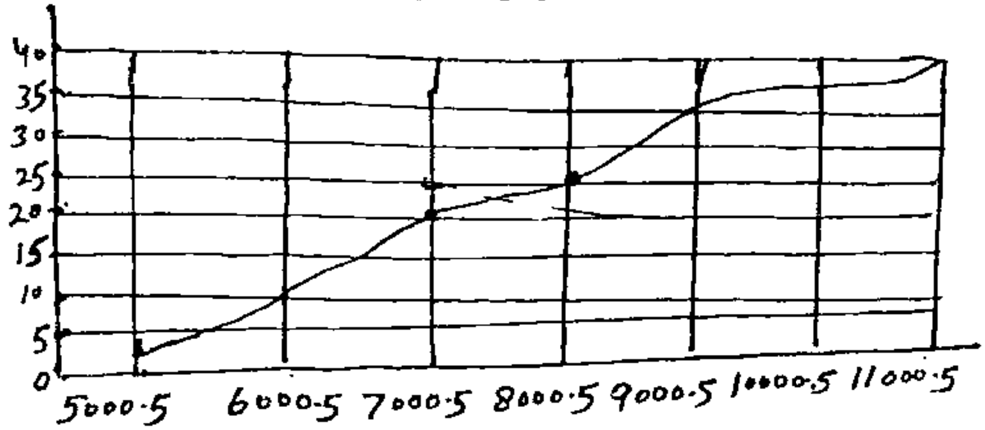
Salary	Workers
4000-5000	3
5001-6000	5
6001-7000	12
7001-8000	9
8001-9000	5
9001-10000	4
10001-11000	2

Solution: For an ogive we construct a cumulative frequency table

Salary	Upper Class boundaries	frequency	Cumulative frequency
4000-5000	5000.5	3	3
5001-6000	6000.5	5	3 +5 = 8
6001-7000	7000.5	12	8 +12 = 20
7001-8000	8000.5	9	20 +9 = 29
8001-9000	9000.5	5	29 + 5 = 34
9001-10000	10000.5	4	34 +4 = 38
10001-11000	11000.5	2	38 + 2 = 40

We take upper class boundaries along x – axis

And Cumulative frequency along y –axis .



Arithmetic Mean: (For simple data):

If $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ are n observations then Arithmetic mean is denoted by $\bar{x}$ and given by

$$\bar{x} = \frac{x_1 + x_2 + \ldots\ldots + x_n}{n}$$

$$\Rightarrow \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

By short cut method :

formula for short cut method is

$$\bar{x} = a + \frac{\sum D_x}{n}$$

Where $\bar{x}$ **= Arithmetic mean ,**

a **= provisional mean (P .M)**

$D_x = x - a$ **(deviation from P . M.)**

$\sum D_x$ **= sum of deviation from P.M.**

n = total number of values in the data .

Arithmetic Mean:(for group data):

If $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ are n observations and their Frequencies are $f_1, f_2, f_3, \ldots\ldots\ldots, f_n$ respectively .

Then Arithmetic Mean is given by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + ...+ f_nx_n}{f_1 + f_2 + ...+ f_n}$$

$$= \frac{\sum fx}{\sum f}$$

By short cut method :

formula for short cut method is

$$\bar{x} = a + \frac{\sum fD_x}{\sum f}$$

Calculating Mean by assuming average: If "a" is assumed Mean which is taken approximately from the center.

Then for ungrouped data Mean is given by

$$\bar{x} = a + \frac{\sum D}{n}$$

For grouped data Mean is given by

$$\bar{x} = a + \frac{\sum fD}{\sum f}$$

Where $D = x_i - a$ and x_i are the average of each class.

Median: (For ungrouped data): If the data are arranged in descending or ascending then central value of the data is called median . If the number of observations "n" are odd then the median is the value of $\frac{(n+1)}{2}th$ observation.

If the number of observations "n" are even then median is the Average of values of $\frac{n}{2}th$ *and* $\frac{(n+2)}{2}th$ observations.

Median for discrete data: In a frequency distribution if $\sum f$ is odd then the values of x opposite to $\sum\left(\frac{f+1}{2}\right)$ th item in the cumulative frequency column is its medium .

if $\sum f$ is odd then the values of x opposite to $\sum\left(\frac{f}{2}\right)$th item in the cumulative frequency column is its medium .

Median for continous data:

If $x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n$ are the mid values of given groups and $f_1, f_2, f_3, \ldots\ldots\ldots, f_n$ are their frequencies . Then the median is given by

$$Meadian = l + \frac{h}{f}\left(\frac{n}{2} - c\right),$$

Where $l = lower\ limit\ of\ class\ containg\ medain$

h = class interval , f = frequency of class containing median.

n = total number of values $\left(n = \sum f\right)$

Mode: The value which occurs most in a set of observations is Called Mode .The data in which no value is repeated has no mode and if more than one value is equally repeated then data has more than one mode.

Mode for grouped data: The Mode for grouped data is given by

$$Mode = l + \frac{(f_m - f_o)}{(2f_m - f_o - f_1)} \times h$$

where l = lower boundary of model class.

$f_m = frequency\ of\ Model\ class$.

f_o = frequency of class preceding Model class

f_1 = frequency of class following the model class.

The class whose frequencies are large is called Model class.

Geometric Mean: It is defined as the nth root of product of n values . Let the values are $a_1, a_2, a_3, \ldots\ldots, a_n$. Then

Geometric mean = $\sqrt[n]{a_1 \times a_2 \times a_3 \times \ldots\ldots \times a_n}$

If number of observations are large then

Geometric Mean = $Anti\log\left(\frac{1}{n}\sum \log x\right)$

Geometric Mean from grouped data: G . M . from grouped data

is find by $G.M. = Anti\log\left(\frac{\sum f_i \log x_i}{\sum f}\right)$

Hormonic Mean: it is the reciprocal of arithmetic mean of

observations . For ungrouped data H . M. is

$$H.M. = \frac{n}{\sum \frac{1}{x_i}}$$

For grouped data the H . M . is

$$H.M = \frac{\sum f}{\sum f_i \times \frac{1}{x_i}}$$

Exercise 6.3:

Q.1: The following are the weights (in kg) of students of 10^{th} grads . 45 , 30 , 25 , 36 , 42 , 27 , 31 , 43 , 49 , 50 Calculate mean of the weight .

Solution: let $\bar{x}$ is the mean .

Total number of weight = n = 10

Then

$$\bar{x} = \frac{45+30+25+36+42+27+31+43+49+50}{10}$$

$$=> \bar{x} = \frac{378}{10} = 37.8$$

Q.2: Find the mean of the weights given in question 1 by using short cut method .

Solution: let we take mean a from the middle .

Let we take $a = 42$

x	$D_x = x - a$
45	45 – 42 = 3
30	30 – 42 = -12

25	25 – 42 = - 17
36	36 - 42 = -6
42	42 - 42 = 0
27	27 - 42 = -15
31	31 - 42 = -11
43	43 – 42 = 1
49	49 - 42 = 7
50	50 - 42 = 8
	$\sum D_x = -42$

As $\bar{x} = a + \frac{\sum D_x}{n}$ put $a = 42$, $n = 10$, $\sum D_x = -42$

=> $\bar{x} = 42 + \frac{-42}{10} = 42 - 4.2 = 37.8$ **Ans.**

Q.3: Using assumed mean find the mean of the following Numbers . 1242, 1248 ,1252 , 1244 , 1249

Solution: Here $n = 5$ *and let assumed mean* $= a = 1244$

x	D = x – a
1242	1242 – 1244 = -2
1248	12448 – 1244 = 4
1252	1252 – 1244 = 6
1244	1244 – 1244 = 0
1249	1249 – 1244 = 5
	$\sum D = 15$

As $\bar{x} = a + \frac{\sum D}{n}$......(*i*) put values in (i)

=> $\bar{x} = 1244 + \frac{15}{5}$

$= 1244 + 3$

=> $\bar{x} = 1247$

The mean is 1247 (for assumed mean 1244) .

Q.4: Find the mean of the following frequency distribution .
Marks obtained by students of 10th class in mathematics

Score(out of 375)	0 – 15	16 – 31	32 – 47	48 – 63	64- 75
	0	10	40	70	45

Solution:

Class interval	Mid value(x)	Frequency(f)	f. x
0 – 15	7.5	0	(7.5)(0) = 0
16 – 31	23.5	10	(23.5)(10) = 235
32 – 47	39.5	40	(39.5)(40)= 1580
48 – 63	55.5	70	(55.5)(70) = 3885
64 – 75	69.5	45	(69.5)(45) = 3127.5
		$\sum f = 165$	$\sum fx = 8827.5$

As $As\ \ Mean = \dfrac{\sum fx}{\sum f}$ ....(i)

Put values in (i) .

$$Mean = \frac{8827.5}{165}$$

$$= 53.5\ \ marks.$$

Q.5: Find the median of the following data .

(a) . Heights of boys in inches . 64 , 65 , 65 , 66 , 66 , 67 .

(b) Salaries of 9 workers of a factory .

7000, 6600 ,8000, 4500 , 7500 ,11000, 9000, 7500

Solution: (a): We write in descending order .

67 , 66 , 66 , 65 , 64 .

Since number of observations are n = 5 .

Thus the third observation is median whose value is 66 .

i.e. Median = 66

(ii) Write in descending form

11000 , 9000 , 8000 , 7500 , 7500 . 7000 , 6600 , 4500

As number of observations are n = 8 even .

Thus median is the average of $\frac{n}{2}$ *and* $\frac{n+2}{2}$ **.Put n = 8**

then median is the average of $\frac{8}{2}=4$ **and** $\frac{10}{2}=5$ **observations**

Whose values are 7500 , 7500 .

Thus median $=\frac{7500+7500}{2}$

$=\frac{1500}{2}$

$=7500$

Q.6: Find the Arithmetic mean , Geometric mean , Median and Mode of the following data.

58 , 59 , 60 , 62, 64 , 64 , 65 , 67 , 67 , 68 , 70 , 71

71 , 71, 73

Solution: Arithmetic mean:

$$\bar{x}=\frac{1}{14}\begin{pmatrix}58+59+60+62+64+64+65+67+67\\+68+70+71+71+71+73\end{pmatrix}$$

$$=\frac{919}{15}=66$$

Geometric mean: As $G.M.=anti\log\left(\frac{\sum \log x}{n}\right).....(i)$

put values

$$G.M.=anti\log\frac{1}{15}\begin{pmatrix}\log 58+\log 59+\log 60+\log 62+\log 64\\+\log 64+\log 65+\log 67+\log 67+\log 68\\+\log 70+\log 71+\log 71+\log 71+\log 73\end{pmatrix}$$

$$= anti\log\frac{1}{15}\begin{pmatrix}1.7634+1.7708+1.7781+1.7924+1.8062\\+1.8062+1.8129+1.8261+1.8261+1.8325\\1.8451+1.8512+1.8512+1.8512+1.8633\end{pmatrix}$$

$$= anti\log\left(\frac{27.2767}{15}\right)$$

$$G.M. = anti\log(1.8184)$$

$$= 64.44$$

Median : First we write the given data in descending order.

58 ,59 , 60 , 62 ,64 ,64,65,67,67,68,70 ,71,71,71,73

The number of observations are n = 15 , then the 8^{th} observation is median which is 67 .

Thus median = 67 .

Mode: The number which occur most is 71 .

Thus mode = 71 .

<u>Q.7</u>: A set of data contains the values as 148 , 145 , 160 , 157 ,156 and 160 . Show that mode > median > mean .

<u>Solution</u>: Mean:

$$\bar{x} = \frac{148+145+160+157+156+160}{6}$$

$$= \frac{926}{9} = 154.33$$

Median:

First we write in descending order

160 , 160 , 157 ,156 , 148 , 144

Since the number of observations are n = 6 which are even

Then average of 3^{rd} and 4^{th} observations is median . Then

$$Median = \frac{157+156}{2}$$

$$= \frac{313}{2}$$

$$= 156.5$$

Mode: The number 160 is repeated most time . Thus

Mood = 160

Thus it is proved Mood > Median > Mean .

Q.8: From the following distribution

Daily wages (in Rs)	112 – 116	117 – 121	122 – 126	127 – 131	132 – 136
No. of workers	3	20	11	4	5

(i) Construct a frequency table .

(ii) : Find the class – boundaries for each group .

(iii) : Calculate Median , Mode , Hormonic Mean and Geometric Mean for the table .

Solution: frequency table and class boundaries .

Class limits	Frequency	Class boundaries	Cumulative frequency
112 – 116	3	111.5 – 116.5	3
117 – 121	20	116.5 - 121.5	3 + 20 = 23
122 – 126	11	121.5 – 1126.5	23 + 11 = 34
127 – 131	4	126.5 – 131.5	34 + 4 = 38
132 - 136	5	131.5 – 136.5	38 + 5 = 43
	$n = \sum f = 43$		

(iii) Median $= \frac{n}{2} th\ item = \frac{43}{2} = 21.5$.

Hence the median lies in the group 116.5 – 121 .5 .

As median $= l + \frac{h}{f}\left(\frac{n}{2} - c\right)$ **.....(i)**

Put $l = 116.5$, **h** $= 121.5 - 116.5 = 5$, **f = 20 .**

c = 3 (c .f . of preceding group)

$n = \sum f = 43$ **in (i)**

$$median = 116.5 + \frac{5}{20}\left(\frac{43}{2} - 3\right) = 116.5 + 0.25(21.5 - 3)$$

$$= 116.5 + 4.625 = 121.125 \ \ Ans.$$

Mode: **As the number of workers are most in $117 - 121$ which is 20.**

Thus the model group is $117 - 121$.

As Mode = $l + \left(\frac{f_m - f_o}{2f_m - f_o - f_1}\right) \times h$ **......(ii)**

Put $l = 117$, $f_m = 20$, $f_o = 3$ $f_1 = 11$,
$h = 121 - 117 = 4$ **in (i) .**

$$Mode = 117 + \left(\frac{20-3}{2(20)-3-11}\right) \times 4 = 117 + \left(\frac{17}{40-14}\right) \times 4$$

$$= 117 + \left(\frac{17}{26}\right) \times 4 = 117 + 2.6154$$

=> $Mode = 119.6535$

Hormonic Mean :

Classes	Mid point of Interval (x)	Frequency (f)	$\frac{f}{x}$
112 – 116	**114**	**3**	$\frac{3}{114} = 0.026$
117 – 121	**119**	**20**	$\frac{20}{119} = 0.168$
122 – 126	**124**	**11**	$\frac{11}{124} = 0.887$
127 – 131	**129**	**4**	$\frac{4}{129} = 0.031$

132 – 136	134	5	$\frac{5}{134}=0.037$
		$\sum f=43$	$\sum\frac{f}{x}=1.149$

As $H.M=\frac{\sum f}{\sum\frac{f}{x}}=\frac{43}{1.149}=37.4238$

Geometric Mean :

Classes	Mid point of Interval (x)	Freque - ncy (f)	Log x	f × log x
112 – 116	114	3	2.0569	6.1707
117 – 121	119	20	2.0755	41.51
122 – 126	124	11	2.0934	23.0274
127 – 131	129	4	0.6020	2.408
132 – 136	134	5	0.6989	3.4945
		$\sum f$ $=43$		$\sum f\log x$ $=76.6106$

$G.M=Anti\log\left(\frac{\sum f\log x}{\sum f}\right)$ put values

=> $G.M=Anti\log\left(\frac{76.6106}{43}\right)=Anti\log(1.7816)$

=> $G.M=60.48$

Q.9: Find Median , Q_1 , Q_3 and Mode from the following distribution graphically .

classes	10 -14	15 - 19	20 - 24	25- 29	30 -34
frequency	1	3	7	12	2

Solution:

Class limits	Frequency (f)	Class boundaries	Cumulative Frequency (c.f)
10 – 14	1	9.5 – 14.5	1
15 – 19	3	14.5 - 19.5	1 + 3 = 4
20 – 24	7	19.5 – 24.5	4 + 7 = 11
25 – 29	12	24.5 – 29.5	11 + 12 = 23
30 - 34	2	29.5 – 34.5	23 + 2 = 25
	$n=\sum f$ $= 25$		

Median $= \frac{n}{2} th\ item = \frac{25}{2} = 12.5$.

Hence the median lies in the group 24.5 – 29.5

As median $= l + \frac{h}{f}\left(\frac{n}{2} - c\right)$ .....(i)

Put $l = 24.5$, h = $29.5 - 24.5 = 5$, f = 12

c = 7 (c .f . of preceding group)

$n = \sum f = 25$ in (i)

$$median = 24.5 + \frac{5}{12}\left(\frac{25}{2} - 7\right) = 24.5 + 0.42(12 - 7)$$

$$= 24.5 + 2.1 = 26.6 \ Ans.$$

$\underline{Q_1}$: As location for $Q_1 = \frac{n+1}{4} th\ item = \frac{25+1}{4} = 6.5 th item$

Hence Q_1 is lies in the interval is 20 – 24 .

Now $Q_1 = l + \frac{h}{f}\left(\frac{n+1}{4} - c.f\right)$ …..(i)

Put $l = 20$, $n = 25$, $f = 7$, ,

$h = 24.5 - 19.5 = 5$

$c.f = 4 = \text{cumulative frequency of the group above the group in which } Q_1 \text{ lies.}$

put in (i)

$$Q_1 = 20 + +\frac{5}{7}(6.5 - 4)$$

$$\Rightarrow Q_1 = 20 + 0.72(2.5)$$

$$= 20 + 1.78 = 21.78$$

Q_3**: As location for**

$$Q_3 = 3\left(\frac{n+1}{4}\right)th\ item = 3\left(\frac{25+1}{4}\right) = 3 \times 6.5$$

$$= 19.5\ th\ item$$

Hence Q_3 is lies in the interval is 25 – 29 .

Now $Q_3 = l + \frac{h}{f}\left(3\left(\frac{n+1}{4}\right) - c.f\right)$ …..(i)

Put $l = 25$, $n = 25$, $f = 12$, $h = 5$,

$c.f = 11 = \text{cumulative frequency of the group above the group in which } Q_3 \text{ lies.}$

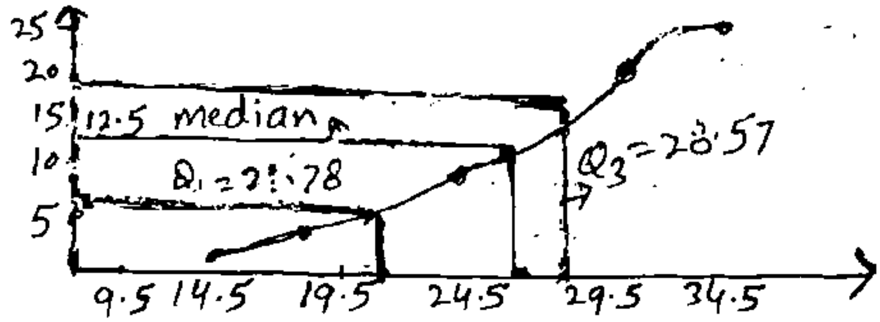
put in (i)

$$Q_3 = 25 + +\frac{5}{12}(19.5 - 11)$$

$$\Rightarrow Q_3 = 25 + 0.42(8.5)$$

$$= 25 + 3.57 = 28.57$$

Graph of Median , Q_1 , Q_3 and Mode



Range: In given data the difference between the largest number and smallest number is called Rang.

Variance: Variance is defined as the sum of average of squares of difference of all the observations from average. It is denoted by S^2 and given by

$$S^2=\frac{\sum\left(x_i-\bar{x}\right)^2}{n} \quad \text{or}$$

$$S^2=\frac{\sum x^2}{n}-\left(\frac{\sum x}{n}\right)^2 \quad \textbf{for ungroup data}$$

And $S^2=\frac{\sum f\left(x-\bar{x}\right)^2}{n}$ *Or* $\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2$ **for group and continuous data .**

Standard Deviation (S .D.): The positive square root of variance Is called standard Deviation .

$$S.D.=\sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n}} \quad \text{For ungrouped data}$$

$$S.D=\sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2} \quad \text{For grouped data}$$

Or $S.D=\sqrt{\frac{\sum f\left(x-\bar{x}\right)^2}{\sum f}}$

Exercise 6.4:

Q.1: Find the range for the following items.

11 , 13 , 15 , 21 , 19 , 23 .

Solution: Here the largest value = 23

The smallest value = 11

Range = 23 – 11 = 12

Q.2: A bank branch manger , interested in waiting times of customers carried out a survey. A random sample of 12 customers is selected and yielded the following

5.90 ,9.66 ,5.79 ,8.02 ,8.73 ,8.01 , 10.49 ,8.35, 6.68,5.64

5.47,9.91

Find values for the following.

(a) Average (b) Median (c) standard deviation .

Solution: (a) Average: Here n = 12

$$\bar{x} = \frac{1}{12}\begin{pmatrix} 5.90+9.66+5.79+8.02+8.73+8.01+10.49+8.35+ \\ 6.68+5.64+5.47+9.91 \end{pmatrix}$$

$$= \left(\frac{92.65}{12}\right)$$

$$= 7.72$$

(b) Median: first we write in descending order.

10.49, 9.91 ,9.66, 8.73 , 8.35 , 8.02, 8.01, 6.68 ,5.90 ,

5.79 , 5.64 ,5.47

Since the number of observations are n = 12 which are even . Then the median are the average of 6^{th} and 7^{th} observations whose values are 8.01 and 8.02 .

$$Then\ Median = \frac{8.01+8.02}{2}$$

$$= \frac{16.03}{2} = 8.01$$

Standard deviation(S.D.):

As $\bar{x} = 7.72$. Then

$S.D.=\sqrt{\frac{\sum(x_i-\bar{x})^2}{n}}$ ....(i) Now

x_i	$x_i-\bar{x}$	$(x_i-\bar{x})^2$
5.90	5.90 – 7.72 = - 1.82	3.31
9.66	9.66 – 7.72 = 1.94	3.76
5.79	5.79 – 7 .72 = - 1.93	3.72
8.02	8.02 – 7.72 = 0.3	0.09
8.73	8.73 – 7.72 = 1.01	1.02
8.01	8.01- 7.72 = 0.29	0.08
10.49	10 .49 – 7.72 = 2.77	7.67
8.35	8.35- 7.72 = 0.63	0.39
6.68	6.64 – 7.72 = - 1.08	1.16
5.64	5.64 – 7.72 = -2.08	4.32
5.47	5.47 – 7 .72 = 2.25	5.06
9.91	9.91 – 7 .72 = 2.19	4.7961
.		$\sum(x_i-\bar{x})^2=35.31$

Put values in (i)

$S.D.=\sqrt{\frac{35.31}{12}}=\sqrt{2.94}$

$=1.72$

Q.3: Calculate the Range ,Variance and Standard Deviation for the following data .

x	5	10	11	13	15
f	2	3	4	1	5

Solution: For range

x	f	xf
5	2	5 × 2 =10
10	3	10 ×3= 30
11	4	11 ×4 = 44
13	1	13 ×1= 13

15	5	$15 \times 5 = 75$
	$\sum f = 15$	$\sum fx = 172$

The largest value = 75
The smallest value = 10
Range = 75 – 10 = 65 .

Now variance : in this case $\bar{x} = \frac{\sum fx}{\sum f}$ => $\bar{x} = \frac{172}{15} = 11.47$

x	f	xf	$x - \bar{x}$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
5	2	10	5 – 11.47= - 6.47	41.86	83.72
10	3	30	10- 11.47=-1.47	2.16	6.48
11	4	44	11- 11.47= -0.47	0.22	0.88
13	1	13	13- 11.47=1. 53	2.34	2.34
15	5	75	15- 11.47=3.53	12.46	62.30

$\sum f = 15$		$\sum f(x - \bar{x})^2 = 155.72$

As variance: $S^2 = \frac{\sum f(x - \bar{x})^2}{\sum f}$ put values

$$S^2 = \frac{155.72}{15} = 10.38$$

Standard deviation = $\sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{155.72}{15}}$

=> $S.D = \sqrt{10.38} = 3.22$

Q.4:Following table shows the marks obtained by 10 students of two students sections of 10^{th} class.

A	7	9	6	9	4	7	5	8	8	7
B	6	10	6	4	2	8	10	6	9	9

Find their (a) Arithmetic Mean (b) Variance

Solution: For section A:

(a) Arithmetic mean: Here n =10

$$\bar{x}=\frac{7+9+6+9+4+7+5+8+8+7}{10}=\frac{70}{10}=7$$

(b) **Variance:**

x	$x-\bar{x}$	$(x-\bar{x})^2$
7	0	0
9	2	4
6	-1	1
9	2	4
4	-3	9
7	0	0
5	-2	4
8	1	1
8	1	1
7	0	0
		$\sum(x-\bar{x})^2=24$

Now Variance is

$$S^2=\frac{\sum(x-\bar{x})^2}{n}\quad ....(i)\quad \textit{put values in}(i)$$

$$=> S^2=\frac{24}{10}=2.4$$

Section B: (a) Arithmetic mean :

$$\bar{x}=\frac{6+10+6+4+2+8+10+6+9+9}{10}$$

$$=\frac{70}{10}$$

$$=> \bar{x}=7$$

(b) Variance:

x	$x-\bar{x}$	$(x-\bar{x})^2$
6	-1	1
10	3	9
6	-1	1
4	-3	9
2	-5	25
8	1	1
10	3	9
6	-1	1
9	2	4
9	2	4
		$\sum(x-\bar{x})^2 = 64$

Now variance:

$$S^2 = \frac{\sum(x-\bar{x})^2}{n} \quad ....(ii) \qquad \text{put values in (ii)}$$

$$\Rightarrow \quad S^2 = \frac{64}{10} = 6.4$$

Q.5: Following are the marks (out of 75) of the eight students in two subjects.

student	A	B	C	D	E	F	G	H
Marks in Maths	54	63	59	45	52	35	61	68
physics	52	55	57	51	56	58	50	59

Compare the standard deviation of the marks and tell that in which Subject students are more consistent.

Solution: **Marks in Maths**: Here n = 8 ,First we find mean.

$$\bar{x} = \frac{54+63+59+45+52+35+61+68}{8}$$

$$= \frac{437}{8} = 54.62$$

students	Marks (x)	$x-\bar{x}$	$(x-\bar{x})^2$
A	54	- 0.62	0.38
B	63	8.38	70.22
C	59	4.38	19.18
D	45	-9.62	92.54
E	52	-2.62	6.86
F	35	19.62	384.94
G	61	6.38	40.7
H	68	13.38	179.02
			$\sum(x-\bar{x})^2 = 793.84$

$$Now \quad S.D. = \sqrt{\frac{(x-\bar{x})^2}{n}} \quad ....(i) \quad \text{Put values in (i)}$$

$$S.D = \sqrt{\frac{793.84}{8}} = \sqrt{99.23}$$

$$\Rightarrow S.D = 9.96$$

(b) Physics marks: first we find mean .

$$\bar{x} = \frac{52+55+57+51+56+58+50+59}{8} = \frac{438}{8} = 54.75$$

Students	Marks(x)	$x-\bar{x}$	$(x-\bar{x})^2$
A	52	-2.75	7.56
B	55	0.25	0.06
C	57	2.25	5.06
D	51	- 3.75	14.06
E	56	1.25	1.56
F	58	3.25	10.56
G	50	- 4.75	22.56
H	59	4.25	18.06
			$\sum(x-\bar{x})^2 = 79.48$

Now standard deviation is

$$S.D. = \sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n}} \text{ ......}(ii) \quad \text{put values in (ii)}$$

$$\Rightarrow S.D = \sqrt{\frac{79.48}{8}} = \sqrt{9.93}$$

$$\Rightarrow S.D = 3.15$$

Thus S.D. of maths is 9.96 and S.D. of physics is 3.15

Thus physics is more consistence .

Q.6: The following is the distribution for the number of defective bulbs in 30 cartons (packs) .

No. of Defective Bulbs (x)	0 - 2	2 - 4	4 – 6	6 - 8	8 - 10
No. of packs(f)	1	3	15	10	2

Solution:

Defective bulbs	f	Mid value (x)	f .x	x^2	$f.x^2$
0 – 2	1	$\frac{0+2}{2}=1$	$1\times1=1$	$(1)^2=1$	$1\times1=1$
2 – 4	3	3	9	9	$3\times9=27$
4 – 6	15	5	75	25	375
6 – 8	10	7	70	49	490
8 – 10	2	9	18	81	162
$\sum f = 31$			$\sum fx = 173$		$\sum fx^2 = 1055$

As variance $S^2 = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2$ **.....(i) Put values**

=> $S^2 = \frac{1055}{31} - \left(\frac{173}{31}\right)^2 = 34.03 - (5.58)^2$

=> $S^2 = 34.03 - 31.14 = 2.89$

S . D = $\sqrt{S^2} = \sqrt{2.89} = 1.7$

=> **S . D = 1.7**

Review Exercise 6:

At the end of each question , four circles are given . Fill in the correct circle only /

(i) The difference between upper limit of two consecutive classes in a frequency table is called .
(a) Class limit (b) Class interval
(c) Class mark (d) all of these

(ii) A cumulative frequency curve is also called
(a) Histogram (b) Pie chart
(c) ogive (d) frequency polygon

(iii) The number of time a value appears on a set of data is called
(a) frequency (b) average
(c) mode (d) median

(iv) The data below represents the number of televisions that 11 students have in their homes . Find the mode of the data 3 , 2 , 1, 1 , 1 ,5 , 3, 1 , 2 , 1, 2 ,
(a) 1 (b) 2 (c) 3 (d) 4

(v) In which data set are mean , median , mode and range all the same number
(a) 1 , 2, 3 , 3 , 2 , 1 , 2 (b) 1 , 2 , 3, 1 , 2 , 3 , 1
(c) 1 , 3 , 3, 3 , 2 , 3 , 1 (d) 2 , 2 , 1 , 2 , 3 , 2 , 3

(vi) The nth root of product of "n" number of values is called
(a) arithmetic (b) geometric mean
(c) Harmonic (d) standard deviation

(vii) In a set of data 63 ,65 ,66,67,69 median is
(a) 63 (b) 66 (c) 67 (8) 69

(viii) In a set of data 41 ,43 ,47, 51 ,57 ,52 ,59 median is
(a) 51 (b) 47 (c) 52 (d) non of these

(ix) In the given data 5, 7 ,7 ,5 ,3 ,7 ,2,8,2 mode is
(a) 9 (b) 5 (c) 2 (d) 7

(x) In the given data 5, 5, 5,5, 5 ,5,5 the standard deviation is
(a) 5 (b) 0 (c) 7 (d) non of these

(xi) The average pocket money of 30 student is Rs 20/. The total amount in class is
(a) Rs. 20/ (b) Rs. 30/ (c) Rs. 300/ (d) Rs. 600/

(xii) The sum of 30 observations is 1500 . Its average will be (a) 1500 (b) 150 (c) 15 (d) none of these

(xiii) The difference of the largest and smallest value in The data is called
(a) Mean (b) Mode (c) Range (d) standard deviation

(xiv) The formula $\frac{\sum x}{n}$ determines .
(a) Arithmetic Mean (b) Median (c) Mode (d) G . M.

(xv) What is the difference between the mean of set B and the median of set A ? .
Set A = { 2 , -1 ,7 , - 4 , 11 , 13 }
Set B = { 12 , 5 , -3 , 4 , 7 , - 7 }
(a) - 0.5 (b) 0 (c) 0.5 (d) 1

(xvi) $\frac{\sum f\left(x-\bar{x}\right)^2}{\sum f}$ is called ……
(a) Range (b) Median (c) S . D . (d) Variance .

(xvii) The most frequent value in the data is called its …
(a) Mean (b) Median (c) Mode (d) G . M.

<u>Answers</u>: (i) b (ii) c (iii) a (iv) a (v) a (vi) b (vii) b
(viii) a (ix) c (x) b (xi) d (xii) d (xiii) c
(xiv) a (xv) c (xvi) d (xvii) d

<u>Q.2</u>: The ages (in years) of 27 students of 10th class are given Prepare a frequency distribution of suitable class intervals.

17 ,17 , 16 ,16 , 17 ,16 ,16 ,17 , 18 ,18 , 15 ,17 ,19 ,18 ,
18 ,17 ,16 ,15 ,16 ,17 ,15 ,19 ,19 ,15 ,15 ,16 ,18,

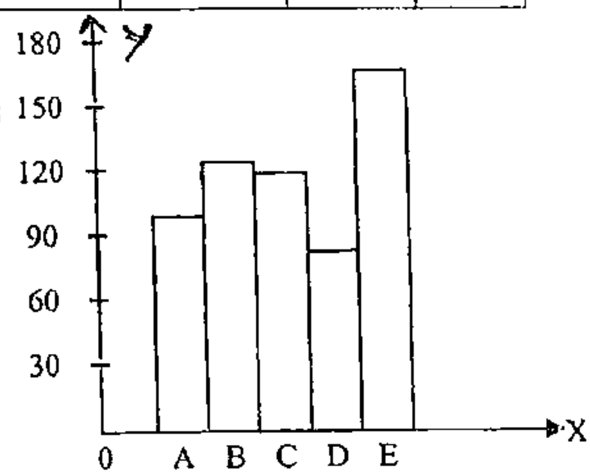
Solution: Here largest number = 19 and smallest number = 15
Then we take class width 1 .
Then frequency distribution is following .

Class intervals	Frequency(f)
15 – 16	12
17 - 18	12
19 - 20	3

Q.3: Prepare a histogram of the following table.

Brand of car	A	B	C	D	E
Sale in 1 month	100	120	110	72	169

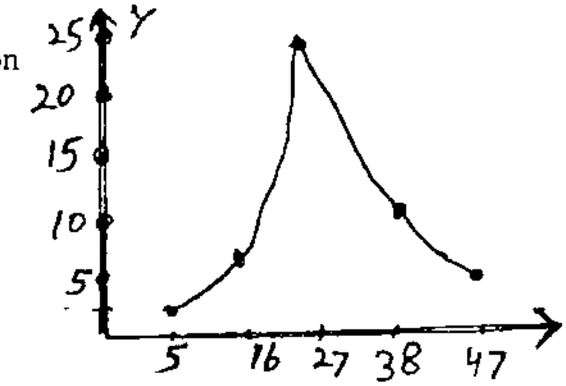
Solution:
We take cars on x –axis
And sales on y –axis



Q.4: Prepare a frequency polygon of the following frequency Distribution.

Score in test	Frequency
0 - 10	2
11 – 21	7
22 – 32	25
33 – 43	11
44 – 50	5

Solution:
The frequency polygon is under .



Class intervals (score in test)	Class boundaries	Mid values x	Frequency
0 – 10	-0.5 – 10.5	5	2
11 – 21	10.5 – 21.5	16	7
22 -32	21.5 – 32 .5	27	25
33 – 43	32.5 – 43.5	38	11
44 – 50	43. 5 – 50.5	47	5

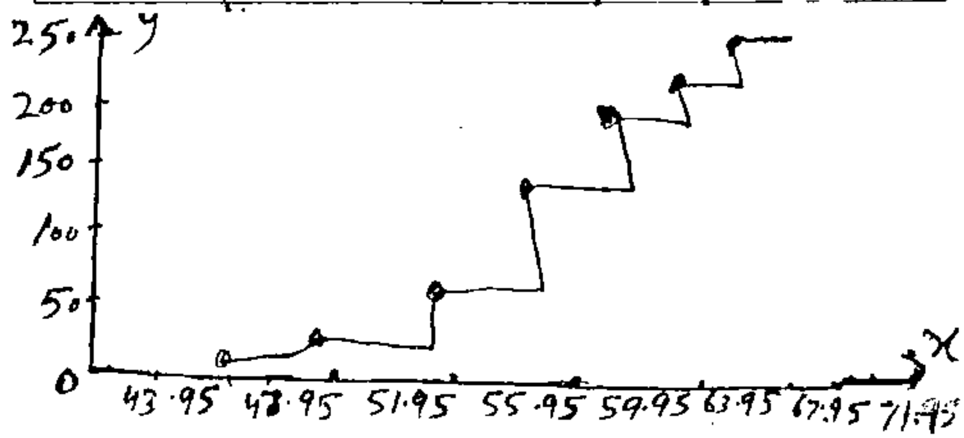
Q.5: This table summarizes the weights in kilogram of 250 boys . Represent these data by means of a cumulative frequency polygon .

Weight (kg)	Number of boys
44.0 – 47.9	3
48.0 – 51.9	17
52.0 – 55.9	50
56.0 – 59.9	81
60.0 – 63.9	57
64.0 – 67.9	23
68.0 – 71.9	9

Solution:

classes	Class boundaries	Mid points x	f	c.f.
44.0 -47.9	43.95- 47.95	45.95	3	3

48.0 –51.9	47.95 51.95	49.95	17	3+17= 20
52.0 –55.9	51.95- 55.95	53.95	50	20+50=70
56.0 –59.9	55.95-59.95	57.95	81	70+81=151
60.0 –63.9	59.95-63.95	61.95	57	151+57=208
64.0 –67.9	63.95-67.95	65.95	23	208+23=231
68.0 –71.9	67.95-71.95	69.95	9	231+9=240



Project: The following scores were made on a 60 – item test .

25	30	34	37	41	42	46	49	53
26	31	34	37	41	42	46	50	53
28	31	35	37	41	43	47	51	54
29	33	36	38	41	44	48	52	54
30	33	36	39	41	44	48	52	55
30	33	37	40	42	45	48	52	

(a) Group the data into class interval of size 2 , beginning With the interval 2 4.5 – 26 .5

(b) Set up the frequency table for the above data .

(c) Build the histogram for the data .

(d) Draw the frequency polygon for this histogram .

(e) Find the cumulative frequency .

(f) Draw the ogive .

(g) Find the range of the data .

(h) Find the mean of the data .

(i) Find the standard deviation of the data .

(j) Find the variance of the data .

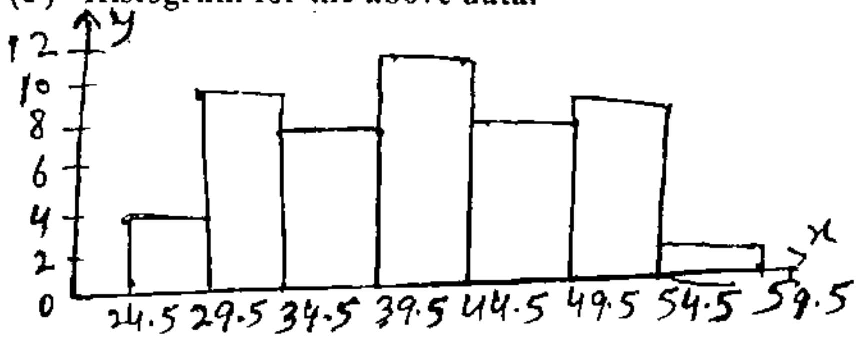
Solution: (a)

Class boundaries	Frequency (f)
24.4 – 26.5	2
26.5 – 28.5	1
28.5 – 30 .5	4
30.5 – 32.5	2
32.5 – 34.5	5
34.5 – 36.5	3
36.5 – 38.5	5
38.5 – 40.5	2
40.5 - 42.5	8
42.5 – 44.5	3
44.5 – 46.5	3
46.5 – 48.5	4
48.5 – 50.5	2
50.5 – 52.5	4
52.5 – 54.5	4
54.5 – 56.5	1

(b) frequency table . we take the width of interval 5.

Class limits	Tailly marks	Frequency (f)
24.4 – 29.5	////	4
29.5 – 34.5	卌 卌	10
34.5 – 39 .5	卌 ////	9
39.5 – 44.5	卌 卌 //	12
44.5 – 49.5	卌 ///	8
49.5 – 54.5	卌 ////	9
54.5 – 59.5	/	1

(c) Histogram for the above data.



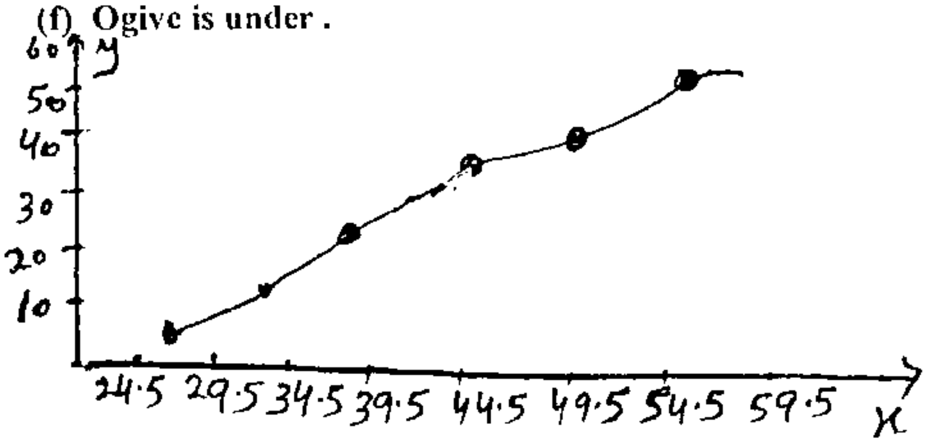
(d) frequency polygon for this histogram .

Class boundaries	Mid point (x)	Frequency (f)
24.4 – 29.5	$\frac{24.5+29.5}{2}=27$	4
29.5 – 34.5	32	10
34.5 – 39 .5	37	9
39.5 – 44.5	42	12
44.5 – 49.5	47	8
49.5 – 54.5	52	9
54.5 – 59.5	57	1

(e) Cumulative frequencies

Class limits	Class boundaries	Frequency (f)	Cumulative frequency
25 – 29	24.4 – 29.5	4	4
30 – 34	29.5 – 34.5	10	4 + 10 = 14
35 – 39	34.5 – 39 .5	9	14 + 9 = 23
40 - 44	39.5 – 44.5	12	23 +12 = 35
45 – 49	44.5 – 49.5	8	35 + 8 = 43
50 – 54	49.5 – 54.5	1	43 + 1 = 44
55 – 59	54.5 – 59.5	9	44 + 9 =53

(f) Ogive is under .



(g) The largest number = 55
The smallest number = 25
Range = 55 – 25 = 30 .

(h)

Class boundaries	Mid point (x)	Frequency (f)	f.x.
24.4 – 29.5	$\frac{24.5+29.5}{2}=27$	4	108
29.5 – 34.5	32	10	320
34.5 – 39.5	37	9	333
39.5 – 44.5	42	12	504
44.5 – 49.5	47	8	376
49.5 – 54.5	52	1	52
54.5 – 59.5	57	9	513
		$\sum f = 53$	$\sum fx = 2206$

As Arithmetic mean $= \frac{\sum fx}{\sum f} = \frac{2206}{53} = 41.62$

(i) Standard deviation :

Class boundaries	Mid point (x)	Frequency (f)	f.x.	x^2	fx^2
24.4 – 29.5	27	4	108	729	19683
29.5 – 34.5	32	10	320	1024	10240
34.5 – 39.5	37	9	333	1369	12321
39.5 – 44.5	42	12	504	1764	21168

44.5 – 49.5	47	8	376	2209	17672
49.5 – 54.5	52	1	52	2704	2704
54.5 – 59.5	57	9	513	3249	29241
		$\sum f = 53$	$\sum fx = 2206$		113029

As S. D . = $\sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$ put values

$$=> S.D.=\sqrt{\frac{113029}{53}--\left(\frac{2206}{53}\right)^2}$$

$$=\sqrt{2132.62-1732.22}=\sqrt{400.40}$$

$$=> S.D=20.009$$

(j) As variance = $\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2$ put values

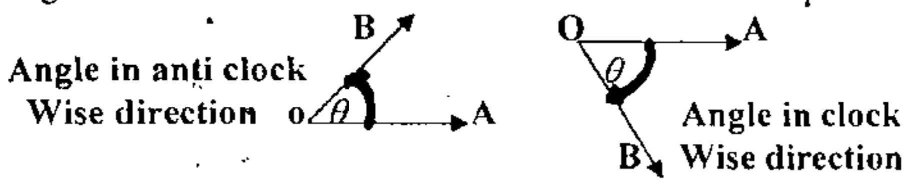
$$=> Variance=\frac{113029}{53}-\left(\frac{2206}{53}\right)^2$$

$$S^2=2132.62-1732.22=400.40$$

Unit 7:

Introduction to Trigonometry:

An angle: The union of two rays which have a common vertex is called angle. Angle measured in clock wise direction is negative and measured in anti clock wise direction is positive.



Sexagesimal system (Degree , Minute and second):

It is a system of measurement of angles in which angles are measured in degrees , minutes and seconds .

one complete rotation of a circle $= 360^o$.

1^o is called one degree .

$1'$ is called one minute and $1''$ is called one second .

Note: $1^o = 60 \text{ minutes} = 60'$

$1' = 60 \text{ second} = 60''$

Thus $1^o = 3600 \text{ second} = 3600''$

Conversion of $D^oM'S''$ form into decimal form and visa versa:

Example 1: Convert $15^o30'25''$ to decimal form.

Solution: As $1' = \left(\frac{1}{60}\right)^o$ *and* $1'' = \left(\frac{1}{3600}\right)^o$. Then

$$15^o30'25'' = \left(15 + 30\times\frac{1}{60} + 25\times\frac{1}{3600}\right)^o$$

$$= (15 + 0.5 + 0.00694)^o$$

$$= 15.50694^o$$

Example 2: Convert 38.39^o to $D^oM'S''$.

Solution: Since $1^o = 60'$ and $1' = 60''$

$$38.39^o = 38^o + (0.39)^o$$
$$= 38^o + (0.39 \times 60)'$$
$$= 38^o + (23.4)'$$
$$= 38^o + 23' + (0.4)'$$
$$= 38^o + 23' + (0.4 \times 60)''$$
$$= 38^o + 23' + 24''$$
$$= 38^o 23' 24''$$

Circular system (radians): In this system unit of measure of angle is radian. The central angle that subtends when length "l" of an arc is equal to radius "r" of the circle.

Relation between degree and radian:

$$2\pi \;\; rad = 360^o$$
$$\Rightarrow \pi \; rad = 180^o$$
$$As \;\; 1 \;\; rad = \left(\frac{180}{\pi}\right)^o = \left(\frac{180}{3.1415}\right)^o = 57.296^o$$
$$\Rightarrow \;\; 1 \;\; rad = 57.3^o$$
$$Similarly \quad 180^o = \pi \;\; rad$$
$$1^o = \frac{\pi}{180} = \frac{3.14159}{180}$$
$$\Rightarrow \;\; 1^o = 0.0175 \, rad$$

Exercise 7.1:

Q.1: Convert the following angles from $D^o M' S''$

(*i*) $80^o 15' 35''$ (*ii*) $39^o 48' 55''$

(*iii*) $84^o 19' 10''$ (*iv*) $18^o 6' 21''$

Solution: (*i*) $80^o 15' 35''$

As

$1' = \left(\frac{1}{60}\right)^o$ *and* $1'' = \left(\frac{1}{3600}\right)^o$. Then

$$80''15'35'' = \left(80 + 15 \times \frac{1}{60} + 35 \times \frac{1}{3600}\right)''$$

$$= (80 + 0.25 + 0.0097)''$$

$$= 80.2597''$$

(*ii*) $39^o48'55''$:

As $1' = \left(\frac{1}{60}\right)^o$ *and* $1'' = \left(\frac{1}{3600}\right)^o$. Then

$$39^o48'55'' = \left(39 + 48 \times \frac{1}{60} + 55 \times \frac{1}{3600}\right)^o$$

$$= (39 + 0.8 + 0.0152)^o$$

$$= 39.8152^o$$

(*iii*) $84^o19'10''$:

As $1' = \left(\frac{1}{60}\right)^o$ *and* $1'' = \left(\frac{1}{3600}\right)^o$. Then

$$84^o19'10'' = \left(84 + 19 \times \frac{1}{60} + 10 \times \frac{1}{3600}\right)^o$$

$$= (84 + 0.3167 + 0.0027)^o$$

$$= 84.3194^o$$

(*iv*) $18^o6'21''$:

As $1' = \left(\frac{1}{60}\right)^o$ *and* $1'' = \left(\frac{1}{3600}\right)^o$. Then

$$18^o6'21'' = \left(18 + 6\times\frac{1}{60} + 21\times\frac{1}{3600}\right)^o$$

$$= (18 + 0.1 + 0.0058)^o$$

$$= 18.1058^o$$

Q.2:**Convert the following angles from decimal forms to** $D^oM'S''$.

(*i*) 42.25^o (ii) 57.325^o (iii) 12.9956^o (iv) 32.625^o

Solution: (*i*) 42.25^o:

Since $1^o = 60'$ and $1' = 60''$

$$42.25^o = 42^o + (0.25)^o$$

$$= 42^o + (0.25\times 60)'$$

$$= 42^o + 15'$$

$$= 42^o15'$$

(ii) 57.325^o: Since $1^o = 60'$ and $1' = 60''$

$$57.325^o = 57^o + (0.325)^o$$

$$= 57^o + (0.325\times 60)'$$

$$= 57^o + (19.5)'$$

$$= 57^o + 19' + (0.5)'$$

$$= 57^o + 19' + (0.5\times 60)''$$

$$= 57^o + 19' + 30''$$

$$\Rightarrow 57.325^o = 57^o19'30''$$

(iii) 12.9956^o: Since $1^o = 60'$ and $1' = 60''$

$$12.9956^o = 12^o + (0.9956)^o$$

$=12^o+(0.9956\times 60)'$

$=12^o+(59.736)'$

$=12^o+59'+(0.736)'$

$=12^o+59'+(0.736\times 60)''$

$=12^o+59'+44''$

$12^o59'44''$

(iv) 32.625^o: Since $1^o=60'$ and $1'=60''$

$32.625^o=32^o+(0.625)^o$

$=32^o+(0.625\times 60)'$

$=32^o+(37.5)'$

$=32^o+37'+(0.5)'$

$=32^o+37'+(0.5\times 60)''$

$=32^o+37'+30''$

$=32^o37'30''$

Q.3: Convert the following radian measures of angles into the measures of degrees.

(i) 2 radians (ii) $\frac{5\pi}{3}$ *radians* **(iii)** $\frac{\pi}{6}$ *radians*

(iv) $-\frac{3\pi}{4}$ *radians*

Solution: (i) 2 radians:

As 1 *rad* = 57.3^o. Then

2 *rad* = 2×57.3^o

$=114.6^o$

(ii) $\frac{5\pi}{3}$ *radians*:

$$As\ \ \pi\ \ radian = 180^o$$

$$1\ \ radian = \left(\frac{180}{\pi}\right)^o$$

$$\frac{5\pi}{3}\ rad = \left(\frac{180}{\pi}\times\frac{5\pi}{3}\right)^o$$

$$= (60\times 5)^o$$

$$= 300^o$$

(iii) $\frac{\pi}{6}\ radians$:

$$As\ \ \pi\ \ radian = 180^o$$

$$1\ \ rad = \left(\frac{180}{\pi}\right)^o$$

$$=> \ \frac{\pi}{6}\ \ rad = \left(\frac{180}{\pi}\times\frac{\pi}{6}\right)^o$$

$$= 30^o$$

(iv) $-\frac{3\pi}{4}\ radians$:

$$As\ \ \pi\ \ radian = 180^o$$

$$1\ rad = \left(\frac{180}{\pi}\right)^o$$

$$=> \ -\frac{3\pi}{4}\ \ rad = -\left(\frac{180}{\pi}\times\frac{3\pi}{4}\right)^o$$

$$= -45^o\times 3$$

$$= -135^o$$

Q.4: **Convert the following angles in terms of radians.**

(i) 45^o **(ii)** 120^o **(iii)** -210^o **(iv)** $60^o35'48''$

Solution: (i) 45^o :

$$As \quad 1^o = \frac{\pi}{180} rad \text{ then}$$

$$45^o = 45 \times \frac{\pi}{180} \; rad$$

$$= \frac{\pi}{4} rad$$

(ii) 120^o:

$$As \quad 1^o = \frac{\pi}{180} rad \text{ then}$$

$$120^o = \frac{\pi}{180} \times 120 \; rad$$

$$=> 120^o = \frac{2\pi}{3} rad$$

(iii) -210^o:

$$As \quad 1^o = \frac{\pi}{180} rad \text{ then}$$

$$-210^o = -\frac{\pi}{180} \times 210 \; rad$$

$$= -\frac{7\pi}{6} \; rad$$

(iv) $60^o 35' 48''$:

$$60^o 35' 48'' = \left(60 + 35 \times \frac{1}{60} + 48 \times \frac{1}{3600}\right)^o$$

$$= (60 + 0.58333 + 0.0133)^o$$

$$= 60.59666^o$$

$$As \; 1^o = 0.0175 \; rad$$

$$60.5933^o = (60.59666 \times 0.0175) \; rad$$

$$= 1.0574 \; rad$$

Length of an arc of a circle: *Let* l be the length of an arc and "r" be the radius of circle and θ be central angle then

$$l = r\theta$$

Area of a sector of a circle: Let "r" be the radius of a circle and θ be central angle of the sector then area of the sector is

$$A = \frac{1}{2}r^2\theta$$

Exercise.7.2:

Q.1: Find l when

(i) $\theta = \frac{\pi}{6}\ radian\ , r = 2cm$

(ii) $\theta = 30^o\ , r = 6cm$

(iii) $\theta = \frac{4\pi}{6}\ radian\ , r = 6cm$

Solution: (i) $\theta = \frac{\pi}{6}\ radian\ , r = 2cm$

$As\ \ l = r\theta$ ....(i) put values in(i).

$$l = \left(\frac{\pi}{6}\right)(2) = \frac{\pi}{3}\ cm$$

$$= \frac{3.14159}{3} = 1.047cm$$

(ii) $\theta = 30^o\ , r = 6cm$.

$$\theta = 30 \times 0.0175 = 0.525\ rad$$

$As\ \ l = r\theta$ ....(i) put values in(i).

$$l = 6(0.525)$$

$$= 3.15\ cm$$

(iii) $\theta = \frac{4\pi}{6}\ radian\ , r = 6cm$

$As\ \ l = r\theta$ ....(i) put values in(i).

$$l = \left(\frac{4\pi}{6}\right)(6)$$
$$= 4\pi \ cm$$
$$= 4(3.14159)$$
$$=> l = 12.57cm$$

Q.2: Find θ when

(i) $l = 5 \ cm \ , \ r = 2cm$

(ii) $l = 30cm \ , r = 6cm$

(iii) $l = 6cm \ , r = 2.87 \ cm$

Solution: (i) $l = 5 \ cm \ , \ r = 2cm$

$$As \ \ \theta = \frac{l}{r} \ ....(i) \quad \text{put values in (i).}$$
$$=> \theta = \frac{5}{2} = 2.5 \ radian$$

(ii) $l = 30cm \ , r = 6cm$

$$As \ \ \theta = \frac{l}{r} \ ....(i) \quad \text{put values in (i).}$$
$$=> \theta = \frac{30}{6} = 5 \ radian$$

(iii) $l = 6cm \ , r = 2.87 \ cm$

$$As \ \ \theta = \frac{l}{r} \ ....(i) \quad \text{put values in (i).}$$
$$=> \theta = \frac{6}{2.87} = 2.09 \ radian$$

Q.3: Find r when

(i) $\theta = \frac{\pi}{6} \ radians \ , \ l = 2cm$

(ii) $\theta = 3\frac{1}{2} \ radians \ , l = \frac{4}{7} cm$

(iii) $\theta = \frac{3\pi}{4} radians \ , l = 15cm$.

Solution: (i) $\theta = \frac{\pi}{6}\ radians\ ,\ l = 2cm$

$As\quad r = \frac{l}{\theta}$ ....(i) put values in(i) .

$$=>\ r = \frac{2}{\frac{\pi}{6}} = \frac{2\times 6}{\pi}$$

$$= \frac{12}{3.14159} = 3.82cm$$

(ii) $\theta = 3\frac{1}{2}\ radians\ , l = \frac{4}{7}cm$

$$=>\ \theta = \frac{7}{2}\ radians$$

$As\quad r = \frac{l}{\theta}$ ....(i) put values in(i) .

$$=>\ r = \frac{\frac{4}{7}}{\frac{7}{2}} = \frac{4\times 2}{7\times 7}$$

$$=>\ r = \frac{8}{49} = 0.1632cm$$

(iii) $\theta = \frac{3\pi}{4} radians\ , l = 15cm$.

$As\quad r = \frac{l}{\theta}$ ....(i) put values in(i) .

$$=>\ r = \frac{15}{\frac{3\pi}{4}}$$

$$=>\ r = \frac{5\times 4}{\pi} = \frac{20}{3.14159}$$

$$=>\ r = 6.366cm$$

Q.4: Find area of sector of a circle whose radius is 4 m , with central angle 12 radians.

Solution: Here $\theta = 12 radians$, $r = 4m$

$$As \quad Area \ of \ \sec tor, \ A = \frac{1}{2}r^2\theta \ .....(i)$$

Put values in (i)

$$A = \frac{1}{2}(4)^2(12) = (16)(6)$$

$$=> \ A = 96 \ m^2$$

Q.5: The area of a circle subtends an angle of 30^o at the center . The radius of circle is 5 cm find ;
(i) Length of the arc (ii) Area of a sector formed.

Solution: $\theta = 30^o$

$$= 30 \times 0.0175 = 0.525 \ radians$$

r = 5 cm

(i) length of the arc.

$As \ l = r\theta$ ....(i) put values in(i).

$$=> l = (5)(0.525)$$

$$= 2.625 \ cm$$

(ii) Area of a sector :

$As \quad A = \frac{1}{2}r^2\theta$.......(ii) put values in (ii).

$$=> \ A = \frac{1}{2}(5)^2(0.525) = \frac{13.125}{2}$$

$$=> \ A = 6.56 \ cm^2$$

Q.6: An arc of a circle subtends an angle of 2 radian at the center . If the area of sector formed is $64\,cm^2$, find the radius of the circle .

Solution: $\theta = 2$ radian , $A = 64 \ cm^2$, $r = ?$.

$$As \quad A = \frac{1}{2}r^2\theta \quad => \quad 2A = r^2\theta$$

=> $r^2 = \frac{2A}{\theta}$ ......(i) , put values

=> $r^2 = \frac{2 \times 64}{2} = 64$

=> $\sqrt{r^2} = \sqrt{64}$ *Take square root of both sides*

=> $r = 8$ Ans.

Q.7: In a circle of radius 10m , find the distance traveled by a point moving on this circle if the point makes 3.5 revolutions (3.5 revolutions = 7π).

Solution: $r = 10$, $\theta = 7\pi$,

Distance travel on the circle = $l = ?$

As $l = r\theta$ .....(i) , put values

=> $l = 10 \times 7\pi = 70\pi$ m

Q.8: What is the circular measure of the angle between the hands of the watch at 3 o,clock .

Solution : At 3 o, clock the minute hand at position 12 . and hour hand at position 3 of a watch .

Then the angle between two hands at this position is equal to 3 parts of the watch .

Since there are 12 equal parts in a watch

And equal to 2π radians

As 12 *parts of watch* $= 2\pi$ *radian*



1 *part of watch* $= \frac{2\pi}{12}$ *radian*

3 *parts of watch* $= \frac{2\pi}{12} \times 3 = \frac{\pi}{2}$ *radians*

Thus $\theta = \frac{\pi}{2}$ **radians .**

Q.9: What is the length of the arc APB ?.



Solution: Here $r = 8$ cm , $\theta = 90''$

Length of arc = l = ?

As $360'' = 2\pi$ radian

$1'' = \frac{2\pi}{360''}$ radian

$90'' = \frac{2\pi}{360} \times 90 = \frac{2\pi}{4} = \frac{\pi}{2}$ *radian*

Then $\theta = \frac{\pi}{2}$

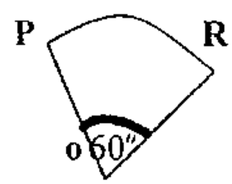
$l = r\theta$ ...(i) put values in (i) .

=> $l = 8 \times \frac{\pi}{2} = 4\pi$ cm .

Q.10: **Find the area of the sector OPR .**

Solution:

Here $r = 6$ cm , $\theta = 60''$, Area = ?



As $360'' = 2\pi$ radian

$1^o = \frac{2\pi}{360''}$ radian

$60'' = \frac{2\pi}{360} \times 60 = \frac{2\pi}{6} = \frac{\pi}{3}$ *radian*

=> $\theta = \frac{\pi}{3}$ radian

As area of the sector $A = \frac{1}{2}r^2\theta$.......(i) put values in (i).

$A = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right)$ => $A = \frac{1}{2} \times 36 \times \frac{\pi}{3}$

=> $A = 6\pi\ cm^2$

Trigonometric Ratios:

(a) **The general angle or Coterminal angles**: Angles having the same initial and terminal sides are general or coterminal angles , and they differ by multiple of 2π or 360^o. For example if $\theta = 30^0$ is a terminal angle then

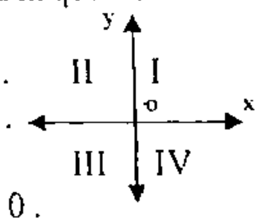
its coterminal angles are

$$30^o + 360^o = 390^o \quad \text{and}$$

$$30^o - 360^o = -330^o .$$

Thus coterminal angles of $\theta = 30^o$ are 390^o *and* -330^o.

Quadrants: If two perpendicular lines $\overline{XOX'}$ and $\overline{YOY'}$ Intersecting XY –Plane at O into four equal parts then each Part is called quadrant. A point is denoted in quadrant in the form of (x , y).



Note: (i) In the first quadrant $x > 0$, $y > 0$.
(ii) In the 2^{nd} quadrant , $x < 0$, $y > 0$.
(iii) In the 3^{rd} quadrant $x < 0$, $y < 0$.
(iv) Im the fourth quadrant $x > 0$. $y < 0$.

Quadrant angles: Those angles whose terminal sides are coincide with coordinate axes i.e. x – axis and y – axis are called quadrant angles. The quadrantal measures of angles are $0^o, 90^o, 180^o, 270^o, 360^o, -90^o, -180^o, -270^o, -360^o$, and $0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi, -\frac{\pi}{2}, -\frac{3\pi}{2}, -2\pi$.

Note:(i) *If* $0 < \theta < 90^o$ *or* $0 < \theta < \frac{\pi}{2}$

then θ lie in first quadrant.

(ii) *If* $90^o < \theta < 180^o$ *or* $\frac{\pi}{2} < \theta < \pi$ then θ lie in second quadrant.

(iii) *If* $180^o < \theta < 270^o$ *or* $\pi < \theta < \frac{3\pi}{2}$ then θ lie in third quadrant.

(iv) *If* $270^o < \theta < 360^o$ *or* $\frac{3\pi}{2} < \theta < 2\pi$ then θ lie in fourth quadrant.

Exercise 7.3:

Q.1: Find coterminal angles of the following angles.

(i) 55^o (ii) -45^o (iii) $\frac{\pi}{6}$ (iv) $-\frac{3\pi}{4}$

Solution: (i) 55^o :

$$As\ \ 55^o + 360^o = 415^o \quad and$$

$$55^o - 360^o = -305^o$$

Thus coterminal angles of 55^o are 415^o and -315^o.

(ii) -45^o:

$$As\ -45^o + 360^o = 315^o$$

$$And\ \ -45^o - 360^o = -405^o$$

Thus coterminal angles -45^o are 315^o and -405^o.

(iii) $\frac{\pi}{6}$:

$$As\ \frac{\pi}{6} + 2\pi = \frac{\pi + 12\pi}{6}$$

$$= \frac{13\pi}{6}$$

$$And\quad \frac{\pi}{6} - 2\pi = \frac{\pi - 12\pi}{6}$$

$$= -\frac{11\pi}{6}$$

(iv) $-\frac{3\pi}{4}$:

$$As\ -\frac{3\pi}{4} = -\frac{3\pi}{4} + 2\pi$$

$$= \frac{-3\pi + 8\pi}{4}$$

$$= \frac{5\pi}{4}$$

And $-\frac{3\pi}{4}-2\pi=\frac{-3\pi-8\pi}{4}$

$=-\frac{11\pi}{4}$

Thus coterminal angles of $-\frac{3\pi}{4}$ are $\frac{5\pi}{4}$ and $-\frac{11\pi}{4}$.

Q.2: In which Quadrant the following angles lie ?

(i) $\frac{8\pi}{5}$ **(ii)** 75^o **(iii)** -818^o **(iv)** $-\frac{5\pi}{4}$

(v) 103^o

Solution: (i) $\frac{8\pi}{5}$ **:** As $\frac{3\pi}{2}<\frac{8\pi}{5}<2\pi$.

Then $\frac{8\pi}{5}$ is lie in third quadrant .

(ii) 75^o**:** *Since* $0^c<75^o<90^o$.Then it is lie in first quadrant.

(iii) -818^o**:**

Since $-818^o=-720^o-98^o$

$=-98^o$

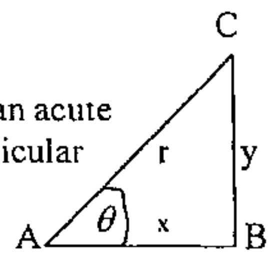
It is in clock wise direction .It is in 3rd quadrant .

(iv) $-\frac{5\pi}{4}$**:**

It is in clock wise direction .It is in 2nd quadrant .

(v) 103^o**:** it is in 2nd quadrant .

Trigonometric ratios of a unit circle:



In a right angle triangle ABC . Let θ be an acute angle . AB is a base and BC is a perpendicular and AC is a hypotenuse . Then let $AB=x$, $BC=y$, $AC=r$

(i) $Sin\theta=\frac{perpendicular}{hypotenous}=\frac{BC}{AC}=\frac{y}{r}$

(ii) $\cos\theta = \frac{base}{hypotenous} = \frac{AB}{AC} = \frac{x}{r}$

(iii) $\tan\theta = \frac{perpendicular}{base} = \frac{BC}{AB} = \frac{y}{x}$

(iv) $\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{r}{y}$ (v) $\sec\theta = \frac{AC}{AB} = \frac{r}{x}$

(vi) $\cot\theta = \frac{AB}{BC} = \frac{x}{y}$.

Values of trigonometric ratios:

θ	30^o	45^o	60^o	90^o	180^o	270^o	360^o
Sin	$\frac{1}{2} = 0.5$	$\frac{1}{\sqrt{2}}$ $= 0.707$	$\frac{\sqrt{3}}{2}$ $= 0.866$	1	0	-1	0
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$ $= 0.707$	$\frac{1}{2} = 0.5$	0	-1	0	1
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	$-\infty$	0

Reciprocal trigonometric ratios:

$\operatorname{cosec}\theta$, $\sec\theta$ $\cot\theta$ are called reciprocal trigonometric ratios

and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$ and $\cot\theta = \frac{1}{\tan\theta}$

The values of reciprocal trigonometric ratios can be find by using these equations.

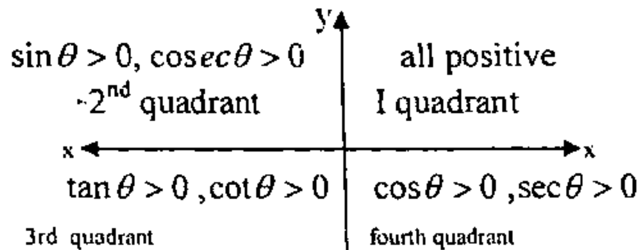
Sings of trigonometric ratios in different quadrants:

(i) In first quadrant all trigonometric ratios are positive.

(ii) In 2nd quadrant $\sin\theta$ *and* $\operatorname{cosec}\theta$ are positive.

(iii) In 3rd quadrant $\tan\theta$ *and* $\cot\theta$ are positive.

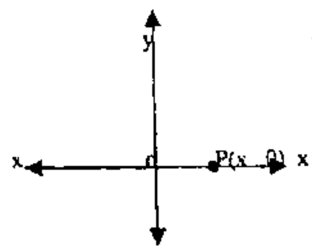
(iv) In fourth quadrant $\cos\theta$ *and* $\sec\theta$ are positive.



Calculating the values of trigonometric ratios of

$0^o, 90^o, 180^o, 270^o$ *and* 360^o

Trigonometric ratios of 0^o :



In XY –Plane the terminal side $\overline{OP}$ of an angle θ is coincide with $\overline{OX}$. There fore radius vector

$\overline{OP} = r = x$ i.e.

x = r and y = 0

$\sin 0^o = \frac{y}{r} = \frac{0}{x} = 0$; $\cos 0^o = \frac{x}{r} = \frac{x}{x} = 1$

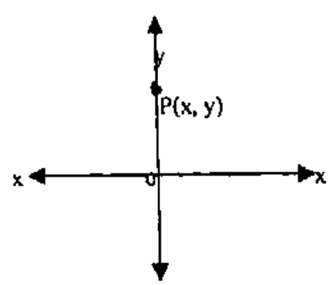
$\tan 0^o = \frac{y}{x} = \frac{0}{x} = 0$

$\operatorname{cosec} 0^o = \frac{r}{y} = \frac{x}{0} = \infty = undefined$

$\sec 0^o = \frac{r}{x} = \frac{x}{x} = 1$

$\cot 0^o = \frac{x}{y} = \frac{x}{0} = \infty = undefined$

Trigonometric ratios of 90^o :



In XY –Plane the terminal side $\overline{OP}$ of an angle θ is coincide with $\overline{OY}$. There fore radius vector

$\overline{OP} = r = y$ i.e.

y = r and x = 0

$$\sin 90^o = \frac{y}{r} = \frac{y}{y} = 1$$

$$\cos 90^o = \frac{x}{r} = \frac{0}{y} = 0$$

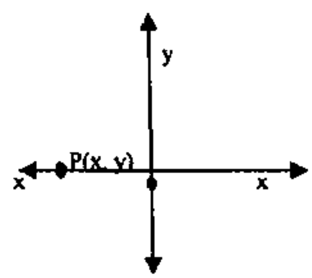
$$\tan 90^o = \frac{y}{x} = \frac{y}{0} = \infty$$

$$\operatorname{cosec} 90^o = \frac{r}{y} = \frac{y}{y} = 1$$

$$\sec 90^o = \frac{r}{x} = \frac{y}{0} = \infty = undefined$$

$$\cot 90^o = \frac{x}{y} = \frac{0}{y} = 0$$

Trigonometric ratios of 180^o :



In XY –Plane the terminal side $\overrightarrow{OP}$ of an angle θ is coincide with $\overrightarrow{OX'}$. There fore radius vector

$\overline{OP} = r = -x$ i.e.

$y = 0$ and $x = -r$

$$\sin 180^o = \frac{y}{r} = \frac{0}{-x} = 0$$

$$\cos 180^o = \frac{x}{r} = -\frac{r}{r} = -1$$

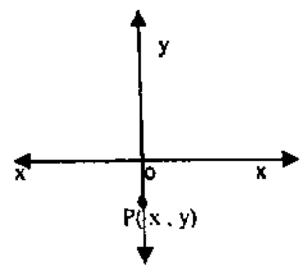
$$\tan 180^o = \frac{y}{x} = \frac{0}{-x} = 0$$

$$\operatorname{cosec} 180^o = \frac{r}{y} = \frac{-x}{0} = -\infty$$

$$\sec 180^o = \frac{r}{x} = -\frac{x}{x} = -1$$

$$\cot 180^o = \frac{x}{y} = \frac{x}{0} = \infty$$

Trigonometric ratios of 270^o :



In XY –Plane the terminal side $\overrightarrow{OP}$ of an angle θ is coincide with $\overleftrightarrow{Oy'}$. There fore radius vector

$\overline{OP} = r = -y$ i.e.

x = 0 and y = -r

$$\sin 270^o = \frac{y}{r} = -\frac{r}{-r} = -1$$

$$\cos 270^o = \frac{x}{r} = \frac{0}{r} = 0$$

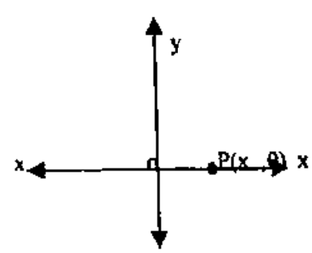
$$\tan 270^o = \frac{y}{x} = -\frac{r}{0} = -\infty$$

$$\operatorname{cosec} 270^o = \frac{r}{y} = \frac{-y}{y} = -1$$

$$\sec 270^o = \frac{r}{x} = -\frac{y}{0} = -\infty = undefined$$

$$\cot 270^o = \frac{x}{y} = \frac{0}{y} = 0$$

Trigonometric ratios of 360^o:



In XY –Plane the terminal side $\overrightarrow{OP}$ of an angle θ is coincide with $\overrightarrow{OX}$. There fore radius vector

$\overline{OP} = r = x$ i.e.

x = r and y = 0

$$\sin 360^o = \frac{y}{r} = \frac{0}{x} = 0$$

$$\cos 360^o = \frac{x}{r} = \frac{x}{x} = 1$$

$\tan 360^o = \frac{y}{x} = \frac{0}{x} = 0$

$\operatorname{cosec} 360^o = \frac{r}{y} = \frac{x}{0} = \infty = undefined$

$\sec 360^o = \frac{r}{x} = \frac{x}{x} = 1$

$\cot 360^o = \frac{x}{y} = \frac{x}{0} = \infty = undefined$

Exercise 7.4:

Q.1: Find the signs of the following trigonometric ratios and tell in which quadrant they lie?.

(i) $Sin98^o$ (ii) $Sin160^o$ (iii) $Tan200^o$ (iv) $Sec120^o$

(v) $\operatorname{cosec}198^o$ (vi) $Sin460^o$

Solution: (i) $Sin98^o$:

Since 98^o is in the 2^{nd} quadrant then sign of $Sin98^o$ is positive.

(ii) $Sin160^o$:

Since 160^o is in the 2^{nd} quadrant then sign of $Sin160^o$ is positive.

(iii) $Tan200^o$:

Since 200^o is in the 3^{rd} quadrant then sign of $Tan200^o$ is positive.

(iv) $Sec120^o$:

Since 120^o is in 2^{nd} quadrant then sign of $Sec120^o$ is negative

(v) $\operatorname{cosec}198^o$:

Since 198^o is in the 3^{rd} quadrant then sign of $\operatorname{cosec}198^o$ is negative

(vi) $Sin460^o$: Since

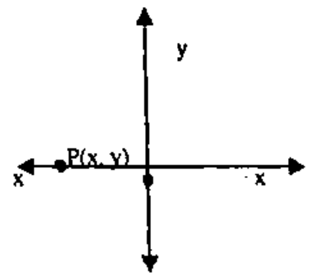
$$460^o = 360^o + 100^o$$
$$= 100^o$$

As 100^o is in the 2^{nd} quadrant then sign of $Sin460^o$ is positive.

Q.2. Find the trigonometric ratios of the following angles

(i) -180^o (ii) -270^o (iii) 720^o (iv) 1170^o

Solution: (i)



Trigonometric ratios of -180^o:

In XY –Plane the terminal side $\overline{OP}$ of an angle θ is coincide with $\overline{OX'}$ in clock wise direction.

There fore radius vector

$\overline{OP} = r = -x$ i.e.

y = 0 and x = -r

$$\sin(-180^o) = \frac{y}{r} = \frac{0}{-x} = 0$$

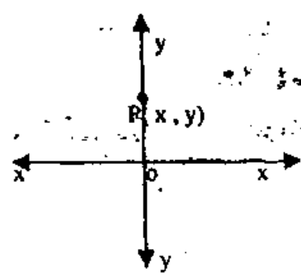
$$\cos(-180^o) = \frac{x}{r} = -\frac{r}{r} = -1$$

$$\tan(-180^o) = \frac{y}{x} = \frac{0}{-x} = 0$$

$$\operatorname{cosec}(-180^o) = \frac{r}{y} = \frac{-x}{0} = -\infty = \text{undefined}$$

$$\sec(-180^o) = \frac{r}{x} = -\frac{x}{x} = -1$$

$$\cot(-180^o) = \frac{x}{y} = \frac{x}{0} = \infty$$



Trigonometric ratios of -270^o :

In XY –Plane the terminal side $\overline{OP}$ of an angle θ is coincide with $\overline{Oy'}$ in clock wise direction.

There fore radius vector

$\overline{OP} = r = y$ i.e.

x = 0 and y = r

$$\sin(-270)^o = \frac{y}{r} = \frac{y}{y} = 1$$

$$\cos(-270^o) = \frac{x}{r} = \frac{0}{y} = 0$$

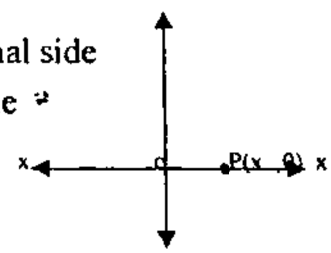
$\tan(-270^o) = \frac{y}{x} = \frac{y}{0} = \infty = undefined$

$\cos ec(-270^o) = \frac{r}{y} = \frac{y}{y} = 1$

$\sec(-270^o) = \frac{r}{x} = \frac{y}{0} = \infty = undefined$

$\cot(-270^o) = \frac{x}{y} = \frac{0}{y} = 0$

(iii) Trigonometric ratios of 720^o**:**



Since 720^o and 0^o have same terminal side $\overline{OP}$ in XY –Plane . The terminal side of an angle θ is coincide with $\overline{OX}$. There fore radius vector

$\overline{OP} = r = x$ i.e.

x = r and y = 0

$\sin 720^o = \frac{y}{r} = \frac{0}{x} = 0$

$\cos 720^o = \frac{x}{r} = \frac{x}{x} = 1$

$\tan 720^o = \frac{y}{x} = \frac{0}{x} = 0$

$\cos ec 720^o = \frac{r}{y} = \frac{x}{0} = \infty = undefined$

$\sec 720^o = \frac{r}{x} = \frac{x}{x} = 1$

$\cot 720^o = \frac{x}{y} = \frac{x}{0} = \infty = undefined$

(iv) Trigonometric ratios of 1470^o**:**

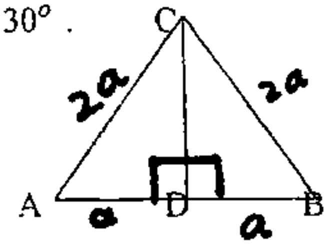
Since $1470^o = 1440^o + 30^o$

$= 4(320^o) + 30^o = 30^o$

As 1470^o *and* 30^o *have same terminal side* .

Let ABC be an equilateral triangle in which each side is equal to "2*a*" . Let draw a bisector CD of angle < C and meet AB at a point D . Then ACD is a right angle triangle in which

$m < ADC = 90^o$ *and* $m < ACD = 30^o$.



Since AB = 2a then AD = a .

By pathagourse theorem

$$(AC)^2 = (AD)^2 + (DC)^2$$

Put values

$$(2a)^2 = a^2 + (DC)^2$$

$$=> \ 4a^2 - a^2 = (DC)^2$$

$$=> \ (DC)^2 = 3a^2 \quad => \sqrt{(DC)^2} = \sqrt{3a^2}$$

$$=> \ DC = 3\sqrt{a}$$

In triangle ACD for $m < ACD = 30^o$, AD is a base , BC is Perpendicular and AC is a hypoteneous .Then

$$\sin 1470^o = \sin 30^o = \frac{AD}{AC} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 1470^o = \cos 30^o = \frac{DC}{AC} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$Tan 1470^o = Tan 30^o = \frac{AD}{DC} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$$

$$Co\sec 1470^o = \operatorname{cosec} 30^o = \frac{AC}{AD} = \frac{2a}{a} = 2$$

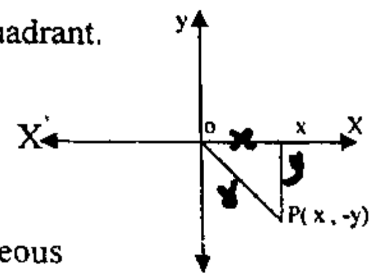
$$Sec 1470^o = Sec 30^o = \frac{AC}{DC} = \frac{2a}{\sqrt{3}a} = \frac{2}{\sqrt{3}}$$

$$\cot 1470^o = \cot 30^o = \frac{DC}{AD} = \frac{\sqrt{3}a}{a} = \sqrt{3}$$

Q.3: If $Sec\theta = 2$ where θ lies in 4th quadrant , find the other Values of trigonometric ratios .

Solution:

As $Sec\theta = 2$ and θ lie in 4th quadrant.



As we know that

$\sec\theta = \frac{r}{x} = \frac{2}{1}$. Then

=> $x = 1, r = 2$,

Where x is base and r is hypoteneous

And y is perpendicular.

Then by pataghourse theorem

$x^2 + y^2 = r^2$ put values

=> $(1)^2 + y^2 = (2)^2$

=> $1 + y^2 = 4$

=> $y^2 = 4 - 1 = 3$

=> $y = -\sqrt{3}$ (*since θ is in 4th quadrant*)

Now $Sin\theta = \frac{y}{r} = -\frac{\sqrt{3}}{2}$

$Cos\theta = \frac{x}{r} = \frac{1}{2}$

$\tan\theta = \frac{y}{x} = -\frac{\sqrt{3}}{1} = -\sqrt{3}$

$\operatorname{cosec}\theta = \frac{r}{y} = -\frac{2}{\sqrt{3}}$

$\cot\theta = \frac{x}{y} = -\frac{1}{\sqrt{3}}$

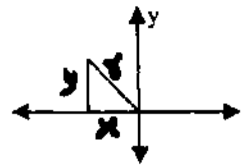
Q.4: If $\sin\theta = \frac{4}{5}$ and $\frac{\pi}{2} < \theta < \pi$ then find other trigonometric ratios:

Solution: If $\sin\theta = \frac{4}{5}$ and $\frac{\pi}{2} < \theta < \pi$.

Then θ lie in 2nd quadrant.

then x is negative.

Here r is hypotenous, x is a base
And y is perpendicular.



$As\ \sin\theta = \frac{y}{r} = \frac{4}{5}$

=> $y = 4\ ,\ r = 5$

Then by pathagourse theorem

$x^2 + y^2 = r^2$ put values

=> $x^2 + (4)^2 = (5)^2$

=> $x^2 = 25 - 16 = 9$

=> $x = -3$ $(because\ in\ 2nd\ quadrant\ \ x\ is\ negative)$

$Cos\theta = \frac{x}{r} = -\frac{3}{5}$

$\tan\theta = \frac{y}{x} = \frac{4}{-3} = -\frac{4}{3}$

$\operatorname{cosec}\theta = \frac{r}{y} = \frac{5}{4}$

$\sec\theta = \frac{r}{x} = -\frac{5}{3}$

$\cot\theta = \frac{x}{y} = -\frac{3}{4}$

Q.5: Find the values of

(i) $2\sin 45^o \cos 45^o$ **(ii)** $\frac{\tan 60^o - \tan 30^o}{1 + \tan 60^o \tan 30^o}$

(iii) $\frac{\cos 45^o}{\sin 45^o + \tan 45^o}$ **(iv)** $\tan 30^o \tan 60^o + \tan 45^o$

(vi) $\cos\frac{\pi}{3}\cos\frac{\pi}{6} - \sin\frac{\pi}{3}\sin\frac{\pi}{6}$

Solution: (i) $2\sin 45^o \cos 45^o$

Put $\sin 45^o = \frac{1}{\sqrt{2}}$ *and* $\cos 45^o = \frac{1}{\sqrt{2}}$

$$2\sin 45^o \cos 45^o = 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$= 2\left(\frac{1}{2}\right) = 1$$

(ii) *As* $\tan 30^o = \frac{1}{\sqrt{3}}$ *and* $\tan 60^o = \sqrt{3}$,Then

$$\frac{\tan 60^o - \tan 30^o}{1+\tan 60^o \tan 30^o} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1+\left(\sqrt{3}\right)\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1+1} = \frac{1}{2}\left(\frac{(\sqrt{3})(\sqrt{3}) - 1}{\sqrt{3}}\right)$$

$$= \frac{3-1}{2\sqrt{3}} = \frac{2}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}$$

(iii) $\sin 45^o = \frac{1}{\sqrt{2}}$ *and* $\cos 45^o = \frac{1}{\sqrt{2}}$, $\tan 45^o = 1$

$$\frac{\cos 45^o}{\sin 45^o + \tan 45^o} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}+1} = \frac{\frac{1}{\sqrt{2}}}{\frac{1+\sqrt{2}}{\sqrt{2}}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{1+\sqrt{2}}$$

$$= \frac{1}{1+\sqrt{2}}$$

(iv)

$As\ \tan 30^o = \frac{1}{\sqrt{3}}$ and $\tan 60^o = \sqrt{3}$, $\tan 45^o = 1$

$$\tan 30^o \tan 60^o + \tan 45^o = \left(\frac{1}{\sqrt{3}}\right)\left(\sqrt{3}\right) + 1$$

$$= 1 + 1 = 2$$

(v) $As\ \cos\frac{\pi}{3} = \frac{1}{2}$,$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin\frac{\pi}{6} = \frac{1}{2}$

Then

$$\cos\frac{\pi}{3}\cos\frac{\pi}{6} - \sin\frac{\pi}{3}\sin\frac{\pi}{6} = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$= 0$$

Q.6: In which quadrant θ is lie?

(i) $\sin\theta > 0$, $\tan\theta > 0$ **(ii)** $\sin\theta < 0$, $\cot\theta > 0$

(iii) $\sin\theta > 0$, $\cos\theta < 0$ **(iv)** $\cos\theta > 0$, $\cos ec\theta < 0$

(v) $\tan\theta < 0$, $\sec\theta > 0$ **(vi)** $\cos\theta < 0$, $\tan\theta < 0$

Solution: (i) $\sin\theta > 0$, $\tan\theta > 0$

Then θ lie in first quadrant.

(ii) $\sin\theta < 0$, $\cot\theta > 0$

Then θ lie in 3rd quadrant.

(iii) $\sin\theta > 0$, $\cos\theta < 0$

Then θ lie in 2nd quadrant.

(iv) $\cos\theta > 0$, $\cos ec\theta < 0$

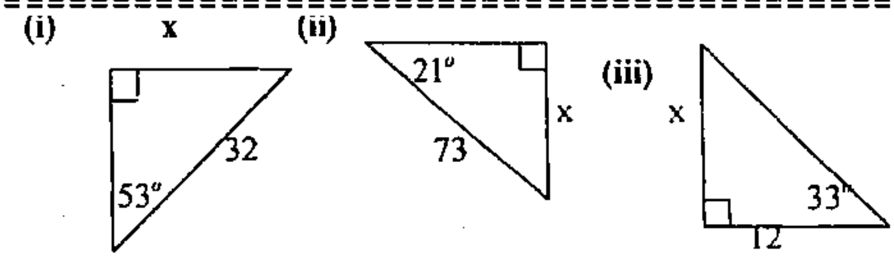
Then θ lie in 4th quadrant.

(v) $\tan\theta < 0$, $\sec\theta > 0$

Then θ lie in 4th quadrant.

(vi) $\cos\theta < 0$, $\tan\theta < 0$ .Then θ lie in 2nd quadrant.

Q.7: For each triangle , find each missing measure to two decimal places .



Solution: **(i) From figure** , $\theta = 53^o$

perpendicular = x , hypotenuse = 32

$As \quad \sin\theta = \frac{perpendicular}{hypotenouse}$ **put values .**

$\sin 53^o = \frac{x}{32} \quad => \quad 0.7986 = \frac{x}{32}$ **use calculator**

$=> \quad 0.7986 \times 32 = x \quad => \quad x = 25.56$

(ii) From figure , $\theta = 21^o$

perpendicular = x , hypotenuse = 73

$As \quad \sin\theta = \frac{perpendicular}{hypotenouse}$ **put values .**

$\sin 21^o = \frac{x}{73} \quad => \quad 0.3583 = \frac{x}{73}$ **use calculator**

$=> \quad 0.3583 \times 73 = x \quad => \quad x = 26.16$

(iii) From figure , $\theta = 33^o$

perpendicular = x , base = 32

$As \quad \tan\theta = \frac{perpendicular}{base}$ **put values .**

$\tan 33^o = \frac{x}{12} \quad => \quad 0.6494 = \frac{x}{12}$ **use calculator**

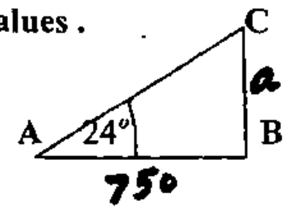
$=> \quad 0.6494 \times 12 = x \quad => \quad x = 7.79$

Q.8: The irregular blue shape in the diagram represents a lake. The distance across the lake "a" is unknown. To find this distance , surveyor took the measurements shown in the figure. What is the distance across the lake.

Solution: **From figure** , $\theta = 24^o$

perpendicular = a **, base = 724 yards**

$As \quad \tan\theta = \dfrac{perpendicular}{base}$ **put values .**



$\tan 24^{\circ} = \dfrac{a}{750}$ **use calculator**

$\Rightarrow \quad 0.4452 = \dfrac{a}{750}$

$\Rightarrow \quad 0.4452 \times 750 = a$

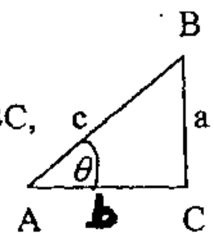
$\Rightarrow \quad a = 333.92 \quad yards$

Trigonometric identities: For any real number θ we have the following trigonometric identities .

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \cos ec^2\theta$



Solution: Consider a right angled triangle ABC, in which $m\angle ACB = 90^{o}$ and $m\angle BAC = \theta$

$m\overline{AB} = c\ , m\overline{BC} = a\ , m\overline{AC} = b$

Then by pathagourse theorem

$a^2 + b^2 = c^2$ ....(*i*) . Divided by c^2

$\Rightarrow \quad \dfrac{a^2}{c^2} + \dfrac{b^2}{c^2} = \dfrac{c^2}{c^2}$

$\Rightarrow \quad \left(\dfrac{a}{c}\right)^2 + \left(\dfrac{b}{c}\right)^2 = 1$ ......(*ii*)

put $\dfrac{a}{c} = \sin\theta \quad , \dfrac{b}{c} = \cos\theta$ in (ii)

$\Rightarrow \quad \sin^2\theta + \cos^2\theta = 1$ proved.

Now divided (i) by b^2

$$=> \frac{a^2}{b^2}+\frac{b^2}{b^2}=\frac{c^2}{b^2}$$

$$=> \left(\frac{a}{b}\right)^2+1=\left(\frac{c}{b}\right)^2 \text{ ....}(iii)$$

$As\ \frac{a}{b}=\tan\theta\ ,\ \frac{c}{b}=\sec\theta$ put in (iii)

$$=> \tan^2\theta+1=\sec^2\theta$$

$or\ \sec^2\theta=1+\tan^2\theta$ proved.

Now divided (i) by a^2

$$=> \frac{a^2}{a^2}+\frac{b^2}{a^2}=\frac{c^2}{a^2}$$

$$=> 1+\left(\frac{b}{a}\right)^2=\left(\frac{c}{a}\right)^2 \text{ .....}(iv)$$

$As\ \frac{b}{a}=\cot\theta\ ,\ \frac{c}{a}=\operatorname{cosec}\theta$ put in (iv)

$=> 1+\cot^2\theta=\operatorname{cosec}^2\theta$ proved.

Example 1: Show that $(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta$

Proof: Take L . H . S.

$$(\sin\theta+\cos\theta)^2=\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta \text{.....}(i)$$

But $\sin^2\theta+\cos^2\theta=1$ ......(ii) Put (ii) in (i)

$$(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta$$

Hence proved.

Exercise 7.5:

Prove the following trigonometric identities .

Q.1: $(\sec^2\theta-1)\cos^2\theta=\sin^2\theta$

Proof: Take L .H . S.

$$(\sec^2\theta-1)\cos^2\theta=\left(\frac{1}{\cos^2\theta}-1\right)\cos^2\theta \qquad (As\ \sec\theta=\frac{1}{\cos\theta})$$

$$=\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right)\cos^2\theta$$

$$= 1-\cos^2\theta \;.....(i)$$

But $\sin^2\theta+\cos^2\theta=1 \quad => \quad \sin^2\theta=1-\cos^2\theta....(ii)$

$=> \left(\sec^2\theta-1\right)\cos^2\theta=\sin^2\theta$ **proved.**

Q.2: $Tan\theta+\sec\theta=\dfrac{1+\sin\theta}{\cos\theta}$

Solution: As $\quad Tan\theta=\dfrac{\sin\theta}{\cos\theta}$ *and* $\quad \sec\theta=\dfrac{1}{\cos\theta}$

Take L . H .S.

$$Tan\theta+\sec\theta=\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}$$

$$=\frac{\sin\theta+1}{\cos\theta}$$

$$=\frac{1+\sin\theta}{\cos\theta}=R.H.S.$$

Hence proved.

Q.3: $(\cos\theta-\sin\theta)^2=1-2\sin\theta\cos\theta$

Proof: Take L . H . S.

$$(\cos\theta-\sin\theta)^2=\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta.....(i)$$

But $\sin^2\theta+\cos^2\theta=1 \;......(ii)$ Put (ii) in (i)

$=> (\cos\theta-\sin\theta)^2=1-2\sin\theta\cos\theta$

Hence proved.

Q.4: $\cos^2\theta-\sin^2\theta=2\cos^2\theta-1$

Solution: As $\cos^2\theta+\sin^2\theta=1$

$=> \sin^2\theta=1-\cos^2\theta \;.....(i)$

Take L .H.S.

$$\cos^2\theta - \sin^2\theta = \cos^2\theta - \left(1-\cos^2\theta\right) \qquad (put\ (i))$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$= 2\cos^2\theta - 1 \quad \textit{Hence proved}$$

Q.5: $\tan\theta + \cot\theta = \sec\theta\ \operatorname{cosec}\theta$

Solution: As $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ *and* $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Take L,. H.S.

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} \ .....(i)$$

As $\cos^2\theta + \sin^2\theta = 1$ put in (i)

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta}.\frac{1}{\cos\theta} \qquad \left(\frac{1}{\sin\theta} = \operatorname{cosec}\theta\ ;\ \frac{1}{\cos\theta} = \sec\theta\right)$$

$$= \operatorname{cosec}\theta.\sec\theta$$

$$= R.H.S.$$

Hence proved.

Q.6: $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1-\sin\theta}$

Solution: Take L . H . S.

$$\frac{1-\sin\theta}{\cos\theta} = \frac{(1-\sin\theta)}{\cos\theta} \times \frac{\cos\theta}{\cos\theta}$$

$$= \frac{(1-\sin\theta)\cos\theta}{\cos^2\theta} \ .....(i)$$

As $\sin^2\theta + \cos^2\theta = 1$

=> $\cos^2\theta = 1 - \sin^2\theta \ ....(ii)$

Put (ii) in (i)

$$\Rightarrow \frac{1-\sin\theta}{\cos\theta} = \frac{(1-\sin\theta)\cos\theta}{1-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)\cos\theta}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{\cos\theta}{1+\sin\theta}$$

Thus proved.

Q.7: $\sin\theta\sqrt{1+\tan^2\theta} = \tan\theta$

Solution: Take L . H.S.

$$\sin\theta\sqrt{1+\tan^2\theta} = \sin\theta\sqrt{\sec^2\theta}$$

$$= \sin\theta \,.\sec\theta$$

$$= \frac{\sin\theta}{\cos\theta} \qquad \left(As \quad \sec\theta = \frac{1}{\cos\theta}\right)$$

$$= \tan\theta$$ Hence proved .

Q.8: $\cos\theta = \sqrt{1-\sin^2\theta}$

Proof: $As \quad \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \quad \cos^2\theta = 1-\sin^2\theta$ Take square root of both sides

$$\Rightarrow \quad \sqrt{\cos^2\theta} = \sqrt{1-\sin^2\theta}$$

$$\Rightarrow \quad \cos\theta = \sqrt{1-\sin^2\theta}$$

Hence proved .

Q.9: $(1+\cos\theta)(1-\cos\theta) = \frac{1}{\operatorname{cosec}^2\theta}$

Proof: Take L .H .S.

$$(1+\cos\theta)(1-\cos\theta) = (1)^2 - \cos^2\theta$$

$$= 1-\cos^2\theta$$

$$= \sin^2\theta$$

$$= \frac{1}{\operatorname{cosec}^2\theta}$$

Thus it is proved.

Q.10: $\cos x - \cos x \sin^2 x = \cos^3 x$

Proof: **Take L . H . S.**

$\cos x - \cos x \sin^2 x = \cos x(1 - \sin^2 x)$ **put** $1 - \sin^2 x = \cos^2 x$

$= \cos x . \cos^2 x$

$= \cos^3 x = R.H.S.$

Q.11: $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x} = 2\operatorname{cosec} x$

Proof: **Take L . H . S.**

$$\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = \frac{\sin x . \sin x + (1+\cos x)(1+\cos x)}{(1+\cos x)\sin x}$$

$$= \frac{\sin^2 x + (1+\cos x)^2}{(1+\cos x)\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x + 1 + 2\cos x}{(1+\cos x)\sin x}$$

$$= \frac{1+1+2\cos x}{(1+\cos x)\sin x} \quad \textbf{put } \sin^2 x + \cos^2 x = 1$$

$$= \frac{2+2\cos x}{(1\div\cos x)\sin x}$$

$$= \frac{2(1+\cos x)}{(1-\cos x)\sin x}$$

$$= \frac{2}{\sin x}$$

$= 2\operatorname{cosec} x$ = R . H . S.

Q.12: $\dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}$

Proof : **Take L . H . S.**

$\dfrac{\sin x}{1+\cos x} = \dfrac{\sin x}{1+\cos x} \times \dfrac{\sin x}{\sin x}$ **multiplied and divided by sinx**

$$= \frac{\sin^2 x}{(1+\cos x)\sin x} \qquad \text{put } \sin^2 x = 1-\cos^2 x$$

$$= \frac{1-\cos^2 x}{(1+\cos x)\sin x}$$

$$= \frac{(1-\cos x)(1+\cos x)}{(1+\cos x)\sin x}$$

$$= \frac{1-\cos x}{\sin x} = R.H.S.$$

Q.13: $\frac{1}{1+\cos a} + \frac{1}{1-\cos a} = 2 + 2\cot^2 a$

Proof: **Take L . H . S.**

$$\frac{1}{1+\cos a} + \frac{1}{1-\cos a} = \frac{1-\cos a + 1 + \cos a}{(1+\cos a)(1-\cos a)}$$

$$= \frac{2}{1-\cos^2 a} \qquad \text{put } 1-\cos^2 a = \sin^2 a$$

$$= \frac{2}{\sin^2 a} \qquad \text{Put } \frac{1}{\sin^2 a} = \text{cosec}^2 a$$

$$= 2\,\text{cosec}^2 a \qquad \text{put } \text{cosec}^2 a = 1 + \cot^2 a$$

$$= 2(1+\cot^2 a)$$

$$= 2 + 2\cot^2 a = R.H.S$$

Q.14: $\cos^4 b - \sin^4 b = 1 - 2\sin^2 b$

Proof: **Take L . H . S.**

$$\cos^4 b - \sin^4 b = (\cos^2 b)^2 - (\sin^2 b)^2$$

$$= (\cos^2 b - \sin^2 b)(\cos^2 b + \sin^2 b)$$

$$= (\cos^2 b - \sin^2 b)(1) \qquad \text{put } \cos^2 b + \sin^2 b = 1$$

$$= \cos^2 b - \sin^2 b$$

$$= 1 - \sin^2 b - \sin^2 b \qquad \text{put } \cos^2 b = 1 - \sin^2 b$$

$$= 1 - 2\sin^2 b = \text{R . H . S.}$$

Q.15: $\frac{\sin y + \cos y}{\sin y} + \frac{\sin y - \cos y}{\cos y} = \sec y . \text{cosec}\, y$

Solution: Take L .H .S.

$$\frac{\sin y+\cos y}{\sin y}+\frac{\sin y-\cos y}{\cos y}=\frac{1}{\sin y\cos y}\begin{pmatrix}\cos y(\sin y+\cos y)\\+\sin y(\sin y-\cos y)\end{pmatrix}$$

$$=\frac{1}{\sin y\cos y}\begin{pmatrix}\cos y\sin y+\cos^2 y\\+\sin^2 y-\sin y\cos y\end{pmatrix}$$

$$=\frac{\cos^2 y+\sin^2 y}{\cos y\sin y}\quad \text{put } \sin^2 y+\cos^2 y=1$$

$$=\frac{1}{\cos y\sin y}$$

$$=\frac{1}{\cos y}.\frac{1}{\sin y}$$

$$=\sec y.\operatorname{cosec} y=R.H.S.$$

Q.16: $(\sec x-\tan x)^2=\dfrac{1-\sin x}{1+\sin x}$

Solution: **Take L . H . S.**

$$(\sec x-\tan x)^2=\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)^2 \quad \textbf{put } \sec x=\frac{1}{\cos x},\tan x=\frac{\sin x}{\cos x}$$

$$=\left(\frac{1-\sin x}{\cos x}\right)^2$$

$$=\frac{(1-\sin x)^2}{\cos^2 x}\qquad \textbf{put } \cos^2 x=1-\sin^2 x$$

$$=\frac{(1-\sin x)^2}{1-\sin^2 x}$$

$$=\frac{(1-\sin x)(1-\sin x)}{(1-\sin x)(1+\sin x)}$$

$$=\frac{1-\sin x}{1+\sin x}\quad = \textbf{R . H .S.}$$

Q.17: $\sin x.\tan x+\cos x=\sec x$.

Solution: **Take L . H . S.**

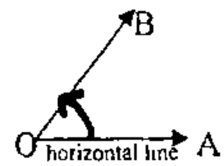
$$\sin x.\tan x+\cos x=\sin x.\frac{\sin x}{\cos x}+\cos x$$

$$=\frac{\sin^2 x+\cos^2 x}{\cos x} \quad \text{put } \sin^2 x+\cos^2 x=1$$

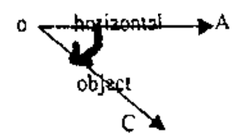
$$=\frac{1}{\cos x}$$

$$=\sec x = R.H.S.$$

Angle of Elevation: If an object is above the level of object observer , then the angle between the horizontal line and observer's line of sight is called angle of Elevation .
From the figure $m<AOB$ is called angle of Elevation.

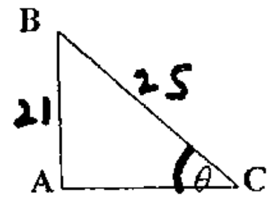


Angle of depression: If an object is below the level of Observer , then the angle between the observer and horizontal line is called angle of depression .
m< AOC is angle of depression.



Exercise 7.6:

Q.1: A building that is 21 meters tall casts a shadow 25 meters long . Find the angle of elevation of the sun to the nearest degree .



Solution:

Height of the building = AB = 21
Shadow of the building = AC = 25
Angle of elevation = < ACB = θ

As $\tan\theta=\frac{AB}{AC}$ **Put values**

=> $\tan\theta=\frac{21}{25}=0.84$

=> $\theta=\tan^{-1}(0.84)$ => $\theta=40''$ **Ans.**

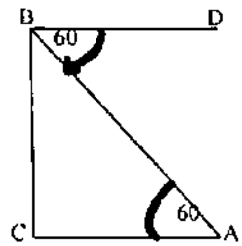
Q.2: A light house is 150 m above the sea level . Angle of

depression of a boat from its top is 60^o. Find the distance between the boat and the light house.

Solution: from the figure

Height of the light house = BC = 150 m

Distance between light house and boat = AC



As angle of depresion is $< ABD = 60^o$

Then $m < BAC = 60^o$ **(alternate angles)**

Then from triangle ABC ,

$$Tan60^o = \frac{BC}{AC} .....(i)$$ put value of BC .

$$=> \sqrt{3} = \frac{150}{AC}$$

$$=> AC = \frac{150}{\sqrt{3}} = \frac{50\times 3}{\sqrt{3}}$$

$$=> AC = \frac{50\times\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$$

$$=> AC = 50\sqrt{3}$$

Thus horizontal distance between boat and light house

$= 50\sqrt{3}\,m$

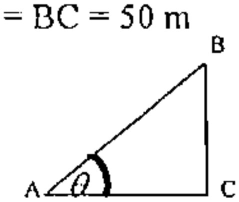
Q.3: A tree is 50 m high .Find the angle of elevation of its top to a point. On the ground 100 m away from the foot of tree.

Solution: Solution: Let height of the tree = BC = 50 m

The Horizontal distance = AC = 100 m

Let θ be the angle of elevation of its top to a point A on the ground.

Then ABC is a right angle of triangle.



$$Then \quad Tan\theta = \frac{BC}{AC} ....(i)$$

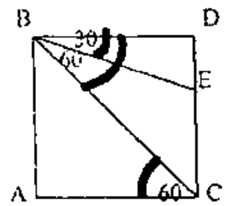
Put values in (i)

$$Tan\theta = \frac{50}{100} = 0.5$$

$\Rightarrow \ \theta = Tan^{-1}(0.5) = 26.56^o$

Q.4: From the top of hill 240 m high , measure of the angles of depression of the top and bottom of minaret are 30^o and 60^o respectively . Find the height of minaret.

Solution: From the figure



Let height of the hill = AB = 240 m

Height of the minaret = CE m

$As\ \ m < CBD = 60^o$ then

$m < ACB = 60^o$

Form triangle ABC .

$\tan 60^o = \dfrac{AB}{AC}$ .....(i) put values in (i)

$\Rightarrow \sqrt{3} = \dfrac{240}{AC} \quad \Rightarrow AC = \dfrac{240}{\sqrt{3}}$ ......(ii)

From right angle triangle BED .

$\tan 30^o = \dfrac{DE}{BD}$ .....(iii)

As DE = AC Put values in (iii) .

$\Rightarrow \ \dfrac{1}{\sqrt{3}} = \dfrac{DE}{AC}$ ......(iv) put value of AC .

$\Rightarrow \ \dfrac{1}{\sqrt{3}} = \dfrac{DE}{\frac{240}{\sqrt{3}}} \quad \Rightarrow \ \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}\ DE}{240}$

$\Rightarrow \ \dfrac{240}{\sqrt{3}.\sqrt{3}} = DE \quad \Rightarrow \ DE = \dfrac{240}{3}$ ....(v)

Sine $\quad AB = CE + ED$

$Then \quad CE = AB - ED$ ......(vi)

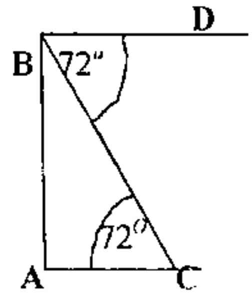
put values of AB and ED in(vi)

$\Rightarrow \ CE = 160$

Thus height of the minaret = 160 m .

Q.5: A police helicopter is flying at 800 feet . A stolen car

is sighted at an angle of depression of 72 ? . Find the distance of the stolen car , to the nearest foot , from a point directly below the helicopter .



Solution:

Highet = AB = 800 ft

Angle of depression = < DBC = 72°

Distance from the foot of Helicopter to the car = AC = ?

Then the angle of evelotion

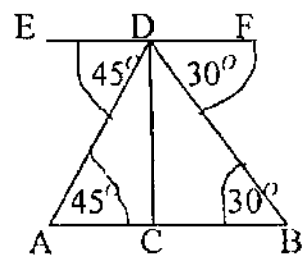
< ACB = 72^o (alternate angles) .

Then $\tan 72^o = \dfrac{AB}{AC}$ => $3.07768 = \dfrac{800}{AC}$

=> $AC = \dfrac{800}{3.07768} = 259.93\ ft$

AC = 260 ft (Approximately) .

Q.6: A light house is 300 m above the sea level .the angle of depression of two boats from the top of light house are 30^o and 45^o respectively . If the line of joining of the boats passes through the foot of light house , find the distance between two boats when they are on opposite side of the light house.



Solution: In the figure

Let height of light house = CD = 300 m

Let the two boats at points A and B, Where A and B are on opposite sides of light house. Since $m < EDA = 45^o$ is angle of depression with A , and $m < BDF = 30^o$ is angle of depression at B .

But (<CAD and < EDA) and (<BDF and <DBC) are alternate angles .

Thus $m < CAD = 45^o$ and $m < DBC = 30^o$.

From right angle triangle ACD ,

$Tan45^o = \frac{CD}{AD}$ ....(i) put $CD = 300$ in(i)

=> $1 = \frac{300}{AC}$ => $m\overline{AC} = 300m$

From right angle triangle BCD ,

$Tan30^o = \frac{CD}{BD}$ ......(ii)

=> $\frac{1}{\sqrt{3}} = \frac{300}{BD}$ => $m\overline{BD} = 300\sqrt{3}\ m$

Thus distance between the two boats is

$m\ \overline{AB} = m\overline{AD} + m\overline{BD}$ ......(iii) put values

=> $m\overline{AB} = 300 + 300\sqrt{3} = 300(1+\sqrt{3})\ m$

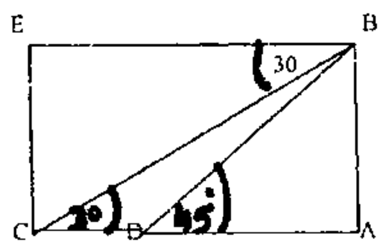
Thus distance between the two boats is $300(1+\sqrt{3})$

Q.7: The angle of elevation of the top of a cliff is 30^o. Walking 210 meter from the point towards the cliff , the angle of Elevation becomes 45^o . Find the height of cliff.

Solution: From figure

The height of the cliff = AB and B is the top of the cliff .

Let C and D are two points such that m CD = 210 m



At point C ,
the angle of elevation is

$m < ACB = 30^o$ .Then also

$m < CBE = 30^o$ (alternate angles)

Also at point D ,

The angle of elevation is $m < ADB = 45^o$.

As $m < ACB = m < CBE$, then $mAB = mCE$

Let AD = x .

Then $m\,BE = m\,AC = mCD + mAD$

=> $m\overline{BE} = 210 + x$ ......(i)

From triangle BEC ,

$$Tan30^{o} = \frac{m\overline{CE}}{m\overline{BE}} \text{ ......}(ii) \qquad \text{put (i) in (ii)}.$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{m\overline{CE}}{210+x} \text{ ....}(iii)$$

From right angle triangle BAD ,

$$Tan45^{o} = \frac{m\overline{AB}}{m\overline{AD}} \qquad put \ \ value$$

$$\Rightarrow 1 = \frac{m\overline{AB}}{x}$$

$$\Rightarrow m\overline{AB} = x$$

As $m\overline{AB} = m\overline{CE}$ $\quad then \quad m\overline{CE} = x \quad put \ \ in(iii)$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{210+x}$$

$$\Rightarrow 210 + x = \sqrt{3}\ x$$

$$\Rightarrow 210 = \sqrt{3}x - x$$

$$\Rightarrow x\left(\sqrt{3}-1\right) = 210$$

$$\Rightarrow x = \frac{210}{\sqrt{3}-1}$$

Hence height of the cliff = $\frac{210}{\sqrt{3}-1}$ m

Review Exercise 7:

Q.1: At the end of each question ,four circles are given . Fill in the correct circle only .

(i) If an object is above the level of observer than angle formed between the horizontal line and observer's line of sight is called ....

(a) angle of depression (b) angle of elevation

(c) Obtuse angle (d) non of these

(ii) $\cot\theta = ...$

(a) $\frac{\sin\theta}{\cos\theta}$ (b) $\frac{1}{\cos\theta}$ (c) $\frac{\cos\theta}{\sin\theta}$ (d) $\frac{1}{\sin\theta}$

(iii) $1+\tan^2\theta = ...$

(a) $\sin^2\theta$ (b) $\cos^2\theta$ (c) $\operatorname{cosec}^2\theta$ (d) $\sec^2\theta$

(iv) If $\tan\theta = 1$ then $\sin\theta = ...$ when θ lies in 3rd quadrant.

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $-\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$

(v) $\sin(-350^\circ)$ lies in ....

(a) 1st quadrant (b) 2nd quadrant
(c) 3rd quadrant (d) 4th quadrant

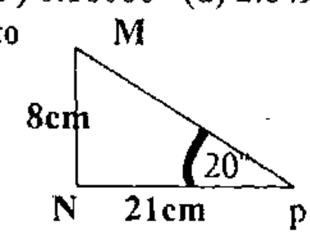
(vi) $45^\circ = ... \textit{radian}$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

(vii) If the measure of the hypotenuse of a right triangle is 5 feet and m<B = 58°, what is the measure of the leg adjacent to <B .

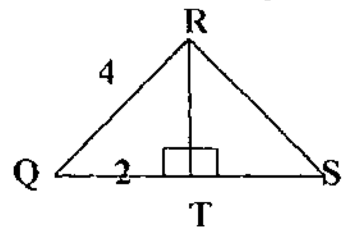
(a) 4.2402 (b) 8.0017 (c) 0.10060 (d) 2.6496

(viii) Find the value of tanP to the nearest tenth .



(a) 2.6 (b) 0.5
(c) 0.4 (d) 0.1

(ix) RT is equal TS. What is RS ? .



(a) $2\sqrt{6}$ (b) $2\sqrt{3}$
(c) $4\sqrt{3}$ (d) $2\sqrt{2}$

(x) RT is equal to TS . What is < S ? .

(a) 25° (b) 30° (c) 45° (d) 60°

<u>Answers</u>: (i) b (ii) c (iii) d (iv) c (v) a (vi) b
(vii) d (viii) c (ix) a (x) d

Q.2: Convert $45^o35'30''$ into decimal form.

Solution: As we know that

$$1' = \left(\frac{1}{60}\right)^o \quad and \quad 1'' = \left(\frac{1}{3600}\right)^o$$

$$Then \quad 45^o35'30'' = \left(45 + 35\times\frac{1}{60} + 30\times\frac{1}{3600}\right)^o$$

$$= (45 + 0.5833 + 0.0083)^o$$

$$= 45.5916^o$$

Q.3: Convert 216.67^o into $D^oM'S''$.

Solution: $As\ 1^o = 60'$ and $1' = 60''$ .Then

$$216.67^o = 216^o + (0.67)^o$$

$$= 216^o + (0.67\times 60)'$$

$$= 216^o + (40.2)'$$

$$= 216^o + 40' + (0.2)'$$

$$= 216^o + 40' + (0.2\times 60)''$$

$$= 216^o + 40' + 12''$$

$$= 216^o40'12''$$

Q.4: Through how many radians does a minute hand of a clock turn in (i) 45 minutes (ii) in one hour.

Solution: Since the minute hand turn one cycle in one hour.
But one cycle is equal to 2π radian. Thus

(i) $60\ minutes = 2\pi\ radian$

$$1\quad minute = \frac{2\pi}{60}$$

$$\Rightarrow 45\quad minute = \frac{2\pi}{60}\times 45 = \frac{3\pi}{2}\ radian$$

(ii) In one hour = 2π *radian.*

Q.5: Find coterminal angle of 190^o *and* -250^o

Solution: 190^o:

$$Since\ \ 190^o + 360^o = 550^o \quad and$$

$$190^o - 360^o = -170^o$$

Thus terminal angles of 190^o are 550^o and -170^o.

-250^o:

$$Since\ \ -250^o + 360^o = 110^o \quad and$$

$$-250^o - 360^o = -610^o$$

Thus terminal angles of -250^o are 110^o and -610^o.

Q.5: Find the trigonometric ratios of (*i*) 390^o (ii) -240^o

Solution: (i) Trigonometric ratios of 390^o :

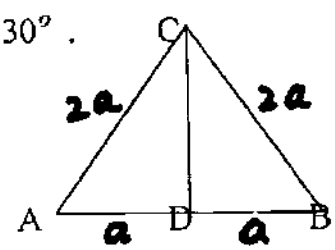
$$As\ \ 390^o = 360^o + 30^0$$

$$= 30^o$$

As 390^o *and* 30^o *have same terminal side* .

Let ABC be an equilateral triangle in which each side is equal to "$2a$" . Let draw a bisector CD of angle $< C$ and meet AB at a point D . Then ACD is a right angle triangle in which

$m < ADC = 90^o$ *and* $m < ACD = 30^o$.



Since AB = 2a then AD = a .

By pathagourse theorem

$$(AC)^2 = (AD)^2 + (DC)^2$$

Put values

$$(2a)^2 = a^2 + (DC)^2$$

$$=> \ 4a^2 - a^2 = (DC)^2$$

$=> (DC)^2 = 3a^2 \quad => \sqrt{(DC)^2} = \sqrt{3a^2}$

$=> DC = 3\sqrt{a}$

In triangle ACD for $m < ACD = 30^o$. AD is a base , BC is Perpendicular and AC is a hypoteneous .Then

$$\sin 390^o = \sin 30^o = \frac{AD}{AC} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 390^o = \cos 30^o = \frac{DC}{AC} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$Tan 390^o = Tan 30^o = \frac{AD}{DC} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$$

$$Co\sec 390^o = \cos ec 30^o = \frac{AC}{AD} = \frac{2a}{a} = 2$$

$$Sec 390^o = Sec 30^o = \frac{AC}{DC} = \frac{2a}{\sqrt{3}a} = \frac{2}{\sqrt{3}}$$

$$\cot 390^o = \cot 30^o = \frac{DC}{AD} = \frac{\sqrt{3}a}{a} = \sqrt{3}$$

Trigonometric ratios of -240^o

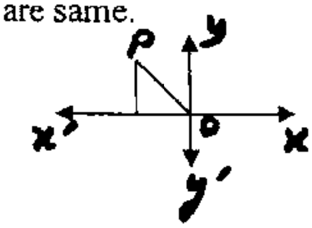
AS -240^o *is in clock wise dirction.* It is in 2^{nd} quadrant .

The terminal side of -240^o and 120^o are same.

Then 60^o is the associated angle .

Then $\overline{OP}$ is in the 2^{nd} quadrant .

Then the sin and cosec ratios in the 2^{nd} quadrant are positive. Thus



$$\sin(-240^o) = \sin 120^o = \sin 60^o = \frac{\sqrt{3}}{2}$$

$$\cos(-240^o) = \cos 120^o = -\cos 60^o = -\frac{1}{2}$$

$$Tan(-240^o) = Tan 120^o = -Tan 60^o = -\sqrt{3}$$

$$Co\sec(-240^o) = \cos ec120^o = \cos ec60^o = \frac{2}{\sqrt{3}}$$

$$Sec(-240^o) = Sec120^o = -Sec60^o = -2$$

$$\cot(-240^o) = \cot 120^o = -Cot60^o = -\frac{1}{\sqrt{3}}$$

Q.7: Prove thst (*i*) $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

(ii) $2\cos\theta\sec\theta - \tan\theta\cot\theta = 1$

Solution: **(i)** $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

Take L . H .S.

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} \times \sqrt{\frac{1-\sin\theta}{1-1\sin\theta}}$$

$$= \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

$$= \frac{\sqrt{(1+\sin\theta)^2}}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{1+\sin\theta}{\sqrt{\cos^2\theta}} = \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta + \tan\theta$$

Hence proved .

(ii) $2\cos\theta\sec\theta - \tan\theta\cot\theta = 1$ Take L .H .S .

$$2\cos\theta\sec\theta - \tan\theta\cot\theta = 2\cos\theta.\frac{1}{\cos\theta} - \tan\theta.\frac{1}{\tan\theta}$$

$$= 2 - 1$$

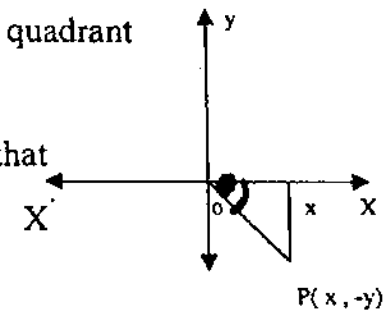
$$= 1$$

Hence proved.

Q.7: If $Sec\theta = 2$ where θ does not lies in first quadrant , Find remaining trigonometric ratios .

Solution:

As $Sec\theta = 2$ and θ is not in the first quadrant
Then θ lie in 4^{th} quadrant.
because sec θ is only positive
in first and 4^{th} quadrant As we know that



$$\sec\theta = \frac{r}{x} = \frac{2}{1} \text{ . Then}$$

=> $x = 1$, $r = 2$,

Where x is base and r is hypoteneous
And y is perpendicular .
Then by pataghourse theorem

$$x^2 + y^2 = r^2 \quad \text{put values}$$

$$=> (1)^2 + y^2 = (2)^2$$

$$=> 1 + y^2 = 4$$

$$=> y^2 = 4 - 1 = 3$$

$$=> y = -\sqrt{3} \qquad (since\ \theta\ is\ in\ 4th\ quadrant)$$

$$Now \quad Sin\theta = \frac{y}{r} = -\frac{\sqrt{3}}{2}$$

$$Cos\theta = \frac{x}{r} = \frac{1}{2}$$

$$\tan\theta = \frac{y}{x} = -\frac{\sqrt{3}}{1} = -\sqrt{3}$$

$$\operatorname{cosec}\theta = \frac{r}{y} = -\frac{2}{\sqrt{3}}$$

$$\cot\theta = \frac{x}{y} = -\frac{1}{\sqrt{3}}$$

Q.9: Refer to the diagram shown. What is the height of the building ?

Solution:

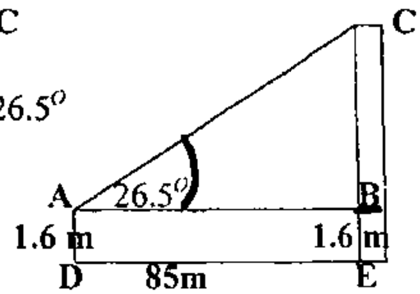
Let height of the building = EC

AB = 85 m

Angle of elevation = $< BAC = 26.5^O$

Also from the figure

AD = BE = 1.6 m .



From triangle ABD ,

$Tan 26.5^o = \frac{BC}{AB}$ **, Put values**

$=> 0.5 = \frac{BC}{85}$ $=> 0.5 \times 85 = BC$

$=> BC = 42.4$

Total height = BC = EB + BC = 1.6 + 42.4 = 44 m

Q.10: From a point on level ground 125 feet from the base of a tower , the angle of elevation is 57.2^o **. Approximate the height of the tower to the nearest foot .**

Solution: from figure

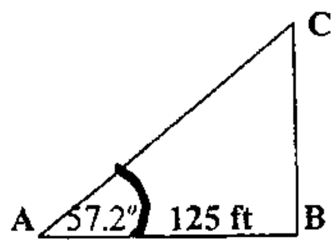
let height of the building = BC

Base = AB = 125

< BAC = 57.2^O

From triangle ABC ,

$Tan 57.2^o = \frac{BC}{AB}$ **, put values**



$=> 1.55 = \frac{BC}{125}$ $=> 1.55 \times 125 = BC$

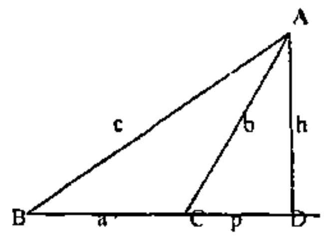
$=> BC = 193.75 \; ft$

Height of the building = 194 ft approximately .

Unit 8:

Projection of a side of a Triangle:

Theorem 8.1: In an obtuse – angled triangle, the square on the side opposite to the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle together with twice the rectangle contained by one of the sides, and the projection on it of the other.



Given: In triangle ABC,
$\angle C$ is the obtuse angle.
Let $m\overline{AB} = c, m\overline{BC} = a, m\overline{AC} = b$

Procedure: From point A draw a Perpendicular $\overline{AD}$ on $\overline{BC}$ such that $\overline{CD}$ is the projection of $\overline{AC}$ on $\overline{BC}$.
Let $m\overline{CD} = p$ and $m\overline{AD} = h$.

To prove: $(m\overline{AB})^2 = (m\overline{BC})^2 + (m\overline{CA})^2 + 2(m\overline{BC})(m\overline{CD})$

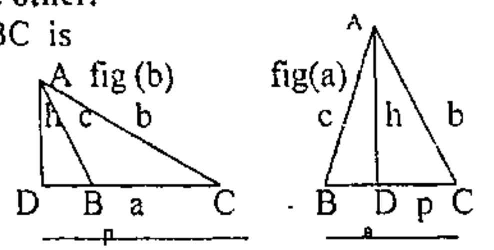
Proof:

Statements	Reasons
As ABD is a right angle triangle Then $(m\overline{AB})^2 = (m\overline{BD})^2 + (m\overline{AD})^2$ Or	Pythagoras theorem
$(m\overline{AB})^2 = (m\overline{BC} + m\overline{CD})^2 + (m\overline{AD})^2$	$m\overline{BD} = m\overline{BC} + m\overline{CD}$
or $(m\overline{AB})^2 = (m\overline{BC})^2 + (\overline{CD})^2 + 2(m\overline{BC})(m\overline{CD}) + (m\overline{AD})^2 \ldots(i)$	$(a+b)^2 = a^2 + b^2 + 2ab$
As ADC is a right angle triangle Then $(m\overline{AC})^2 = (m\overline{CD})^2 + (m\overline{AD})^2$	Pythagoras theorem
$or (m\overline{AC})^2 - (m\overline{CD})^2 = (m\overline{AD})^2 \ldots(ii)$ So that (i) becomes	From both sides equal values are subtracted.
$(m\overline{AB})^2 = (m\overline{BC})^2 + (\overline{CD})^2 + 2(m\overline{BC})(m\overline{CD}) + (m\overline{AC})^2 - (m\overline{CD})^2$	Put (ii) in (i)

$\left(m\overline{AB}\right)^2=\left(m\overline{BC}\right)^2+\left(m\overline{AC}\right)^2$ $+2\left(m\overline{BC}\right)\left(m\overline{CD}\right)$	The difference between two equal numbers are zero
Or $c^2=a^2+b^2+2ap$	
Thus proved.	

Theorem 8.2: In any triangle, the square on the side opposite to an acute angle is equal to the sum of the squares on the sides containing the acute angle diminished by twice the rectangle contained by one of those sides and projection on it of the other.

Given: The given triangle ABC is either acute as in figure (a) or obtuse as in figure (b).
In the ΔABC,
m<C is acute angle.



Let $m\overline{AB}=c$, $m\overline{BC}=a$, $m\overline{AC}=b$

From point A draw Perpendicular $\overline{AD}$ on $\overline{BC}$.
such that $\overline{CD}$ is the projection of $\overline{AC}$ on $\overline{BC}$.
Let $m\overline{CD}=p$ and $m\overline{AD}=h$.

To prove: $\left(m\overline{AB}\right)^2=\left(m\overline{BC}\right)^2+\left(m\overline{AC}\right)^2-2\left(m\overline{BC}\right)\left(m\overline{CD}\right)$
or $c^2=a^2+b^2-2ap$

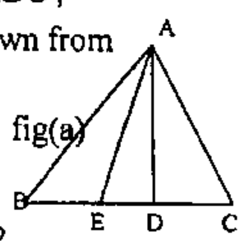
Proof:

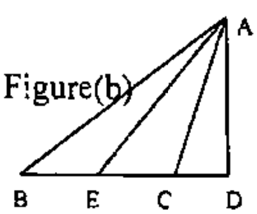
Statement	Reasons
As ABD is a right angle triangle	As $\overline{AD}\perp\overline{BC}$
Then $\left(m\overline{AB}\right)^2=\left(m\overline{BD}\right)^2+\left(m\overline{AD}\right)^2$ ..(i)	Pythagoras theorem
Also $m\overline{BD}+m\overline{CD}=m\overline{BC}$ Then	segment addition postulate(fig a)
$\left(m\overline{AB}\right)^2=\left(m\overline{BC}-m\overline{CD}\right)^2+\left(m\overline{AD}\right)^2$ ..(ii)	$m\overline{BD}=m\overline{BC}-m\overline{CD}$
Also $m\overline{BD}+m\overline{BC}=m\overline{CD}$ => $m\overline{BD}=m\overline{CD}-m\overline{BC}$ ...(iii)	segment addition (fig b)

$(m\overline{AB})^2 = (m\overline{CD} - m\overline{BC})^2 + (m\overline{AD})^2$ ..(iv)	
From (ii) and (iv) the R. H .S. are same	$(m\overline{BC} - m\overline{CD})^2 = (m\overline{CD} - m\overline{BC})^2$
Then we take one of the equation (ii). Then from (ii)	
$(m\overline{AB})^2 = (m\overline{BC})^2 + (\overline{CD})^2 - 2(m\overline{BC})(m\overline{CD}) + (m\overline{AD})^2$ ...(v)	$(a+b)^2 = a^2 + b^2 - 2ab$
or $(m\overline{AB})^2 = (m\overline{BC})^2 + (\overline{CD})^2 + (m\overline{AD})^2 - 2(m\overline{BC})(m\overline{CD})$ ..(vi)	
But $(m\overline{CD})^2 + (m\overline{AD})^2 = (m\overline{AC})^2$ ..(vii)	by Pythagoras theorem
$(m\overline{AB})^2 = (m\overline{BC})^2 + (m\overline{AC})^2 - 2(m\overline{BC})(m\overline{CD})$	put (vii) in (vi)
or $c^2 = a^2 + b^2 - 2ap$	Hence proved.

Theorem 8.3: In any triangle , the sum of the squares on any two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side (Apollonious theorem) :

Given: In the triangle ABC , $\overline{AE}$ is the median drawn from the vertex A to $\overline{BC}$.





To prove:

$$(m\overline{AB})^2 + (m\overline{AC})^2 = 2(m\overline{EB})^2 + 2(m\overline{EA})^2$$

Construction: From A , draw $\overline{AD} \perp \overline{BC}$

Poof:

Statement	Reason
Sinc ΔABD and ΔADC are right angle triangles.	construction
$(m\overline{AB})^2 = (m\overline{AD})^2 + (m\overline{BD})^2$ ....(i)	by Pythagoras theorem
$(m\overline{AC})^2 = (m\overline{AD})^2 + (m\overline{DC})^2$ ..(ii)	by Pythagoras theorem
Therefore	
$(m\overline{AB})^2 + (m\overline{AC})^2 = 2(m\overline{AD})^2 + (m\overline{BD})^2 + (m\overline{DC})^2$ ...(iii)	adding (i) and (ii)
But $(m\overline{BD})^2 = (m\overline{BE})^2 + (m\overline{ED})^2$ ..(iv)	segment addition postulate
$(m\overline{AB})^2 + (m\overline{AC})^2 = 2(m\overline{AD})^2 + (m\overline{BE} + m\overline{ED})^2 + (m\overline{DC})^2$ ...(v)	put (iv) in (iii)
Since $m\overline{DC} = m\overline{EC} - m\overline{ED}$ ...(vi)	from figure (a)
$m\overline{DC} = m\overline{ED} - m\overline{EC}$ ...(vii)	from figure (b)
$(m\overline{DC})^2 = (m\overline{EC} - m\overline{ED})^2$	$from\,(vi)\ and\,(vii)$
$= (m\overline{ED} - m\overline{EC})^2$ ...(viii)	$(a-b)^2 = (b-a)^2$
Then from (viii) put one value of $(m\overline{CD})^2$ from (viii) in (v)	
$(m\overline{AB})^2 + (m\overline{AC})^2 = 2(m\overline{AD})^2 + (m\overline{BE} + m\overline{ED})^2 + (m\overline{EC} - m\overline{ED})^2$	
or $(m\overline{AB})^2 + (m\overline{AC})^2 = 2(m\overline{AD})^2 + (m\overline{BE})^2 + (m\overline{ED})^2 + 2(m\overline{BE})(m\overline{ED}) + (m\overline{EC})^2 + (m\overline{ED})^2 - 2(m\overline{EC})(m\overline{ED})$	$(a+b)^2 = a^2 + b^2 + 2ab$ $(a-b)^2 = a^2 + b^2 - 2ab$

or $(m\overline{AB})^2 + (m\overline{AC})^2 = 2(m\overline{AD})^2$ $+ (m\overline{BE})^2 + (m\overline{ED})^2 + (m\overline{EC})^2 + (m\overline{ED})^2$ $+ 2(m\overline{BE})(m\overline{ED}) - 2(m\overline{EC})(m\overline{ED})$	AS $x - x = 0$
Or $(m\overline{AB})^2 + (m\overline{AC})^2 = 2(m\overline{BE})^2$ $2\left[(m\overline{AD})^2 + (m\overline{ED})^2\right]$ ....(ix)	As $m\overline{BE} = m\overline{EC}$
But $(m\overline{AD})^2 + (m\overline{ED})^2 = (m\overline{AE})^2$....(x)	As ADE is right triangle
$(m\overline{AB})^2 + (m\overline{AC})^2$ $= 2(m\overline{BE})^2 + 2(m\overline{AE})^2$ Hence proved .	$put(x)$ $in(ix)$

Rev iew Exercise 8 .

Q.1: At the end of each question , four circles are given . Fill in the correct circle only .

(i) Pythagoras was a ....mathematician .
(a) Indian (b) Greek
(c) Pakistani (d) Chinese

(ii) Apollonius was born in ....
(a) Perga (b) Istanbol
(c) Islamabad (d) Kabul .

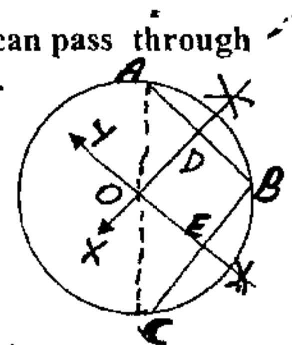
(iii) Apollonius is the name of
(a) City (b) Town
(c) Country (d) Mathematician

Answers: (i) b (ii) a (c) d

Unit. No: 9.

Chords of a circle:

Theorem 9.1: One and only one circle can pass through three non collinear points.



Given: A , B and C are three non- collinear points.

To prove: One and only one circle can pass through the points A , B and C .

Construction:

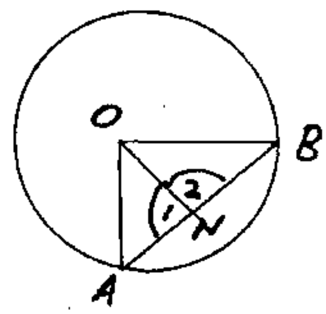
Join B to A and C . Draw perpendicular bisectors $\overrightarrow{XD}$ and $\overrightarrow{YE}$ of $\overline{AB}$ and $\overline{BC}$ respectively with intersect at O . Join O with A , B and C .

Proof:

Statement	Reasosn
In $\Delta OAD \leftrightarrow \Delta OBD$	
$< ODA \cong < ODB$	Both are right angles
$\overline{AD} \cong \overline{BD}$	D is the mid point of $\overline{AB}$
$\overline{OD} \cong \overline{OD}$	Common
Thus $\Delta OAD \cong \Delta OBD$	S .A .S. Postulate
Thus $\overline{OA} \cong \overline{OB}$ ....(i)	corresponding sides of congruent triangles .
Again in $\Delta OBE \leftrightarrow \Delta OCE$	
$< OEB \cong < OEC$	Both are right angles
$\overline{BE} \cong \overline{EC}$	E is the mid point of $\overline{BC}$.
$\overline{OE} \cong \overline{OE}$	Common
Thus $\Delta OBE \cong \Delta OCE$	S.A.S. postulate.
$\overline{OB} \cong \overline{OC}$ ....(ii)	corresponding sides of congruent triangles .
From (i) and (ii)	
$\overline{OA} \cong \overline{OB} = \overline{OC}$	
=> $m\overline{OA} \cong m\overline{OB} = m\overline{OC}$	
It means that the point O is equidistance from the three	

points A , B and C . Therefore a circle with center O and radius $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$ will pass through the points A , B and C. Since O is the only point which is equidistant from the points A , B and C. Therefore one and only one circle can Pass through these three non – collinear Points.	

Theorem 9.2: A straight line , drawn from the center of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord .



Given: A circle with enter O ,
$\overline{AB}$ is a chord of the circle .
N is the mid point of $\overline{AB}$
Which is joined to O.

To Prove: $\overline{ON} \perp \overline{AB}$

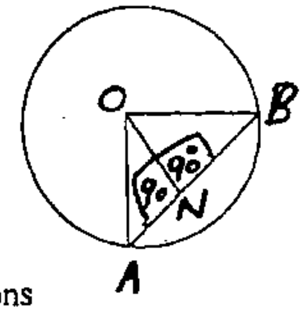
Construction: Join O to A and B.

Proof:

Statement	Reasons
In $\Delta OAN \leftrightarrow \Delta OBN$	
$\overline{OA} \cong \overline{OB}$	Radii of the same circle
$\overline{AN} \cong \overline{BN}$	given
$\overline{ON} \cong \overline{ON}$	common
Thus $\Delta OAN \cong \Delta OBN$	$S.S.S. \cong S.S.S.$
Thus $<1 \cong <2$	Corresponding angles of the congruent triangles.
But $m<1+m<2=180^o$ ..(i)	Supplementary angles postulate
Then $m<1=m<2=90^o$	$<1 \cong <2$
Thus $\overline{ON} \perp \overline{AB}$ proved.	

Theorem 9.3: Perpendicular from the center of a circle on a

chord bisects it .



Given: A circle with center at O.
$\overline{AB}$ is a chord. $\overline{ON} \perp \overline{AB}$.

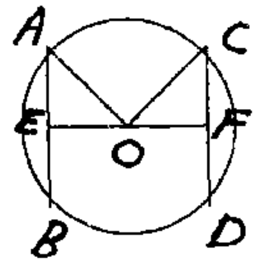
Prove: $\overline{AN} \cong \overline{BN}$

Construction: Join O to A and B.

Proof:

Statements	Reasons
In $\Delta OAN \leftrightarrow \Delta OBN$	
$< ONA \cong < ONB$	Right angles
$\overline{ON} \cong \overline{ON}$	common
$\overline{OA} \cong \overline{OB}$	Radii of the same circle
Thus $\Delta OAN \cong \Delta OBN$	$H.S. \cong H.S.$
Thus $\overline{AN} \cong \overline{BN}$	Corresponding sides of congruent triangles .

Theorem 9.4:If two chords of a circle are congruent then they will be equidistance from the center.



Given: A circle with center O ,
$\overline{AB}$ and $\overline{CD}$ are two congruent chords of the circle.

To prove: $\overline{AB}$ and $\overline{CD}$ are equidistant from the center O .

Construction: Join O to A and C .
Also draw perpendicular $\overline{OE}$ and $\overline{OF}$ on the given chords $\overline{AB}$ and $\overline{CD}$ respectively .

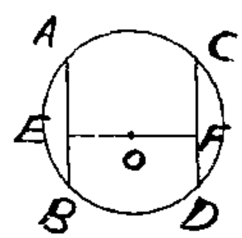
Proof:

Statement	Reasons
Since $\overline{OE} \perp \overline{AB}$ and $\overline{OF} \perp \overline{CD}$	construction
Thus $\overline{AE} \cong \overline{EB}$ and $\overline{CF} \cong \overline{DF}$	perpendicular from the center of a circle on a chord bisects it .

or $m\overline{AE} \cong m\overline{EB}$..(i) and $m\overline{CF} \cong m\overline{DF}$....(ii)	
But $m\overline{AB} \cong m\overline{CD}$	Given
$m\overline{AE} + m\overline{EB} \cong m\overline{CF} + m\overline{DF}$....(iii)	segment addition
$m\overline{AE} + m\overline{AE} \cong m\overline{CF} + m\overline{CF}$	$m\overline{EB} = m\overline{AE}$ and $m\overline{DF} = m\overline{CF}$
=> $2m\overline{AE} \cong 2m\overline{CF}$	
=> $m\overline{AE} \cong m\overline{CF}$	divided both sides by 2
or $\overline{AE} \cong \overline{CF}$ ......(iv)	
Now in $\Delta AOE \leftrightarrow \Delta COF$	
$\overline{OA} \cong \overline{OC}$	Radii of the same circle
$\overline{AE} \cong \overline{CF}$	proved .
$< AEO \cong < COF$	Right angles.
Thus $\Delta AOE \cong \Delta COF$	S.A.S. Postulate
Thus $\overline{OE} \cong \overline{OF}$	corresponding sides of
Or $m\overline{OE} \cong m\overline{OF}$	congruent triangles.
Hence proved.	

Theorem: 9.5:Two chords of a circle which equidistance from the center are congruent.

Given: A circle with center at O ,
$\overline{AB}$ and $\overline{CD}$ are two chords of the circle.
$\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{CD}$ *and* $\overline{OE} \cong \overline{OF}$



To prove: $\overline{AB} \cong \overline{CD}$

Construction: Join O with A and C.

Proof :

Statements	Reasons
$\overline{OE} \perp \overline{AB}$, $\overline{OF} \perp \overline{CD}$	Given

$m\overline{AE} \cong m\overline{EB}$ $m\overline{CF} \cong m\overline{FD}$	Perpendicular from the center of a circle on a chord bisect it.
Now in $\Delta AEO \leftrightarrow \Delta CFO$	
$< AEO \cong < CFO$	Both are right angles
$\overline{OA} \cong \overline{OC}$	Radii of the same circle
$\overline{OE} \cong \overline{OF}$	Given
Thus $\Delta AEO \cong \Delta CFO$	S.A.S. Postulate
Thus $\overline{AE} \cong \overline{CF}$ Or $m\overline{AE} \cong m\overline{CF}$	corresponding sides of congruent triangles
=> $2m\overline{AE} \cong 2m\overline{CF}$ ....(ii)	both sides multiplied by 2
Also $m\overline{AB} = m\overline{AE} + m\overline{EB}$	segment addition postulate
$m\overline{AB} = m\overline{AE} + m\overline{AE}$	$m\overline{AE} = m\overline{EB}$
$m\overline{AB} = 2m\overline{AE}$ ......(iii)	
Similarly $m\overline{CD} = 2m\overline{CF}$ .....(iv)	
AS from (ii) $2m\overline{AE} \cong 2m\overline{CF}$	
Then $m\overline{AB} \cong m\overline{CD}$	from (iii) and (iv)
or $\overline{AB} \cong \overline{CD}$	
Hence proved.	

Review Exercise 9:

Q.1: At the end of each question, four circles are given. Fill in the correct circle only.

(i) In a circle, two chords are equally distant from the center of the circle. The chords are

(a) congruent (b) not congruent
(c) parallel (d) not parallel

(ii) $\overline{AB}$ and $\overline{CD}$ are two chords of the same circle and AB < CD.

(a) AB is closer to O (b) $\overline{AB}$ must be parallel to $\overline{CD}$
(c) CD is closer to O (d) can't decide

(iii) A chord 5 cm long is contained in a circle of radius 13 . The length of the chord is

(a) 6 cm (b) 12 cm

(c) 24 cm (d) 30 cm

(iv) A chord 40 units long is contained in a circle of radius 25 . The distance of the chord from the center of the circle is

(a) 15 units (b) 31.2 units

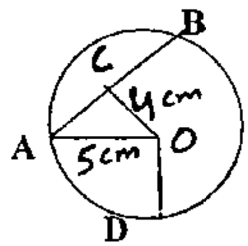
(c) 47.1 units (d) 50.1 units

(v) A chord $8\sqrt{3}$ units long is 4 from the center of a circle. The length of the circle is

(a) 14.4 units (b) 8 units

(c) $8\sqrt{3}$ units (d) $8\sqrt{2}$ units

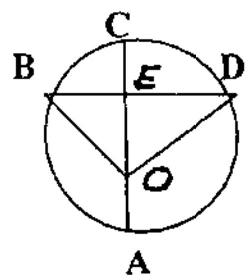
(vi) In adjacent circular figure with center o at radius 5 cm , the length of the chord intercepted at 4cm away from the center of this circle.



(a) 4cm (b) 6 cm

(c) 7 cm (d) 9 cm

(vii) In the diagram , circle O has a radius of 5 , and CE = 2 . Diameter $\overline{AC}$ is Perpendicular to chord $\overline{BD}$ at E .What is the length of $\overline{BD}$.



(a) 12 (b) 8

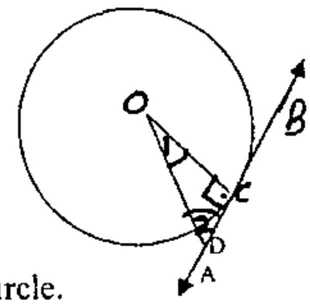
(c) 10 (d) 4

<u>Answers</u>: (i) a (ii) c (iii) c (iv) a (v) a (vi) b (vii) b

Unit 10:

Tangent to a circle:

Theorem 10.1: If a line is drawn perpendicular to a radial Segment of a circle at its outer end point, it is tangent to the circle at that point.



Given: A circle having center O with a line $\overleftrightarrow{AB}$ which is perpendicular to the radial segment $\overline{OC}$ at its outer end C.

i.e. $\overleftrightarrow{AB} \perp \overline{OC}$

To Prove: $\overleftrightarrow{AB}$ is tangent to the circle.

Construction: Take any point D on $\overleftrightarrow{AB}$ except other than C. Join O to D.

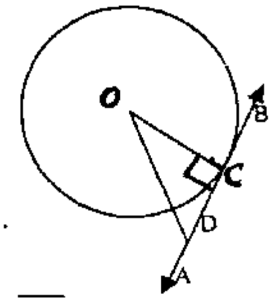
Proof:

Statements	Reasons
As $\overleftrightarrow{AB} \perp \overline{OC}$	Given
So the ΔOCD is a right triangle	As $m\angle OCD = 90^o$
As $\angle 1$ and $\angle 2$ are acute Thus $m\angle OCD > m\angle 2$	i.e. $\angle 1 + \angle 2 = 90^o$ As $\angle OCD$ is a right angle And $\angle 2$ is an acute angle.
Thus $m\overline{OD} > m\overline{OC}$ ......(i)	In a triangle, greater angle has greater side opposite to it.
But $\overline{OC}$ is the radial segment with C as its outer end. Thus D lies in the exterior of the circle.	Given
Hence $\overleftrightarrow{AB}$ meets the circle at one and only one point which is C.	
Thus $\overleftrightarrow{AB}$ is tangent to the circle.	Definition of tangent to a circle.

Theorem 10.2: The tangent to a circle and the radial segment Joining the point of contact and the center are Perpendicular to each other.

Given:

A circle with center O, $\overline{AB}$ is tangent to the circle at point C. $\overline{OC}$ is the radial segment which is obtained by joining O with the point of contact C of the tangent $\overline{AB}$.



To prove: $\overleftrightarrow{AB} \perp \overleftrightarrow{OC}$.

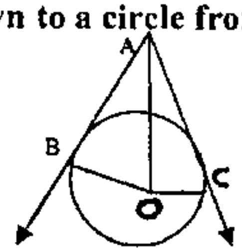
Construction: Take any point D on $\overline{AB}$ except C. Join O and D.

Proof:

Statements	Reasons
As $\overline{AB}$ is tangent to the circle at the point C. So C is the only point common to the circle and the line $\overline{AB}$.	Given
D is a point on $\overline{AB}$ other than C. Thus D is an exterior point of the circle.	Construction. Except C every point of $\overline{AB}$ is outside the circle.
It means $m\overline{OD} > m\overline{OC}$	In a triangle greater angle has greater side opposite to it.
Hence m $\overline{OC}$ is the shortest distance between the point O and line $\overline{AB}$. Hence $\overleftrightarrow{AB} \perp \overleftrightarrow{OC}$	By definition of shortest distance.

Theorem 10.3: The two tangents drawn to a circle from a point out side it, are equal in length.

Given: A circle with center O. A is any point outside the circle. $\overline{AB}$ and $\overline{AC}$ are two tangents drawn from point A.



To prove: $m\overline{AB} \cong m\overline{AC}$.

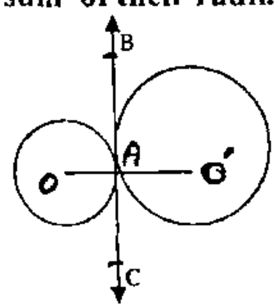
Construction: Join O to A , B and C .

Proof:

Statements	Reasons
In $\Delta AOB \leftrightarrow \Delta AOC$	
$\overline{AO} \cong \overline{AO}$	common
$\overline{OB} \cong \overline{OC}$	Radial segments .
$< ABO \cong < ACO$	Right angles because tangent to a circle is perpendicular to the radial segment at the point of contact .
Thus $\Delta AOB \cong \Delta AOC$	$H.S. \cong H.S.$
Thus $\overline{AB} \cong \overline{AC}$ Or $m\overline{AB} \cong m\overline{AC}$	corresponding sides of two congruent triangles .

Theorem 10.4(a): If two circles touch externally , the distance between their centers is equal to the sum of their radii.

Given: Two circles with centers O and O' which are touching each other externally at point A. $\overline{OA}$ *and* $\overline{O'A}$ are the radial Segments of the circles .



To prove: $m\overline{OO'} = m\overline{OA} + m\overline{AO'}$

Construction: Draw a common tangent $\overleftrightarrow{BC}$ at the point A which is the common point of contact of the given two circles.

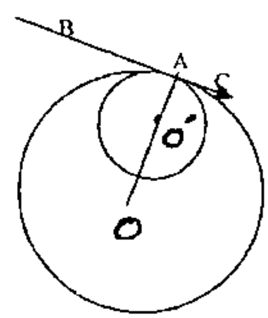
Proof:

Statements	Reasons
$m < BAO = 90^o$ ...(i)	A tangent to a circle is Perpendicular to the radial Segment at the point of contact.
Similarly $< BAO' = 90^o$ ..(ii)	A tangent to a circle is Perpendicular to the radial Segment at the point of contact

Statements	Reasons
Thus $m\angle BAO + m\angle BAO' = 90^o + 90^o = 180^0$	Adding (i) and (ii)
$m\angle BAO$ *and* $m\angle BAO'$ are Supplementary angles with the common vertex A .	Their sum is 180^o and supplementary angles postulate.
$\overline{OA}$ *and* $\overline{O'A}$ are opposite rays.	
This makes O , A and O' , the three different collinear points.	$\overline{OA}$ *and* $\overline{O'A}$ are opposite rays
Thus $m\overline{OO'} = m\overline{OA} + m\overline{AO'}$.	Segment addition

Theorem 10.4(b): If two circles touch internally, the distance between their centers is the difference of their radii.

Given: Two circles with centers O and O' which are touching each other internally at point A. $\overline{OA}$ *and* $\overline{O'A}$ are the radial Segments of the circles .



To prove: $m\overline{OO'} = m\overline{OA} - m\overline{AO'}$

Construction :

Draw a common tangent $\overline{BC}$ at the point A which is the common point of contact of the two circles .

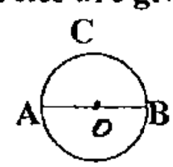
Proof:

Statements	Reasons
Since the circles touch each other at A therefore both $\overline{OA}$ *and* $\overline{O'A}$ are perpendicular to $\overline{BC}$.	A tangent to a circle is perpendicular to the radius drawn through the point of contact .
Thus $m\angle BAO = m\angle BAO' = 90^o$ i.e. $\overline{OA} \perp \overline{BC}$ at A .	

and $\overline{O'A} \perp \overline{BC}$ at A .	
Therefore O, O' and A lie on the same straight line .	At a point on a line a unique perpendicular can be drawn on it .
Thus $m\overline{OA} = m\overline{OO'} + m\overline{O'A}$	segment addition postulate
$m\overline{OO'} = m\overline{OA} - m\overline{O'A}$	From a law of equation .

Review Exercise10:

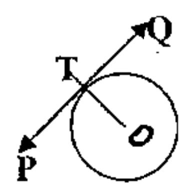
Q.1: At the end of each question , four circles are given . Fill in the correct circle only .



(i) In the figure , $\overset{\frown}{ABC}$, is called

(a) an arc (b) a secant

(c) a chord (d) a diameter



(ii) In a circle with center O , If $\overline{OT}$ is the radial segment $\overleftrightarrow{PTQ}$ is the tangent line , then

(a) $\overline{OT} \perp \overleftrightarrow{PQ}$ (b) $\overline{OT} \not\perp \overleftrightarrow{PQ}$

(c) $\overline{OT} \parallel \overleftrightarrow{PQ}$ (d) $\overline{OT}$ is a right bisector of $\overleftrightarrow{PQ}$

(iii) Two tangents drawn to a circle from a point outside it are of .... in length .

(a) half (b) equal

(c) double (d) triple

Answers: (i) a (ii) a (iii) b

Unit 11:

Chords and Arcs:

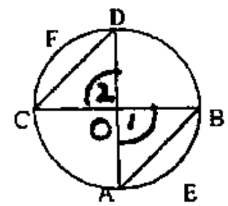
Theorem 11.1: If two arcs of a circle (or of congruent circles) are congruent then the corresponding chords are equal

Note: we will prove the theorem for two cases .

Case I: For one circle.

Case II: For two congruent circles.

Case I: For one circle:



Given: A circle with center O ,

$A\hat{E}B$ and $C\hat{F}D$ are congruent arcs.

i.e. $A\hat{E}B \cong C\hat{F}D$.

$\overline{AB}$ *and* $\overline{CD}$ are the corresponding chords of the given congruent chords.

To prove: $\overline{AB} \cong \overline{CD}$

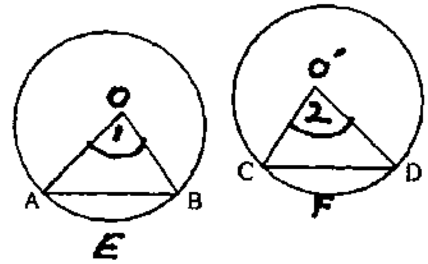
Construction: Join O to A , B , C and D respectively and name Of central angles < 1 and < 2.

Proof:

Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta OCD$	
$\overline{OA} \cong \overline{OC}$	Radii of the same circle
$\overline{OB} \cong \overline{OD}$	Radii of the same circle
$m<1 \cong m<2$	central angles of congruent arcs
Thus $\Delta OAB \cong \Delta OCD$	S .A.S . Postulate.
Thus $\overline{AB} \cong \overline{CD}$	Corresponding sides of Congruent triangles

Case II: For two congruent triangles:

Given: Two congruent circles with centers O and O' respectively . $A\hat{E}B$ and $C\hat{F}D$ are congruent arcs. $\overline{AB}$ *and* $\overline{CD}$ are the corresponding chords



To prove: $\overline{AB} \cong \overline{CD}$

Construction: Join O to A and B and O' to C and D.
Proof:

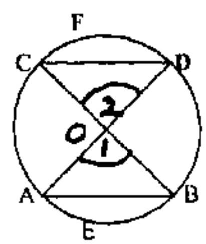
Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta O'CD$	
$\overline{OA} \cong \overline{O'C}$	Radii of two congruent circles
$\overline{OB} \cong \overline{O'D}$	Radii of two congruent circles
$m<1 \cong m<2$	central angles of congruent arcs
Thus $\Delta OAB \cong \Delta O'CD$	S.A.S. Postulate.
Thus $\overline{AB} \cong \overline{CD}$	Corresponding sides of Congruent triangles

Theorem 11.2: If two chords of a circle (or of congruent circles) are equal, then their correspondence (minor, major, semi- circle) congruent.

Note: we will prove the above theorem for two cases.

Case I: For one circle

Case II: For two congruent circles.



Case I: For one circle:

A circle with center O having two chords $\overline{AB}$ *and* $\overline{CD}$ such that $\overline{AB} \cong \overline{CD}$.

To prove: $A\hat{E}B \cong C\hat{F}D$ (These are minor arcs of the chords $\overline{AB}$ *and* $\overline{CD}$ respectively). And

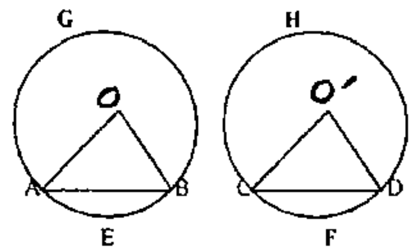
$A\hat{F}B \cong C\hat{E}D$ (These are minor arcs of the chords $\overline{AB}$ *and* $\overline{CD}$ respectively).

Construction: Join O to A, B, C and D respectively

Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta OCD$	
$\overline{OA} \cong \overline{OC}$	Radii of the same circle
$\overline{OB} \cong \overline{OD}$	Radii of the same circle
$\overline{AB} \cong \overline{CD}$	Given

Thus $\Delta OAB \cong \Delta OCD$	S.S.S. Postulate.
$m<1 \cong m<2$	Corresponding angles of two Congruent triangles
But <1 and <2 are central angles. Thus $A\hat{E}B \cong C\hat{F}D$. Thus the corresponding minor arcs of two chords $\overline{AB}$ and $\overline{CD}$ of a circle are congruent.	Definition of two congruent arcs

Case II: Now we prove the same result for two congruent Circles.



Given: Two congruent circles with centers O and O' with chords $\overline{AB}$ and $\overline{CD}$ respectively which Are not the diameter such that $\overline{AB} = \overline{CD}$.

To prove: $A\hat{E}B \cong C\hat{F}D$ (these arc minor arcs corresponding to the chords $\overline{AB}$ and $\overline{CD}$) and

$A\hat{G}B \cong C\hat{H}D$ (these arc major arcs corresponding to the chords $\overline{AB}$ and $\overline{CD}$).

Construction: Join O with A and B and O' with C and D. This gives central angles labeled <1 and <2.

Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta O'CD$	
$\overline{OA} \cong \overline{O'C}$	Radii of two congruent circles
$\overline{OB} \cong \overline{O'D}$	Radii of two congruent circles

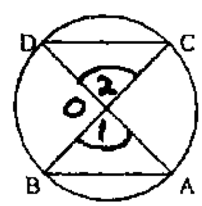
$\overline{AB} \cong \overline{CD}$	Given
Thus $\Delta OAB \cong \Delta O'CD$	S.A.S. Postulate.
Thus $m<1 \cong m<2$	Corresponding angles of Congruent triangles
$A\hat{E}B \cong C\hat{F}D$	condition of equality of two arcs.
Thus the corresponding minor arcs of two equal chords $\overline{AB}$ *and* $\overline{CD}$ of two congruent circles are congruent.	

Theorem 11.3: Equal chords of a circle (or of congruent circles) subtend equal angles at the center (at the corresponding centers).

We will prove the above theorem for two cases.

Case I: For one circle.

Given: A circle with center O.
$\overline{AB}$ *and* $\overline{CD}$ are two chords of the circle (which are not diameters) such that $\overline{AB} \cong \overline{CD}$. Arcs subtend <1 and <2 at the center.



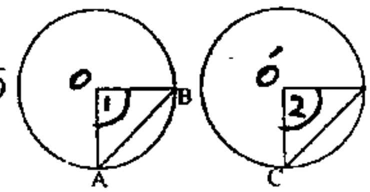
To prove: $<1 \cong <2$

Construction: Join O to A, B, C and D respectively so that
$m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = radii\ of\ the\ circle.$

Proof: Statements	**Reasons**
In $\Delta OAB \leftrightarrow \Delta OCD$	
$\overline{OA} \cong \overline{OC}$	Radii of the same circle
$\overline{OB} \cong \overline{OD}$	Radii of the same circle
$\overline{AB} \cong \overline{CD}$	Given
Thus $\Delta OAB \cong \Delta OCD$	S.S.S. Postulate.
$m<1 \cong m<2$	Corresponding angles of two Congruent triangles

Case (b) : For two circles:

Given: Two congruent circles with centers O and O' having two equal chords $\overline{AB}$ and $\overline{CD}$ i.e. $\overline{AB} \cong \overline{CD}$



To prove: These chords subtend equal Angles at the center. i.e. $m<1 \cong m<2$

Construction: Join O with A and B , and O' with C and D .

Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta O'CD$	
$\overline{OA} \cong \overline{O'C}$	Radii of two congruent circles
$\overline{OB} \cong \overline{O'D}$	Radii of two congruent circles
$\overline{AB} \cong \overline{CD}$	Given
Thus $\Delta OAB \cong \Delta O'CD$	S .A.S . Postulate.
Thus $m<1 \cong m<2$	Corresponding angles of Congruent triangles

Theorem 11.4: If the angles subtended by two chords of a circle (or congruent circles) at the center (corresponding centers) are equal in measures , then the chords are equal in measures .

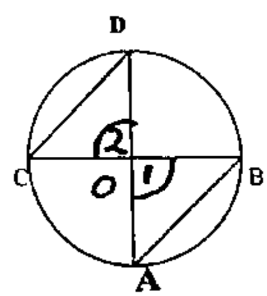
Note: We will this theorem for the following two cases.

Case I: For one circle .

Case II: For two congruent circles.

Case I: For one circle:

Given: A circle with center O . $\overline{AB}$ and $\overline{CD}$ are two chords of the circle and $<1 \cong <2$.



To prove: $\overline{AB} \cong \overline{CD}$.

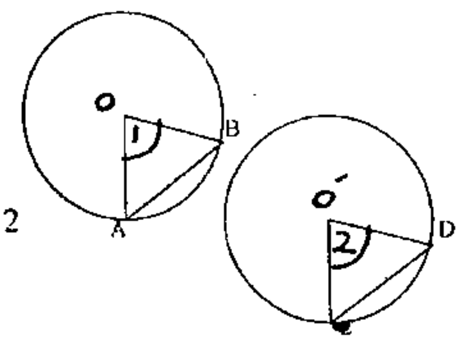
Proof :

Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta OCD$	
$\overline{OA} \cong \overline{OC}$	Radii of the same circle
$\overline{OB} \cong \overline{OD}$	Radii of the same circle
$m\angle 1 \cong m\angle 2$	Given
Thus $\Delta OAB \cong \Delta OCD$	S.S.S. Postulate.
$\overline{AB} \cong \overline{CD}$	Corresponding sides of two Congruent triangles

Case II: For two congruent circles:

Given: Two congruent circles

with centers O and O', $\overline{AB}$ *and* $\overline{CD}$ are two chords of these circles such that they Subtended equal angles at the center i.e. $m\angle 1 \cong m\angle 2$



To prove: $\overline{AB} \cong \overline{CD}$

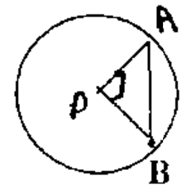
Proof:

Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta O'CD$	
$\overline{OA} \cong \overline{O'C}$	Radii of two congruent circles
$\overline{OB} \cong \overline{O'D}$	Radii of two congruent circles
$m\angle 1 \cong m\angle 2$	Given
Thus $\Delta OAB \cong \Delta O'CD$	S.A.S. Postulate.
Thus $\overline{AB} \cong \overline{CD}$	Corresponding sides of Congruent triangles

Review Exercise 11:

Q.1: At the end of each question , four circles are given . Fill in the correct circle only .

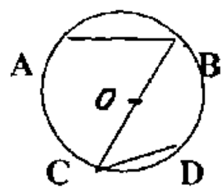
(i) AP has a radius of 3 and $< APB$ has measure of $90°$. What is the length of $\overline{AB}$?



(a) $3\sqrt{2}$ (b) $3\sqrt{3}$

(c) 6 (d) 9

(ii) In the accompanying diagram of circles O , $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ which statement is true .



(a) $\overline{AB} \cong \overline{CD}$ (b) $\overset{\frown}{AB} \parallel \overset{\frown}{CD}$

(c) $\overline{AC} \cong \overline{BD}$ (d) $< ABC \cong < BCD$

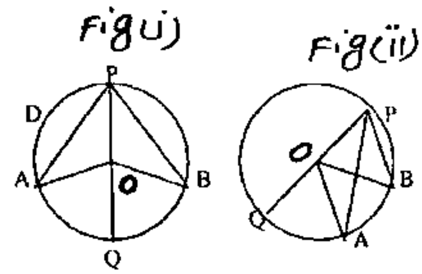
Answers: (i) a (ii) a

Unit 12:

Angle in a segment of circle:

Theorem 12.1: The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

Given: A circle with center O, $\widehat{AB}$ is a minor arc whose Central angles is < AOB. P is any Point on the major arc $\widehat{ADB}$. <APB is the angle subtended by the arc at P.



To prove: m < AOB = 2 m < APB.

Note: We will prove the above theorem for the following cases according to the position of the point on the major arc.

Case1: When P is an arbitrary point of the major arc (General case) figure 1.

Case II: When PQ is extremity of the semi – circle containing the minor arc $\widehat{AB}$ shown in figure (2).

Construction: Draw $\overline{PQ}$ to meet the circumference at Q (For both cases).

Proof:

Statements	Reasons
Case 1. In the ΔOAP	
$m\overline{OA} = m\overline{OP}$	Radii of the same circle
Thus ΔOAP is an isosceles triangle.	Definition of an isosceles Triangle
And $m < OAP = m < OPA$	opposite angles of equal sides
Also $m < AOQ = m < OAP + m < OPA$	The measure of exterior angle of a triangle is equal to the sum of the measure of opposite interior angles.
$\Rightarrow m < AOQ = 2m < OPA ....(i)$	$m < OAP = m < OPA$

Similarly $m < BOQ = 2m < OPB...(ii)$ $m < AOQ + m < BOQ$ $= 2m < OPA + 2m < OPB$	adding (i) and (ii) .
$m < AOB = 2(m < OPA + m < OPB)$ $=> m < AOB = 2m < APB$ Hence proved .	$< AOQ + < BOQ = < AOB$ Postulate of addition of angles.

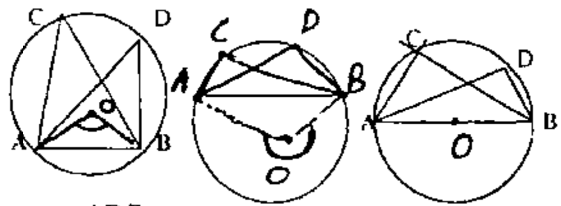
Case. II:

In the ΔOAP $m\overline{OA} = m\overline{OP}$	Radii of the same circle
Thus ΔOAP is an isosceles triangle.	Definition of an isosceles Triangle
And $m < OAP = m < OPA$	opposite angles of equal sides
Also $m < AOQ = m < OAP + m < OPA$	The measure of exterior angle of a triangle is equal to the sum of the measure of opposite interior angles.
$=> m < AOQ = 2m < OPA....(i)$	$m < OAP = m < OPA$
Similarly $m < BOQ = 2m < OPB...(ii)$ Subtract (i) from (ii) $=> m < BOQ - m < AOQ$ $= 2m < OPB - 2m < OPA$	
$=> m < AOB = 2(m < OPB - < OPA)$ $=> m < AOB = 2m < APB$ Hence proved.	postulate of angles Subtraction .

Theorem 12.2: Any two angles in the same segment of a circle are equal.

Given: A circle with center O . <ACB and <ADB are any two

Angles in the same segment ACDB of the circle.



To prove: $m < ACB = m < ADB$

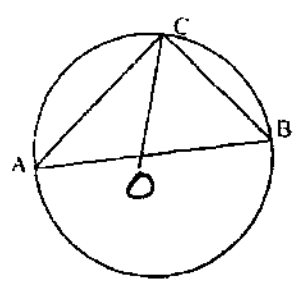
Construction: Join O to A and B respectively.

Proof:

Statements	Reasons
In all the three figures ,we have $m < AOB = 2m < ACB$ ...(i)	The angle which an arc of a circle Subtends at the center is twice that which is subtends at any point on the remaining part of the circumference.
$m < AOB = 2m < ADB$ ...(ii)	The angle which an arc of a circle Subtends at the center is twice that which is subtends at any point on the remaining part of the circumference.
Therefore $2m < ACB = 2m < ADB$	As L . H.S. of (i) and (ii) are Same then the R.H.S. is also same
=> $m < ACB = m < ADB$	half of equal quantities are equal .

Theorem 12.3(a):The angle in a semi – circle is a right angle.

Given: A circle with center O ,
$\overline{AB}$ is a diameter of the circle and <ACB is any angle in the semi – circle.



To prove: $m < ACB = 90^o$

Construction:

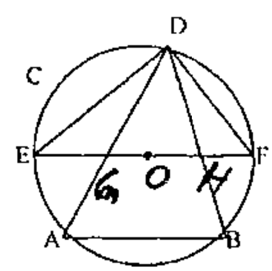
Join O to C .

Proof:

Statements	Reasons
In ΔOAC	
$m\overline{OA} = m\overline{OC}$	Radii of the same circle.
Thus ΔOAC is an isosceles triangle	Definition of an isosceles triangle
and $m<OAC = m<OCA$	opposite angles of equal sides
In ΔOCB	
$m\overline{OB} = m\overline{OC}$	Radii of the same circle.
$m<OBC = m<OCB$ ...(ii)	opposite angles of equal sides
$m<OAC + m<OBC$ $= m<OCA + m<OCB$	adding (i) and (ii).
$m<OAC + m<OBC$ $= m<ACB..(iii)$	$<OCA + <OCB = <ACB$
But	
$<OAC + <OBC + <ACB = 180^o$	Sum of the angles of a triangle are 180^o.
$m<ACB + m<ACB = 180^o$	<OAC + <OBC = <ACB
=> $2m<ACB = 180^o$	adding two equal numbers
=> $m<ACB = 90^o$	
Hence <ACB is a right angle.	

Theorem:12.3(b): The angle in a segment greater than a semi Circle is less than a right angle.

Given: A circle with center O ,
$\overline{AB}$ is a chord and $\overline{EF}$ is the diameter parallel $\overline{AB}$ such that $\overline{AB}$ lies below the diameter $\overline{EF}$.
Thus the segment AECDFB is greater than semi – circular region .
<ADB is any angle in the segment.



To prove: < ADB is less than a right angle. i.e. $m<ADB<90^o$
Construction: Join D with E and F such that <EDF is in the

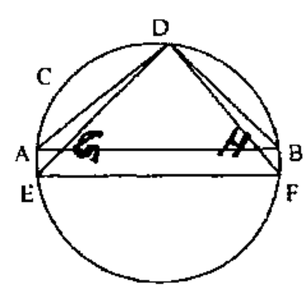
semi – circle .

Proof:

Statements	Reasons
$m < EDF = 90^o$	The measure of an angle in a Semi-circle is a right angle.
Therefore	
$m < EDG + m < GDH + m < HDF = 90^o$	$< EDG + < GDH + < HDF = < EDF$
Thus $m < GDH = 90^o - \begin{pmatrix} m < EDG \\ + m < HDF \end{pmatrix}$	
$\Rightarrow m < GDH < 90^o$	
Or $m < ADB < 90^o$	As $m < ADB = m < GDH$

Theorem:12.3(c): The angle in a segment less than a semi-circle is greater than a right angle.

Given: A circle with center O , $\overline{AB}$ is a chord and $\overline{EF}$ is the diameter parallel $\overline{AB}$ such that $\overline{AB}$ lies above the diameter $\overline{EF}$. Thus the segment ACDB is less than semi – circular region . <ADB is any angle in the segment.



To prove: < ADB is greater than a right angle.

i.e. $m < ADB > 90^o$

Construction: Join D with E and F such that <EDF is in the semi – circle .

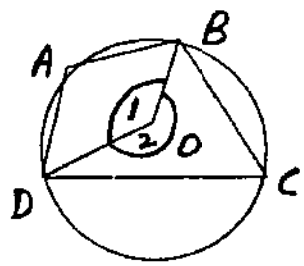
Proof:

Statements	Reasons
$m < EDF = 90^o$	The measure of an angle in a Semi-circle is a right angle.

Or $m<GDH=90^o$ ...(i) Or $m<ADG+m<GDH+m<HDB$ $=m<ADG+90^o+m<HDB$	As $m<EDF=m<GH$ adding both sides equal quantities
Or $m<ADB=90^o+m<ADG$ $+m<HDB$ or $m<ADB>90^o$	Postulate of addition angles

Theorem 12.4: The opposite angles of any quadrilateral inscribed in a circle are supplementary .

Given: A circle with center at O and ABCD is a quadrilateral inscribed in the given circle.



To prove: $m<BCD+m<BAD=180^o$

$m<ABC+m<ADC=180^o$

Construction: Join O to B and C.

Proof:

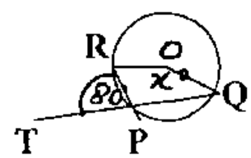
Statements	Reasons
$m<1=2m<BCD$...(i)	The angle which an arc of a circle Subtends at the center is twice that which is subtends at any point on the remaining part of the circumference.
$m<2=2m<BAD$....(ii)	The angle which an arc of a circle Subtends at the center is twice that which is subtends at any point on the remaining part of the circumference.
$m<1+m<2=$ $2m<BCD+2m<BAD$...(iii)	adding (i) and (ii)

But $m<1+m<2=360^o$	circumference of a circle subtends an angle equal to four right angles at the center .
Then (iii) becomes $2(m<BCD+m<BAD)=360^o$	
$m<BCD+m<BAD=\frac{360^o}{2}$	divided by 2 both sides
Or $m<BCD+m<BAD=180^o$ Hence m< BCD and < BAD are Supplementary angles . Similarly it can be proved that m< ABC and <ADC are supplementary angles	

Review Exercise 12:

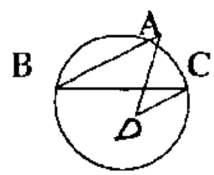
Q.1: At the end of each question , four circles are given . Fill in the correct circle only .

(i) In the figure, o is the center of circle , $<TPR=80^o$ and $<QPS=x^o$, find x^o



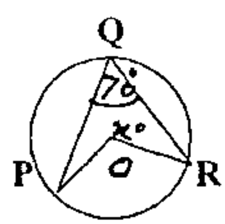
(a) 80^o (b) 160^o
(c) 100^o (d) 120^o

(ii) In the diagram , <ADC is a central angle and $m<ADC=60^o$ What is $m<ABC$?



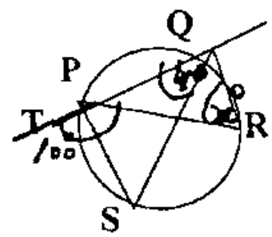
(a) 15^o (b) 60^o
(c) 120^o (d) 30^o

(iii) In the figure , o is a center of the circle and $<PQR=70^o$ calculate the value of x .

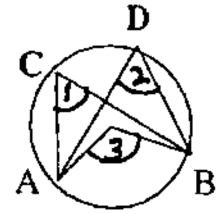


(a) 140^o (b) 220^o
(c) 290^o (d) 110^o

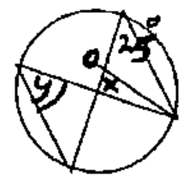
(iv) In the figure, $<SPT = 100^o$,
$<PRQ = x^o$.
Calculate the value of x^o.
(a) 30^o (b) 40^o
(c) 50^o (d) 60^o



(v) In the adjacent figure,
If $m<3 = 75^o$, then find
m < 1 and m < 2
(a) $37\frac{1}{2}^o$, $37\frac{1}{2}^o$ (b) $37\frac{1}{2}^o$, 75^o
(c) 75^o, $37\frac{1}{2}^o$ (d) 75^o, 75^o



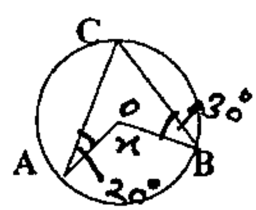
(vi) Given that O is the center of the circle. The angle marked x will be
(a) $12\frac{1}{2}^o$ (b) 25^o
(c) 50^o (d) 75^o



(vii) Given that O is the center of the circle. The angle marked y will be
(a) $12\frac{1}{2}^o$ (b) 25^o
(c) 50^o (d) 75^o



(viii) In the figure, o is the center of the circle then the angle x is:
(a) 50^o (b) 75^o
(c) 100^o (d) 125^o



Answers: (i) b (ii) b (iii) d (iv) b (v) a
(vi) c (viii) b (viii) d

Unit 13:

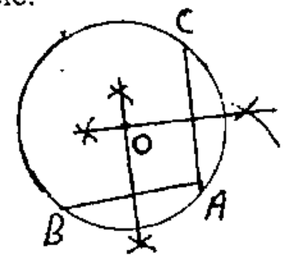
Practical geometry:

13.1: Locate the center of a given circle:

Given: A circle without any mention of its center.

Required : To locate the center of the circle.

Steps of construction:

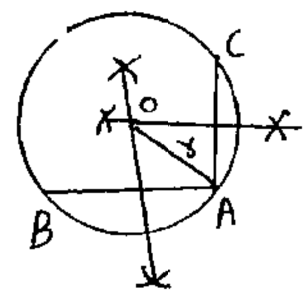


(i) Take any three points on given circle.
(ii) Join A with B and C .
(iii) Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$.
(iv) These bisectors meet at point O .
(v) Thus O is the required center of the given circle.

13.2: Draw a circle passing through three given non – collinear points .

Given: Any three points A , B and C in a plane which are non – collinear .

Required : To draw a circle passing through these points.



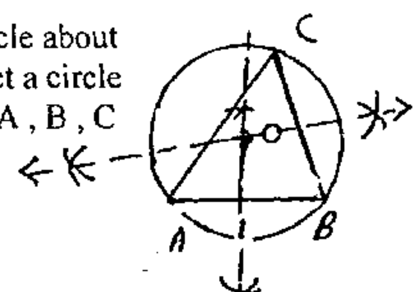
Steps of construction:

(i) Join A with B and C .
(ii) Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$.
(iii) These bisectors meet at point O .
(iv) Taking radius $= m\overline{OA} = m\overline{OB} = m\overline{OC}$,draw a circle.
(v) This is the required circle.

13.3: Circumscribed a circle about a given triangle.

Given: In triangle ABC .

Required : To circumscribe a circle about the given triangle .i.e. to construct a circle which passes through the vertex A , B , C of ABC .



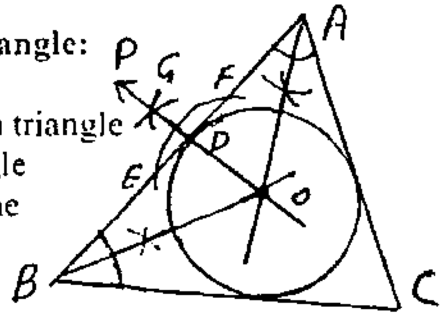
Steps of construction:

(i) Construct triangle ABC .
(ii) Draw perpendicular bisectors of $\overline{AB}$ *and* $\overline{BC}$ which intersect each other at point O.

(iii) Take O is center and draw a circle of radius equal to
$m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$.

(iv) This is the required circumscribe a circle about the given triangle

13.4: Inscribed a circle in a given triangle:



Given: ΔABC

Required: Inscribed a circle in a given triangle i.e. construct a circle inside the triangle which touches all the three sides of the triangle.

Steps of construction:

(i) Construct ΔABC .

(ii) Draw bisectors of < A and < B of given ΔABC , which intersect each other at point O.

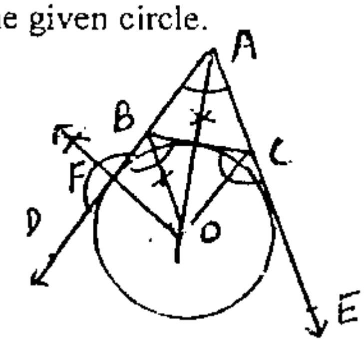
(iii) O is the center of required circle.

(iv) From point O draw $\overline{OD} \perp \overline{AB}$.

(v) Take O is a center and draw a circle of radius $m\overline{OD}$ which touches all the three sides of the given triangle.

(vi) This is the required inscribed circle in the given circle.

13.5:Escribed a circle to a given triangle.



Given : (i) ΔABC.

Required: To draw an escribed circle opposite to vertex A.

Steps of construction:

(i) Construct a ΔABC .

(ii) Produce $\overline{AB}$ and $\overline{AC}$ to form two exterior angles < CBD and <BCE.

(iii) Draw bisectors of < BAC , <CBA and < BCE which intersect each other at point O .

(iv) O is the center of the required circle.

(v) Draw $\overline{OF} \perp \overline{AB}$

(vi) Take O is the center and draw a circle of radius $m\overline{OF}$

which touches $\overline{BC}$, $\overline{BD}$ and $\overline{CE}$.

(vii) This is the required circle.

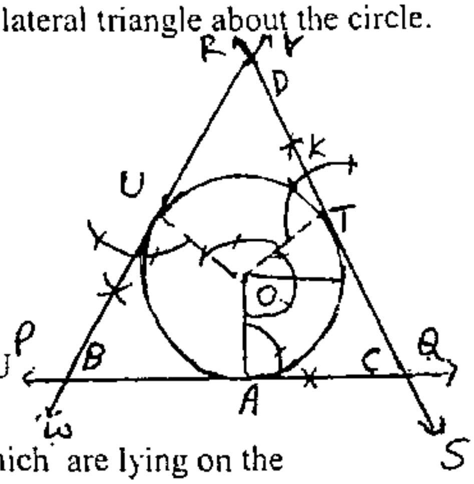
Similarly escribed circles opposite to the vertices B and C can be drawn.

13.6: Circumscribed an equilateral triangle about a given Circle.

Given: A circle with center O .

Required : Circumscribed an equilateral triangle about the circle.

Steps of construction:



(i) Take any point A on the circumference of the circle and join O to A .

$\overline{OA}$ is redial segment.

(ii) Construct an angle <AOT , of measure 120^o.

(iii) construct anther angle < TOU measuring 120^o.

(iv) The three points A , T , U which are lying on the circumference of the given circle are such that they divided the boundary of the circle into three equal arcs $\widehat{AT}$, $\widehat{TU}$ and $\widehat{UA}$.

(v) Draw perpendiculars to $\overline{OA}$, $\overline{OT}$ and $\overline{OU}$ at A , T and U respectively .These perpendiculars lines are $\overline{PQ}$, $\overline{RS}$ and $\overline{VW}$.

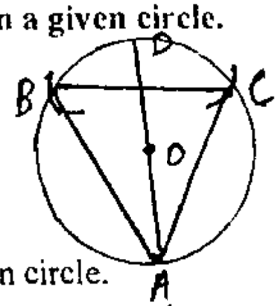
(vi) These lines intersects each other at B , C and D forming a triangle BCD.

(vii) Thus BCD is required triangle.

13.7: Inscribed an equilateral triangle in a given circle.

Given : A circle with center O .

Required: To inscribed an equilateral triangle in the circle.



Steps of construction:

(i) Draw any diameter $m\overline{AD}$ of the given circle.

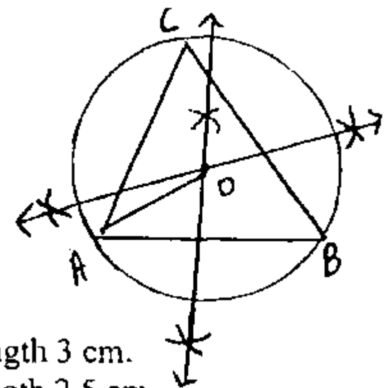
(ii) With center D and radius $\overline{OD}$, draw two arcs which cut the given circle at B and D.

(iii) Join A , B and C .

(iv) Thus ΔABC is required triangle.

Exercise 13.1:

Q.1: Construct a triangle with sides 2cm , 2.5cm and 3 cm . Also draw its circum circle.



Solution: Given: In ΔABC ,

$\overline{AB} = 2cm$, $\overline{BC} = 2.5cm$

And $\overline{AC} = 3cm$

Required: Construct a triangle and its circum circle.

Steps of construction:

(1) Draw $\overline{AB} = 2cm$.

(2) At point A draw an arc of length 3 cm.

(3) At point B draw an arc of length 2.5 cm. which cut each other at point C .

(4) Join C to A and B . Thus ABC is required triangle.

(5) Draw perpendicular bisectors of $\overline{AB}$ *and* $\overline{BC}$ which intersect each other at point O.

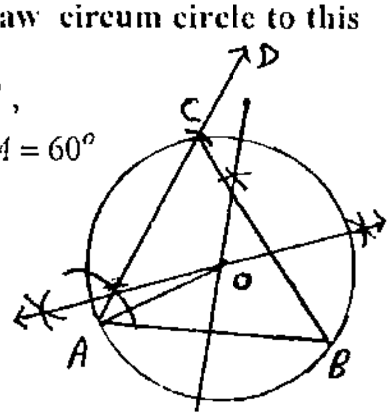
(6) Take O is center and draw a circle of radius equal to $m\overline{OA}$ *or* $m\overline{OB}$ *or* $m\overline{OC}$.

(7) This is the required circumscribe a circle about the given triangle.

Q.2: Construct a triangle ABC such that $m\overline{AB} = 3''$, $m\overline{AC} = 4''$ and $< A = 60^o$ .Draw circum circle to this triangle.

Solution: **Given**: In a triangle ABC ,

$m\overline{AB} = 3''$, $m\overline{AC} = 4''$ and $< A = 60^o$



Required: Construct a triangle and its circum circle.

Steps of construction:

(1) Draw $m\overline{AB} = 3''$.

(2) At point A , draw $m < BAD = 60^o$.

(3) At point A , draw an arc of length $4''$ which cut AD at point C .

(4) Join C to B . Thus ABC is required triangle.

(5) Draw perpendicular bisectors of $\overline{AB}$ *and* $\overline{BC}$ which intersect each other at point O.

(6) Take O is center and draw a circle of radius equal to $m\overline{OA}$ *or* $m\overline{OB}$ *or* $m\overline{OC}$.

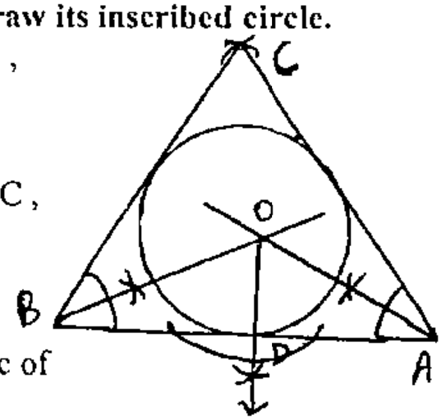
(7) This is the required circumscribe a circle about the given triangle.

Q.3: Suppose we have a triangle whose sides are 3cm , 4cm and 6 cm respectively . Draw its inscribed circle.

Solution: **Given:** In triangle ABC ,

$m\overline{AB} = 3cm$, $m\overline{BC} = 4cm$

and $m\overline{AC} = 6cm$



Required: Construct triangle ABC , and draw inscribed.

Steps of construction:

(1) Draw $m\overline{AB} = 3cm$.

(2) Take A is center draw an arc of length 6 cm.

(3) Take B is center draw an arc of 4 cm , which cut each other at point C.

(4) Join C with A and B .

(5) Thus ΔABC is formed .

(6) Draw bisectors of < A and < B of given ΔABC , which intersect each other at point O.

(7) O is the center of required circle.

(8) From point O draw $\overline{OD} \perp \overline{AB}$.

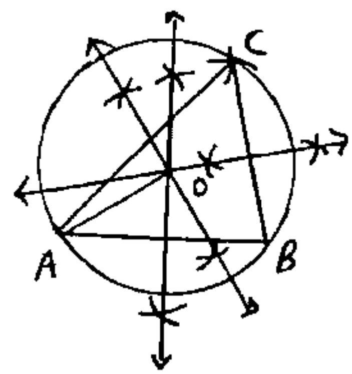
(9) Take O is a center and draw a circle of radius $m\overline{OD}$

which touches all the three sides of the given triangle.

(10) This is the required inscribed circle in the given circle.

Q.4: Construct a triangle ABC with sides $m\overline{AB} = 5cm$, $m\overline{BC} = 6cm$ and $m\overline{AC} = 8cm$. Draw perpendicular bisectors of its sides and then circumscribed a circle.

Solution: **Given**: In triangle ABC, $m\overline{AB} = 5cm$, $m\overline{BC} = 6cm$ and $m\overline{AC} = 8cm$.



Required: Construct triangle ABC. Draw perpendicular bisectors of its sides and then circumscribed a circle.

Steps of construction:

(1) Draw $m\overline{AB} = 5cm$.

(2) At point A draw an arc of length 8 cm.

(3) At point B draw an arc of length 5 cm, which cut each other at point C.

(4) Join C to A and B. Thus ABC is required triangle.

(5) Draw perpendicular bisectors of $\overline{AB}$, $\overline{AC}$ *and* $\overline{BC}$ which intersect each other at point O.

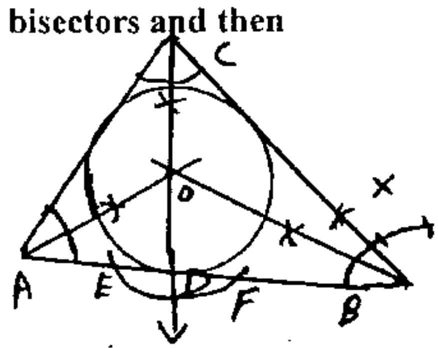
(6) Take O is center and draw a circle of radius equal to $m\overline{OA}$ *or* $m\overline{OB}$ *or* $m\overline{OC}$.

(7) This is the required circumscribe a circle about the given triangle.

Q.5: Draw a triangle ABC with $< A = 60^o$, $m < B = 45^o$ and $m\overline{AB} = 5cm$. Draw three angle bisectors and then inscribed a circle in it.

Solution: **Given**: In triangle ABC, $< A = 60^o$, $m < B = 45^o$ and $m\overline{AB} = 5cm$.

Required: Construct triangle ABC.



and draw three angle bisectors and then draw inscribed circle.

Steps of construction:

(1) Draw $m\overline{AB} = 5cm$.

(2) At point A draw $< BAD = 60^o$

(3) At point B draw $< ABE = 45^o$.

(4) which cut each other at point C.

(5) Thus ΔABC is formed .

(6) Draw bisectors of < A ,< B and < C of given ΔABC , which intersect each other at point O.

(7) O is the center of required circle.

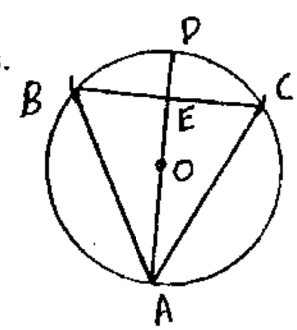
(8) From point O draw $\overline{OD} \perp \overline{AB}$.

(9) Take O is a center and draw a circle of radius $m\overline{OD}$ which touches all the three sides of the given triangle.

(10) This is the required inscribed circle in the given circle.

Q.6: An equilateral triangle in inscribed in a circle. Find the altitude of the triangle if the radius of the circle varies as under . r = 3 units , r = 4 units , r = 6 units ,r = 10 units

Solution: **Given:** A circle of center O,
With radius r = 3 , 4 , 6 and 10 units.



Required: An equilateral triangle in inscribed in a circle .
Also altitude of the triangle.

Steps of construction: For r = 3 units:

(i) Take O is a center and draw a circle of radius 3 units.

(ii) Draw diameter $m\overline{AD}$ of the given circle.

(iii) With center D and radius $\overline{OD}$=3 units, draw two arcs which cut the given circle at B and D.

(iv) Join A , B and C .

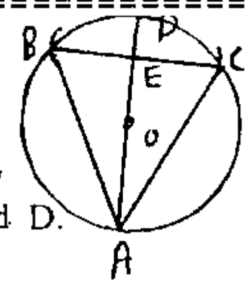
(v) Thus ΔABC is required triangle.

(vi) E is a point to intersect $\overline{AD}$.

(vii)Thus $\overline{AE}$ is the altitude of triangle ABC of measures 4 units.

For r = 4 units:

(i) Take O is a center and draw a circle of radius 4 units.



(ii) Draw diameter $m\overline{AD}$ of the given circle.

(iii) With center D and radius $\overline{OD}$ =4 units, draw two arcs which cut the given circle at B and D.

(iv) Join A , B and C .

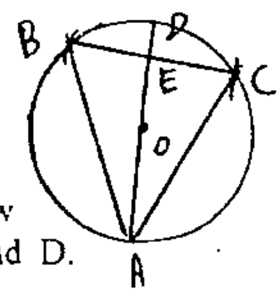
(v) Thus ΔABC is required triangle.

(vi) E is a point to intersect $\overline{AD}$.

(vii)Thus $\overline{AE}$ is the altitude of triangle ABC of measures 6 units.

For r = 6 units:

(i) Take O is a center and draw a circle of radius 6 units.



(ii) Draw diameter $m\overline{AD}$ of the given circle.

(iii) With center D and radius $\overline{OD}$ =6 units, draw two arcs which cut the given circle at B and D.

(iv) Join A , B and C .

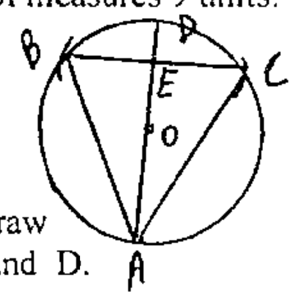
(v) Thus ΔABC is required triangle.

(vi) E is a point to intersect $\overline{AD}$.

(vii)Thus $\overline{AE}$ is the altitude of triangle ABC of measures 9 units.

For r = 12 units:

(i) Take O is a center and draw a circle of radius 12 units.



(ii) Draw diameter $m\overline{AD}$ of the given circle.

(iii) With center D and radius $\overline{OD}$ =12 units, draw two arcs which cut the given circle at B and D.

(iv) Join A , B and C .

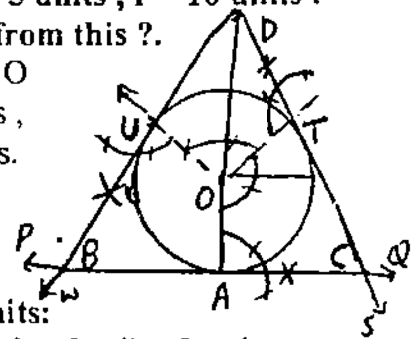
(v) Thus ΔABC is required triangle.

(vi) E is a point to intersect $\overline{AD}$.

(vii)Thus $\overline{AE}$ is the altitude of triangle ABC of measures 18 units.

From this we deduce that E divides diameter $\overline{AD}$ of the circle in the ratios of 3 : 1 .

Q.7: An equilateral triangle is circumscribed about a circle. Find the altitude of the triangle if the radius r of the circle varies as r = 2 units, r = 5 units, r = 10 units. Can you deduce some result from this ?.



Solution: Given: A circle of center O and radius of r = 2 units, r = 5 units and r = 10 units.

Required: Circumscribed an equilateral triangle about the circle. Also find altitude of triangle.

Steps of construction: **For r = 2units:**

(i) Take O is a center and draw a circle of radius 2 units.

(ii) Take any point A on the circumference of the circle and join O to A . $\overline{OA}$ is redial segment.

(iii) Construct an angle <AOT of measure 120^o.

(iv) construct anther angle < TOU measuring 120^o.

(v) The three points A , T , U which are lying on the circumference of the given circle are such that they divided the boundary of the circle into three equal arcs $\widehat{AT}$, $\widehat{TU}$ *and* $\widehat{UA}$.

(vi) Draw perpendiculars to $\overline{OA}$, $\overline{OT}$ and $\overline{OU}$ at A , T and U respectively .These perpendiculars lines are $\overline{PQ}$, $\overline{RS}$ and $\overline{VW}$.

(vii) These lines intersects each other at B , C and D forming a triangle BCD.

(viii) Thus BCD is required triangle.

(ix) Join A and D .

(x) Thus $\overline{AD}$ is the altitude of the given triangle of length 6 units.

For r = 5 units:

(i) Take O is a center and draw a circle of radius 5 units.

(ii) Take any point A on the circumference of the circle and join O to A . $\overline{OA}$ is redial segment.

(iii) Construct an angle <AOT of measure 120^o.

(iv) construct anther angle < TOU measuring 120^o.

(v) The three points A , T , U which are lying on the circumference of the given circle are such that they divided the boundary of the circle into three equal arcs $A\hat{T}$, $T\hat{U}$ and $U\hat{A}$.

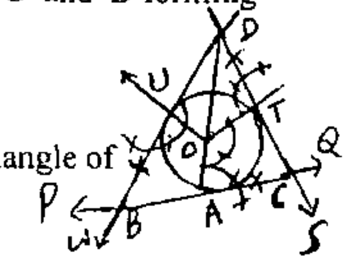
(vi) Draw perpendiculars to $\overline{OA}$, $\overline{OT}$ and $\overline{OU}$ at A , T and U respectively .These perpendiculars lines are $\overline{PQ}$, $\overline{RS}$ and $\overline{VW}$.

(vii) These lines intersects each other at B , C and D forming a triangle BCD.

(viii) Thus BCD is required triangle.

(ix) Join A and D .

(x) Thus $\overline{AD}$ is the altitude of the given triangle of length 7.5 units.



For r = 10 units: .

(i) Take O is a center and draw a circle of radius 10 units.

(ii) Take any point A on the circumference of the circle and join O to A . $\overline{OA}$ is redial segment.

(iii) Construct an angle <AOT of measure 120^o.

(iv) construct anther angle < TOU measuring 120^o.

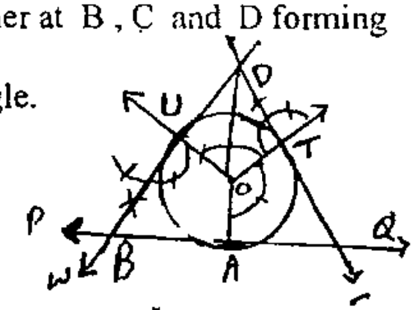
(v) The three points A , T , U which are lying on the circumference of the given circle are such that they divided the boundary of the circle into three equal arcs $A\hat{T}$, $T\hat{U}$ and $U\hat{A}$.

(vi) Draw perpendiculars to $\overline{OA}$, $\overline{OT}$ and $\overline{OU}$ at A , T and U respectively .These perpendiculars lines are $\overline{PQ}$, $\overline{RS}$ and $\overline{VW}$.

(vii) These lines intersects each other at B , C and D forming a triangle BCD.

(viii) Thus BCD is required triangle.

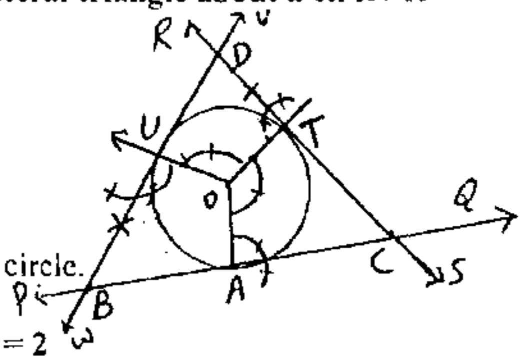
(ix) Join A and D .



(x) Thus $\overline{AD}$ is the altitude of the given triangle of length 15 units.

From this we can deduced that a point on the circumference which intersects $\overline{AD}$ in the ratio of 2 : 1.

Q.8: Circumscribe an equilateral triangle about a circle of radius 2 , 3 and 1 .



Solution: Given:
Circumscribe an equilateral triangle about a circle of radius 2 , 3 and 1 .

Required: Circumscribed an equilateral triangle about the circle. Also find altitude of triangle.

Steps of construction: for **r = 2**

(i) Take O is a center and draw a circle of radius 2 units.

(ii) Take any point A on the circumference of the circle and join O to A . $\overline{OA}$ is redial segment.

(iii) Construct an angle <AOT of measure 120^o.

(iv) construct anther angle < TOU measuring 120^o.

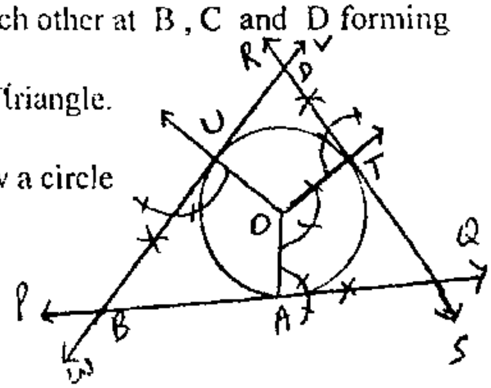
(v) The three points A , T , U which are lying on the circumference of the given circle are such that they divided the boundary of the circle into three equal arcs $\widehat{AT}$, $\widehat{TU}$ *and* $\widehat{UA}$.

(vi) Draw perpendiculars to $\overline{OA}$, $\overline{OT}$ and $\overline{OU}$ at A , T and U respectively .These perpendiculars lines are $\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$ and $\overleftrightarrow{VW}$.

(vii) These lines intersects each other at B , C and D forming a triangle BCD.

(viii) Thus BCD is required triangle.

For r = 3:



(i) Take O is a center and draw a circle of radius 3 units.

(ii) Take any point A on the

circumference of the circle and join O to A .

$\overline{OA}$ is redial segment.

(iii) Construct an angle <AOT of measure 120^o.

(iv) construct anther angle < TOU measuring 120^o.

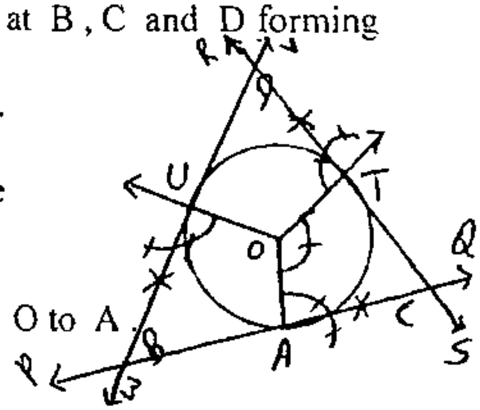
(v) The three points A , T . U which are lying on the circumference of the given circle are such that they divided the boundary of the circle into three equal arcs

$A\hat{T}$, $T\hat{U}$ and $U\hat{A}$.

(vi) Draw perpendiculars to $\overline{OA}$, $\overline{OT}$ and $\overline{OU}$ at A , T and U respectively .These perpendiculars lines are $\overline{PQ}$, $\overline{RS}$ and $\overline{VW}$.

(vii) These lines intersects each other at B , C and D forming a triangle BCD.

(viii) Thus BCD is required triangle.



For r = 1:

(i) Take O is a center and draw a circle of radius 3 units.

(ii) Take any point A on the circumference of the circle and join O to A .

$\overline{OA}$ is redial segment.

(iii) Construct an angle <AOT of measure 120^o.

(iv) construct anther angle < TOU measuring 120^o.

(v) The three points A , T , U which are lying on the circumference of the given circle are such that they divided the boundary of the circle into three equal arcs

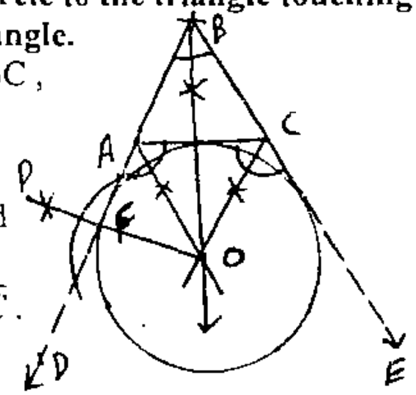
$A\hat{T}$, $T\hat{U}$ and $U\hat{A}$.

(vi) Draw perpendiculars to $\overline{OA}$, $\overline{OT}$ and $\overline{OU}$ at A , T and U respectively .These perpendiculars lines are $\overline{PQ}$, $\overline{RS}$ and $\overline{VW}$.

(vii) These lines intersects each other at B , C and D forming a triangle BCD.

(viii) Thus BCD is required triangle.

Q.9: Draw a triangle with sides 2.5cm , 3.5cm and 4.5 cm long. Draw an escribed circle to the triangle touching the longest side of the triangle.



Solution: Given: A triangle ABC ,

$m\overline{AB} = 2.5cm$, $m\overline{BC} = 3.5cm$

and $m\overline{AC} = 4.5cm$

Required: Draw a triangle and and escribed circle to the triangle touching the side $\overline{AC}$.

Steps of construction:

(1) Draw $m\overline{AC} = 4.5cm$.

(2) Take A is a center and draw an arc of length 2.5 cm .

(3) Take C is a center and draw an arc of length 3.5 cm

(4) Which cut each other at point B . Join B with A and C .

(5) Thus triangle ABC is formed .

(6) Produce $\overline{AB}$ and $\overline{BC}$ to form two exterior angles < ACE and <CAD.

(iii) Draw bisectors of < ABC , <ACE and < CAD which intersect each other at point O .

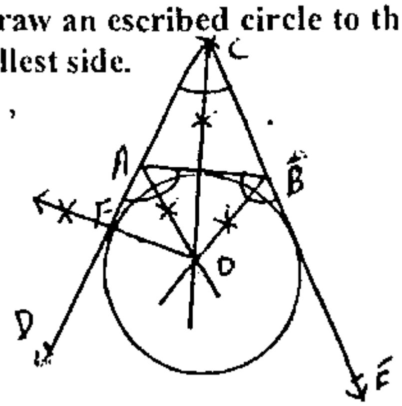
(iv) O is the center of the required circle.

(v) Draw $\overline{OF} \perp \overline{AB}$

(vi) Take O is the center and draw a circle of radius $m\overline{OF}$ which touches $\overline{AC}$, $\overline{AD}$ *and* $\overline{CE}$.

(vii) This is the required circle.

Q.10: For the problem in Q.9 draw an escribed circle to the triangle touching the smallest side.



Solution: Given: A triangle ABC ,

$m\overline{AB} = 2.5cm$, $m\overline{BC} = 3.5cm$

and $m\overline{AC} = 4.5cm$

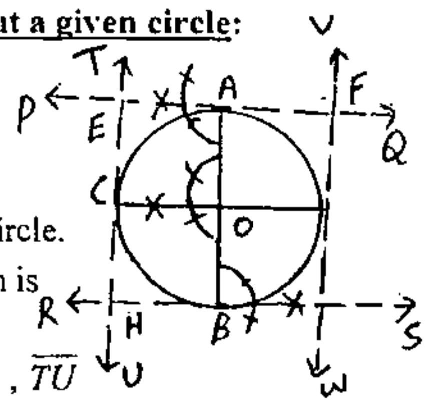
Required: Draw a triangle and and escribed circle to the

triangle touching the side $\overline{AB}$.

Steps of construction:

(1) Draw $m\overline{AB} = 2.5cm$.
(2) Take A is a center and draw an arc of length 4.5 cm .
(3) Take B is a center and draw an arc of length 3.5 cm
(4) Which cut each other at point C . Join C with A and B .
(5) Thus triangle ABC is formed .
(6) Produce $\overline{AC}$ and $\overline{BC}$ to form two exterior angles < ABE and <BAD.
(iii) Draw bisectors of < ACB , <ABE and < BAD which intersect each other at point O .
(iv) O is the center of the required circle.
(v) Draw $\overline{OF} \perp \overline{AC}$
(vi) Take O is the center and draw a circle of radius $m\overline{OF}$ which touches $\overline{AB}$, $\overline{AD}$ *and* $\overline{BE}$.
(vii) This is the required circle.

13.8: Circumscribed a square about a given circle:



Given : A circle at center O .
Required : To circum a square about a given circle.

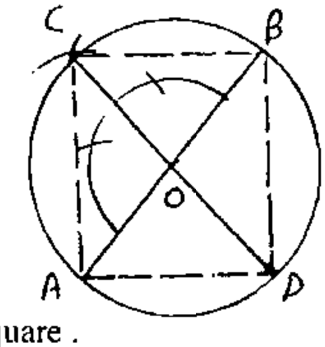
Steps of construction:

(i) Draw any diameter $\overline{AB}$ of the circle.
(ii) Draw other diameter $\overline{CD}$ which is perpendicular to $\overline{AB}$.
(iii) Draw perpendiculars $\overline{PQ}$, $\overline{RS}$, $\overline{TU}$ and $\overline{VW}$ at extremities A , B , C , D of the diameters $\overline{AB}$ and $\overline{CD}$.
(iv) These lines intersect each other at the points E , F , G and H.
(v) EFGH is the required circumscribed square about the given circle.

13.9: Inscribed a square in a given circle.

Given: A circle with center O

Required : To inscribed a square in a given circle.



Steps of construction:

(i) Draw any diameter $\overline{AB}$ of the circle.

(ii) Draw other diameter $\overline{CD}$ which is perpendicular to $\overline{AB}$.

(iii) Draw $\overline{AC}$, $\overline{CB}$, $\overline{BD}$ and $\overline{DA}$.

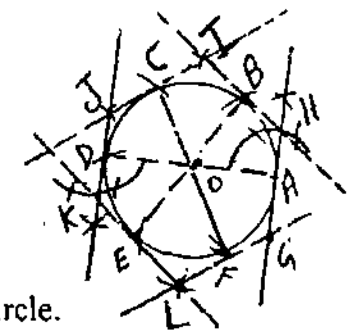
(iv) ABCD is the required inscribed square .

13.10: Circum scribed a regular hexagon about a given circle.

Note: Regular hexagon : It has six sides and each side rotated by 60^o .

Given: A circle with center O .

Required: To circumscribe a regular hexagon about the given circle.



Steps of construction:

(i) Take any point A on the circumference of the given circle.

(ii) With A as a center and $m\overline{OA}$ as radius , draw two arcs which cut the circumference of the circle at B and F.

(iii) Through O , draw $\overline{AD}$, $\overline{BE}$ and $\overline{FC}$.

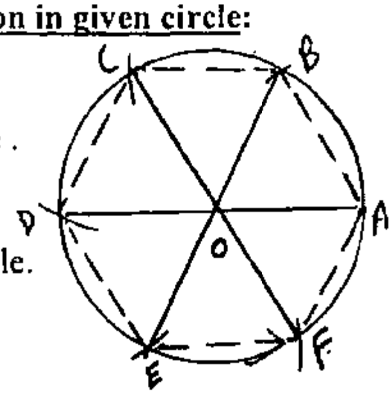
(iv)Draw perpendiculars at the extremities A , B , C, D ,E and F of the diameters $\overline{AD}$, $\overline{BE}$ and $\overline{FC}$ of the circle. These lines cut each other at points G , H , I , J , K and L.

(v) GHIJKL is the required circumscribed hexagon .

13.11: Inscribed a regular hexagon in given circle:

Given: A circle with center O.

Required : To inscribed a regular hexagon in given circle .



Steps of construction:

(i) Take any point A on the circumference of the given circle.

(ii) With A as a center and $m\overline{OA}$ as radius, draw an arc which cuts the circumference of the circle at B .

(iii) Similarly draw successive arcs which cut the circumference of the circle at C , D , E and F.

(iv) Draw $\overline{AB}$, $\overline{BC}$, $\overline{DE}$ and $\overline{EF}$.

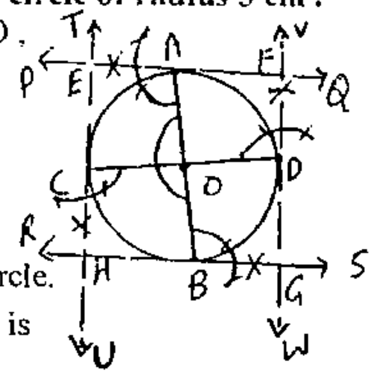
(v) ABCDEF is the required hexagon which is inscribed in the given circle.

Exercise 13.2:

Q.1: Circumscribe a square about a circle of radius 5 cm .

Solution: **Given**: A circle at center O , and a radius of 5 cm .

Required : To circum a square about a given circle.



Steps of construction:

(i) Draw a circle of radius 5 cm .

(ii) Draw any diameter $\overline{AB}$ of the circle.

(iii) Draw other diameter $\overline{CD}$ which is perpendicular to $\overline{AB}$.

(iv) Draw perpendiculars $\overline{PQ}$, $\overline{RS}$, $\overline{TU}$ and $\overline{VW}$ at extremities A , B , C , D of the diameters $\overline{AB}$ and $\overline{CD}$.

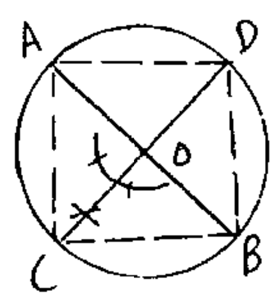
(v) These lines intersect each other at the points E , F , G and H.

(vi) EFGH is the required circumscribed square about the given circle.

Q.2: Inscribe a square in a circle of radius 6 cm .

Solution: Given: A circle with center O and radius 6 cm .

Required : To inscribed a square in a given circle.



Steps of construction:

(i) Draw a circle of radius 6 cm .

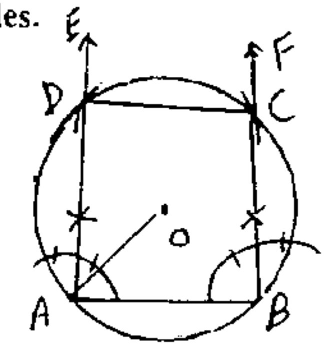
(ii) Draw any diameter $\overline{AB}$ of the circle.

(iii) Draw other diameter $\overline{CD}$ which is perpendicular to $\overline{AB}$.

(iv) Draw $\overline{AC}$, $\overline{CB}$, $\overline{BD}$ and $\overline{DA}$.

(v) ABCD is the required inscribed square .

Q.3: Draw a square of side 6 cm . Circumscribe a circle about that square and the inscribe a circle in the same square measure the radii of these two circles.



__Solution__: **Given:** A square of side 6 cm.

__Required__ : (a) Circumscribe a circle about that square

(b) Inscribe a circle in the same square measure the radii of these two circles.

__Steps of construction__:

(i) Draw $m\overline{AB} = 6cm$

(ii) At points A and B draw $< BAE = 90^o$ and $< ABF = 90^o$ respectively. At A and B draw two arcs of radius 6 cm which cuts $\overline{AE}$ and $\overline{BF}$ at points D and C respectively .

(iii) join C and D . Thus ABCD is a required square .

(iv) Now join A to C and B to D , which cuts each other at point O .

(v) Take O is a center and draw a square of radius $m\overline{OA}$ which intersect the square at points A , B ,C and D .

(vi) This is the required circum circle .

Now we find its radius. By Pythagoras theorem

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$=> (AC)^2 = (6)^2 + (6)^2 = 36 + 36$$

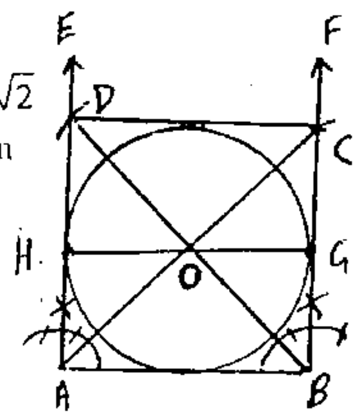
$$=> \sqrt{(AC)^2} = \sqrt{36\times 2} \;=>\; AC = 6\sqrt{2}$$

As O is the mid point of AC . Then

$$m\overline{OA} = \frac{AC}{2} = \frac{6\sqrt{2}}{2}$$

$$=> m\overline{OA} = 3\sqrt{2}$$

Thus radius $= m\overline{OA} = 3\sqrt{2}\,cm$.



Now we find its inscribe circle.

(vii) Let G and H are the mid points of sides $\overline{BC}$ and $\overline{AD}$

then join G and H which intersect AC and BD at O.

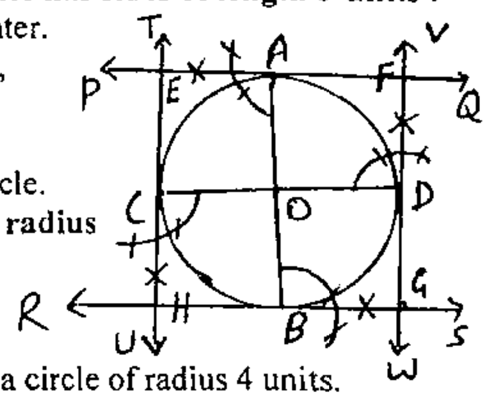
As $\overline{AB} = \overline{HG} = 6cm$

Then $\overline{OG} = \frac{\overline{HG}}{2} = \frac{6}{2} = 3cm$

(viii) Then take O is a center and draw a circle of radius $\overline{OG} = 3cm$ which touches the sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$ at points S, G, R and H respectively.

Thus the required inscribe circle of radius 3cm is formed.

Q. 4: First draw a circle of suitable radius, so that the square circumscribed about that circle has sides of length 8 units.



Solution: Given: A circle of center.
And a square of side = 8 units,

Required: Construct a circle and a square which circumscribed the circle.

Note: Since inscribe circle the radius of the circle is half of the side of the square.

Steps of construction:

(i) Take O is a center and draw a circle of radius 4 units.

(ii) Draw any diameter $\overline{AB}$ of the circle.

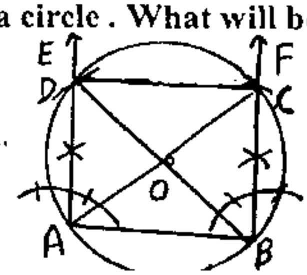
(iii) Draw other diameter $\overline{CD}$ which is perpendicular to $\overline{AB}$.

(iv) Draw perpendiculars $\overline{PQ}$, $\overline{RS}$, $\overline{TU}$ and $\overline{VW}$ at extremities A, B, C, D of the diameters $\overline{AB}$ and $\overline{CD}$.

(v) These lines intersect each other at the points E, F, G and H.

(vi) EFGH is the required circumscribed square about the circle.

Q.5: Inscribe a square of side 10cm in a circle. What will be the size of the radius.



Solution: **Given:** A square of side 10 cm.

Required: (a) Circumscribe a circle

about that square

Steps of construction:

(i) Draw $m\overline{AB} = 10cm$

(ii) At points A and B draw $< BAE = 90^o$ and $< ABF = 90^o$ respectively. At A and B draw two arcs of radius 10 cm which cuts $\overline{AE}$ and $\overline{BF}$ at points D and C respectively.

(iii) join C and D. Thus ABCD is a required square.

(iv) Now join A to C and B to D, which cuts each other at point O.

(v) Take O is a center and draw a square of radius $m\overline{OA}$ which intersect the square at points A, B, C and D.

(vi) This is the required circum circle.

Now we find its radius. By Pythagoras theorem

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow (AC)^2 = (10)^2 + (10)^2 = 100 + 100$$

$$\Rightarrow \sqrt{(AC)^2} = \sqrt{100 \times 2} \Rightarrow AC = 10\sqrt{2}$$

As O is the mid point of AC. Then

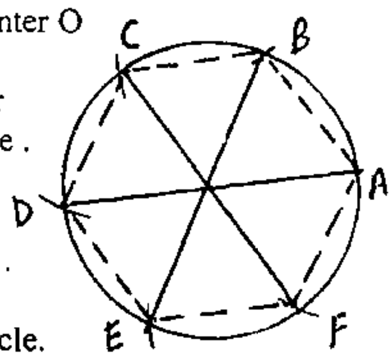
$$m\overline{OA} = \frac{AC}{2} = \frac{10\sqrt{2}}{2}$$

$$\Rightarrow m\overline{OA} = 5\sqrt{2}$$

Thus radius of the circle = $5\sqrt{2}\ cm$

Q.6: Inscribe a regular hexagon in a circle of radius 4 cm.

Solution: **Given**: A circle with center O and radius 4 cm.



Required: To inscribed a regular hexagon in given circle.

Steps of construction:

(i) Take O is a center and draw a circle of radius of 4 cm.

(ii) Take any point A on the circumference of the given circle.

(iii) With A as a center and $m\overline{OA} = 4$ cm as radius, draw an arc

which cuts the circumference of the circle at B.

(iv) Similarly draw successive arcs which cut the circumference of the circle at C, D, E and F.

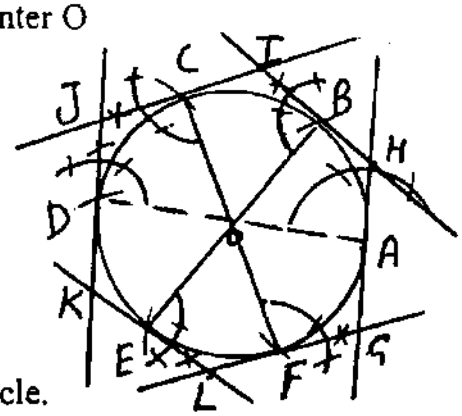
(v) Draw $\overline{AB}$, $\overline{BC}$, $\overline{DE}$ and $\overline{EF}$.

(vi) ABCDEF is the required hexagon which is inscribed in the given circle.

Q.7: Construct a circle of radius 4 cm and circumscribe a regular hexagon.

Solution: Given: A circle with center O and radius 4 cm.

Required: To circumscribe a regular hexagon about the given circle.



Steps of construction:

(i) Take O is a center and draw a circle of radius 4 cm.

(ii) Take any point A on the circumference of the given circle.

(iii) With A as a center and $m\overline{OA}$ as radius, draw two arcs which cut the circumference of the circle at B and F.

(iv) Through O, draw $\overline{AD}$, $\overline{BE}$ and $\overline{FC}$.

(v) Draw perpendiculars at the extremities A, B, C, D, E and F of the diameters $\overline{AD}$, $\overline{BE}$ and $\overline{FC}$ of the circle. These lines cut each other at points G, H, I, J, K and L.

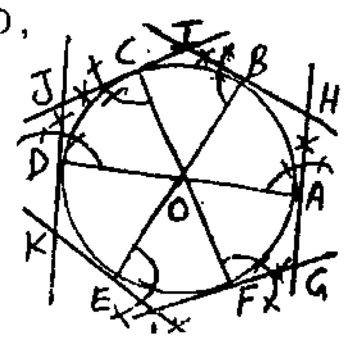
(vi) GHIJKL is the required circumscribed hexagon.

Q.8: Draw a circle of radius 8 cm. circumscribe a regular hexagon about that circle and also inscribe a regular hexagon in the same circle. Find the areas of these Geometrical figures. Comment on the values of these area

Solution: Given: A circle of center O, and radius 8 cm.

Required:

Construct a circumscribe regular Hexagon and also inscribe

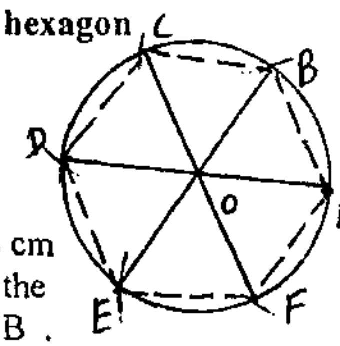


Regular Hexagon.
And also find their areas.

Steps of construction:

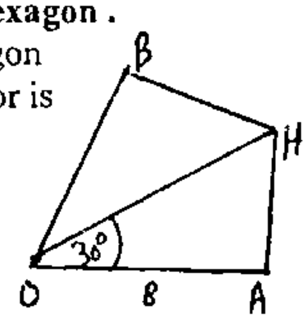
(i) Take O is a center and draw a circle of radius 8 cm .
(ii) Take any point A on the circumference of the given circle.
(iii) With A as a center and $m\overline{OA}$ as radius , draw two arcs which cut the circumference of the circle at B and F.
(iv) Through O , draw $\overline{AD}$, $\overline{BE}$ and $\overline{FC}$.
(v)Draw perpendiculars at the extremities A , B , C, D ,E and F of the diameters $\overline{AD}$, $\overline{BE}$ and $\overline{FC}$ of the circle. These lines cut each other at points G , H , I , J , K and L.
(vi) GHIJKL is the required circumscribed hexagon .

Now we construct inscribe regular hexagon



(i) Take O is a center and draw a circle of radius of 8 cm .
(i) Take any point A on the circumference of the given circle.
(ii) With A as a center and $m\overline{OA} = 8$ cm as radius , draw an arc which cuts the circumference of the circle at B .
(iii) Similarly draw successive arcs which cut the circumference of the circle at C , D , E and F.
(iv) Draw $\overline{AB}$, $\overline{BC}$, $\overline{DE}$ and $\overline{EF}$.
(v) ABCDEF is the required hexagon which is inscribed in the given circle.

Now we find the area of circumscribe hexagon .



Since the GHIJKL is circumscribe hexagon and has six equal sectors and each sector is equal to two right angle triangles.
Let take the sector OAHB.
Join H to O . Thus OAH is right angle triangle in which OH is hypotenuse and OA is base

and angle <AOH $< AOH = 30^o$ and AH is perpendicular

$$Then \quad Tan30^o = \frac{AH}{OA} \quad put \ OA = 8$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AH}{8}$$

$$\Rightarrow AH = \frac{8}{\sqrt{3}}$$

Now area of right angle triangle OAH is

$$Area \ of \ OAH = \frac{1}{2}(OA)(AH) \quad \text{put values}$$

$$= \frac{1}{2}(8)\left(\frac{8}{\sqrt{3}}\right)$$

$$= \frac{32}{\sqrt{3}}$$

Thus area of sector OAHB = $2(Area \ of \ \Delta OAH)$

$$= 2\left(\frac{32}{\sqrt{3}}\right) = \frac{64}{\sqrt{3}}$$

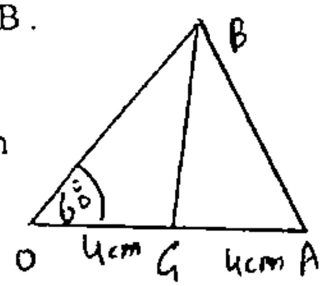
Thus area of hexagon GHIKL = $6\left(\frac{64}{\sqrt{3}}\right) = \frac{384}{\sqrt{3}}$

$$= 221.7 \ cm^2$$

Now we find the area of of inscribed hexagon ABCDEF.

Since in inscribe hexagon there are six equilateral triangles .

Let we take equilateral triangle OAB . Let BG is perpendicular from B on OA. Then OGB is a right angle triangle in which OG = 4 cm is a base and BG is perpendicular and $< GOB = 60^o$. Then



$$Tan60^o = \frac{GB}{OG} \quad \text{put values}$$

=> $\sqrt{3} = \frac{GB}{4}$ => $GB = 4\sqrt{3}$

Area of triangle OGB $= \frac{1}{2}(OG)(GB)$ put values .

$Area\ of\ triangle\ OGB = \frac{1}{2}(4)(4\sqrt{3})$

$= 8\sqrt{3}\ cm^2$

Now area of triangle OAB $= 2(Area\ of\ triangle\ OGB)$

$= 2(8\sqrt{3}) = 16\sqrt{3} cm^2$

Area of hexagon ABCDEF = $6(16\sqrt{3}) = 166.28\ cm^2$

Now area of circle:

$As\ area\ of\ circle = \pi r^2$ put $r = 8$

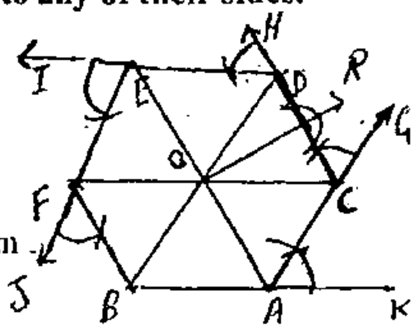
$Area\ of\ circle = \pi(8)^2 = 201.06\ cm^2$

As area inscribe hexagon < area of circle

< area of circumscribe hexagon

Q.9: Draw two regular hexagons of perimeters 6cm and 30cm respectively . Determine their centers . From centers draw perpendicular to any of their sides.

Solution: Given:



Two regular hexagons of Perimeters of 6 cm and 30 cm . Then length of each side of first Hexagon is 1 cm . And the length of each side of 2nd hexagon is 5 cm .

Required: (i) Draw $m\overline{AB} = 1cm$

(ii) At point A draw $m\angle KAG = 60^o$. Take A is a center and draw an arc of length 1 cm which cuts $\overline{AG}$ at point C .

(iii) At point C draw $m\angle GCH = 60^o$. Take C is a center and draw an arc of length 1 cm which cuts $\overline{CH}$ at point D .

v) At point D draw $m\angle HDI = 60^o$. Take D is a center and

draw an arc of length 1 cm which cuts $\overline{DI}$ at point E.

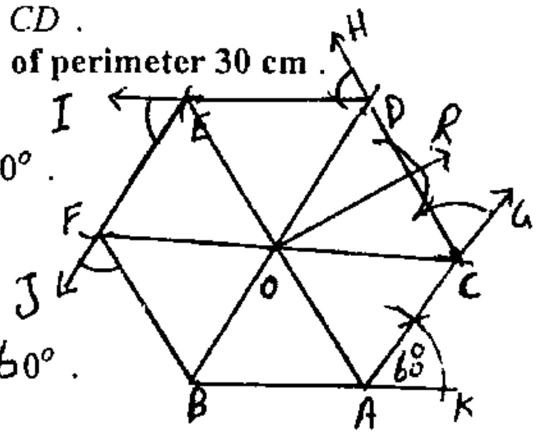
(iv) At point E draw $m\angle IEJ = 60^o$. Take E is a center and draw an arc of length 1 cm which cuts $\overline{EI}$ at point E.

(v) At point F draw $m\angle JFB = 60^o$.

Thus BACDEF is required regular hexagon of perimeter 6 cm.

(vi) Join A to E, B to D and C to F. Which cuts each other At point O. Thus O is the center and at point O draw a perpendicular $\overline{OR}$ to side $\overline{CD}$.

(b) Now we take regular hexagon of perimeter 30 cm.



(i) Draw $m\overline{AB} = 5cm$

(ii) At point A draw $m\angle KAG = 60^o$. Take A is a center and draw an arc of length 5 cm which cuts $\overline{AG}$ at point C.

(iii) At point C draw $m\angle GCH = 60^o$. Take C is a center and draw an arc of length 5 cm which cuts $\overline{CH}$ at point D.

(iv) At point D draw $m\angle HDI = 60^o$. Take D is a center and draw an arc of length 1 cm which cuts $\overline{DI}$ at point E.

(iv) At point E draw $m\angle IEJ = 60^o$. Take E is a center and draw an arc of length 1 cm which cuts $\overline{EI}$ at point E.

(v) At point F draw $m\angle JFB = 60^o$.

Thus BACDEF is required regular hexagon of perimeter 30 cm.

(vi) Join A to E, B to D and C to F. Which cuts each other At point O. Thus O is the center and at point O draw a perpendicular $\overline{OR}$ to side $\overline{CD}$.

Q.10: Can you construct a square whose area equals the area of the given circle . Discus in details .

Solution: As we know that if r is the radius of the circle

Then area of the circle = $\pi r^2\ cm^2$ .....(i)

If x is the length of each side of a square then

Area of the square = $x \times x = x^2\ cm^2$ ......(ii)

If we take r = 1 cm .Then (i) becomes

Area of circle = $\pi\ cm^2$ .....(iii)

If $x = \sqrt{\pi}$ cm then from eq(ii)

The area of square = $\pi\ cm^2$ ......(iv)

From (iii) and (iv) the area of square is equal to the area of circle.

But Since π is an irrational number and its value approximately is equal to 1.1415.....

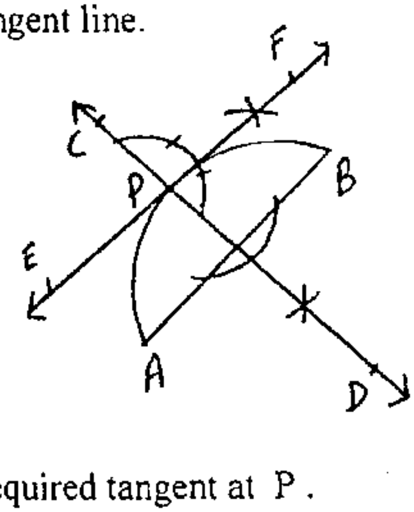
Then $\sqrt{\pi}$ is also irrational and its exact value we cannot find

So we cannot take $\sqrt{\pi}$ is length of a side of square.

Thus it is not possible that we construct a square whose area is equal to the area of given circle.

Tangent to the circle: Definition: A straight line which touch a curve at only one point is called tangent line.

13.12: Draw a tangent to a given arc , with out using the center , through a given point p , when P is the middle point of the arc.



Given: An arc such that P is the mid point of the arc.

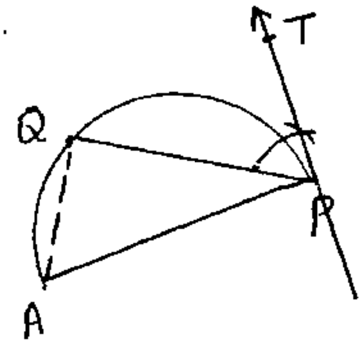
Required: To draw a tangent at P .

Steps of construction: (i) Draw $\overline{AB}$.

(ii) From P, draw $\overline{CD} \perp \overline{AB}$.

(iii) At P , draw $\overline{EF} \perp \overline{CD}$ which is required tangent at P .

13.13: Draw a tangent to a given arc , with out using the center through a given point p , when P is at the end of the arc.



Solution: Given: An arc $\overline{AP}$ of a circle.

Required: To draw a tangent at the end point P.

Steps of construction:

(i) Take any point Q on the arc $\overline{AP}$ And draw the chords $\overline{AP}$ and $\overline{PQ}$.

(ii) Join A and Q.

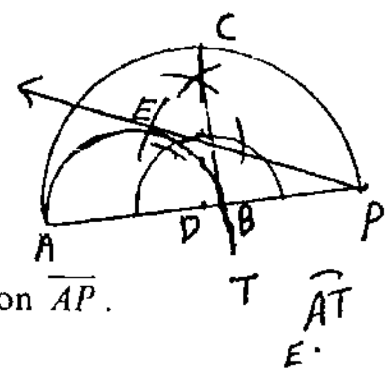
(iii) Now construct $<QPT \cong< PAQ$.

(iv) $\overline{PT}$ is the required tangent at P.

13.14: Draw a tangent to a given arc, with out using the center through a given point p, when P is out side the arc.

Given: $\widehat{AT}$ is an arc and P is a point outside the arc.

Required: To draw a tangent from Point P on the arc.



Steps of construction:

(i) Join A and P, which cuts the arc at B.

(ii) Bisect $\overline{AP}$ at D.

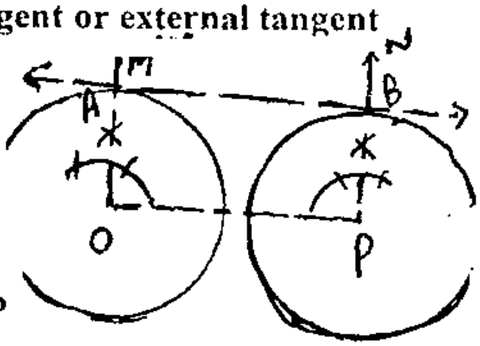
(iii) With D as a center and radius equal to $m\overline{DP}$, draw a semi – circle on $\overline{AP}$.

(iv) Draw $\overline{BC} \perp \overline{AP}$, which intersects the Semi – circle at C.

(v) With P as center and $m\overline{PC}$ as radius draw an arc to cut at point

(vi) Thus $\overline{PE}$, which is the required tangent.

13.15(a): Draw direct common tangent or external tangent to equal circles.



Given: Two equal circles with centers at O and P respectively.

Required: Draw direct common tangent or external tangent to the given circles.

Steps of construction: (i) Join O to P

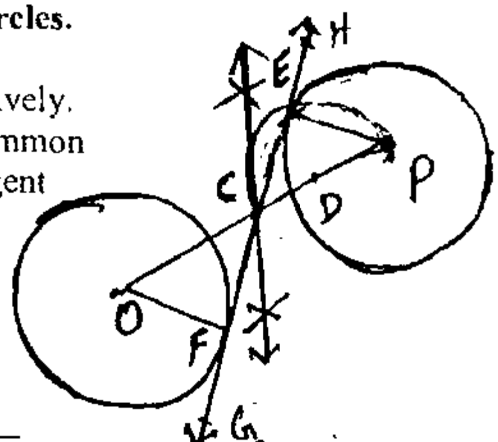
(ii) Now draw two perpendiculars on

$\overline{OP}$ at O and P which cut the circles at A and B respectively.

(iii) Join A and B and produce it towards both ends.

(iv) $\overline{AB}$ is the required direct common tangent Or external tangent to two given equal circles.

13.15(b): Draw transverse common tangent or internal tangent to equal circles.



Given: Two equal circles with centers at O and P respectively.

Required: Draw transverse common tangent or internal tangent to the given circles.

Steps of construction:

(i) Join O to P .

(ii) Bisect $\overline{OP}$ at C .

(iii) Bisect $\overline{CP}$ at D.

(iv) Draw a semi – circle on $\overline{CP}$ diameter which cuts the given circle at E.

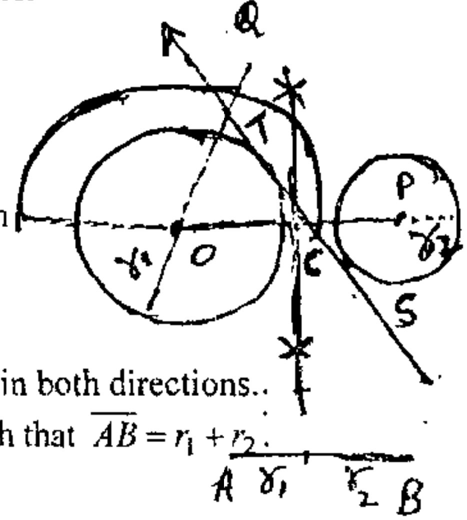
(v) Join E and P .

(vi) Draw $\overline{OF} \| \overline{EP}$.

(vii) E join to F and extend the line to both directions.

(viii) Thus $\overline{GH}$ is the required tangents to both circles.

13.15(b): Draw transverse common tangent or internal tangent to unequal circles.



Given: Two unequal circles with centers at O and P and having radii of r_1 and r_2 . Also $r_2 > r_1$

Required: Draw transverse common tangent or internal tangent to the given circles.

Steps of construction:

(i) Join O to P and produce the line in both directions.. Also draw a line segment $\overline{AB}$ such that $\overline{AB} = r_1 + r_2$.

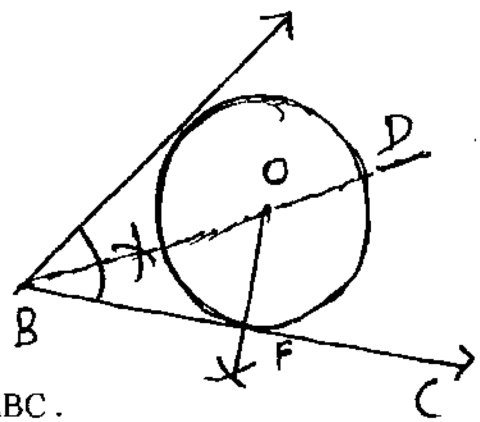
(ii) with center O and radius equal to $\overline{AB}$ draw a Semi – circle .

(iii) Bisect $\overline{OP}$ at C . Then choosing C is a center and draw an arc of radius $m\overline{OC}$ which cuts the semi – circle at point Q .

(iv) Draw $\overline{OQ}$ cutting the bigger circle at T . Now from P, Draw $\overline{PS} \| \overline{OQ}$ in the opposite sense cutting the Smaller circle at S .

(v) Join T and S and produce it in both directions. The line $\overline{TS}$ so obtained is the required transverse tangent Or internal tangent to the given unequal circles.

<u>13.16</u>: Draw a circle which touches both the arms of a Given angle.



<u>Given</u>: An angle <ABC whose vertex is B . And $\overline{AB}$ and $\overline{BC}$ are its arms .

<u>Required</u>: To draw a circle which touches both arms $\overline{AB}$ and $\overline{BC}$ of the given angle.

<u>Steps of construction</u>:

(i) Draw $\overline{BD}$, the bisector of < ABC .

(ii) Take any point O on $\overline{BD}$ and draw a perpendicular $\overline{OF}$ on $\overline{BC}$.

(iii) Taking O is a center and $m\overline{OF}$ as radius draw a circle .

(iv) This is the required circle.

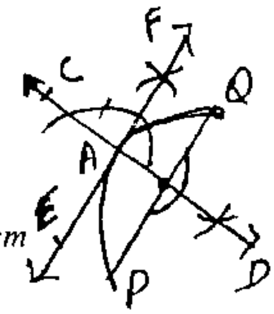
<u>Exercise 13.3</u>:

<u>Q.1</u>: Draw an arc of length 7 cm . Without using the center draw a tangent through a given point A when A is

(I) The middle point of the arc .

(II) End point of the arc.

(III) outside the arc.



Solution: (i) **Given:**

An arc $P\hat{Q} = 7cm$ such that A is the mid point of the arc.

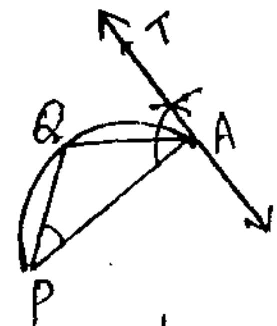
Required: To draw a tangent at A.

Steps of construction: (i) Draw $\overline{PQ} = 7cm$

(ii) From A, draw $\overline{CD} \perp \overline{PQ}$.

(iii) At A , draw $\overline{EF} \perp \overline{CD}$ which is required tangent at A .

(II) Given: An arc $A\hat{P} = 7$ cm of a circle.



Required: To draw a tangent at the end point A.

Steps of construction:

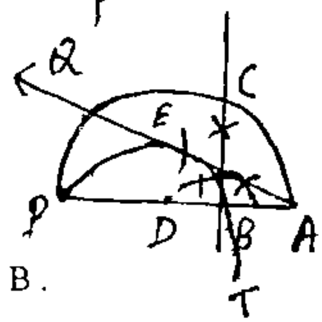
(i) Take any point Q on the arc $A\hat{P}$. And draw the chords $\overline{AP}$ and $\overline{AQ}$.

(ii) Join P and Q .

(iii) Now construct $< QAT \cong < APQ$.

(iv) $\overline{AT}$ is the required tangent at A.

(III) Given: $P\hat{T}$ is an arc and A is a point outside the arc.



Required: To draw a tangent from Point A on the arc.

Steps of construction:

(i) Join P and A, which cuts the arc at B .

(ii) Bisect $\overline{AP}$ at D.

(iii) With D as a center and radius equal to $m\overline{DP}$, draw a semi – circle on $\overline{AP}$.

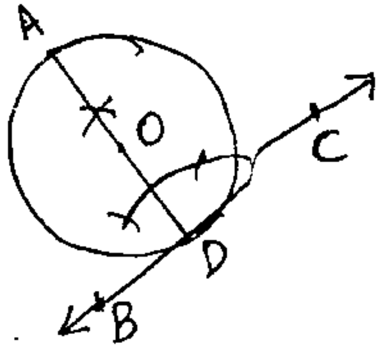
(iv) Draw $\overline{BC} \perp \overline{AP}$, which intersects the Semi – circle at C .

(v) With A as center and $m\overline{AC}$ as radius draw an arc to cut the given arc at E .

(vi) Thus $\overline{AE}$, which is the required tangent.

Q.2: Draw a circle passing through a point A and touching

a given line $\overline{BC}$ at point D.



Solution: Given:

A line $\overline{BC}$ and a point A.

Required: Draw a circle passing through A and touch the line $\overline{BC}$ at D.

Steps of construction:

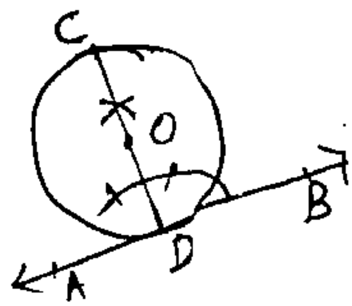
(i) Take the point D on the line $\overline{BC}$.

(ii) Take A is a point out side the line $\overline{BC}$ in such awy that $\overline{AD} \perp \overline{BC}$.

(iii) Take O is the mid point of $\overline{AD}$.

(iv) Now take O is the center and draw a circle of radius of $m\overline{OA}$ which touches the circle at point D. which is the required circle.

Q.3: Describe a circle of radius 4cm, passing through a given point C and touching a given straight line $\overline{AB}$.



Solution: Given: A circle of radius 4 cm and center O.

And a line $\overline{AB}$.

Required: Draw a circle passing through C and touch the line $\overline{AB}$.

Steps of construction:

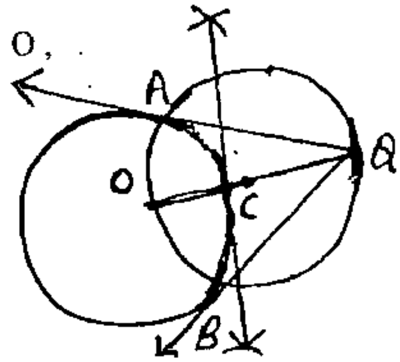
(i) Take a point D on the line $\overline{AB}$.

(ii) Take C is a point out side the line $\overline{AB}$ in such away that $\overline{CD} \perp \overline{AB}$ and $\overline{CD} = 8cm$

(iii) Take O is the mid point of $\overline{CD}$.

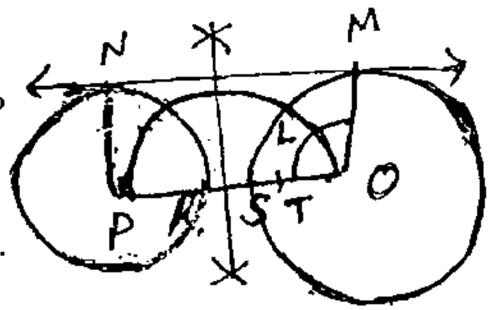
(iv) Now take O is the center and draw a circle of radius of $m\overline{OC} = 4\text{cm}$ which touches the circle at point D. which is the required circle.

Q.4: Radius of a circle of a circle is 2.5 cm. A point Q is at a distance of 5 cm from the center. Draw tangent to the circle from the point Q.



Solution: Given: A circle of center O ,
And radius 2.5 cm
and $m\overline{OQ} = 5cm$.
Required: A tangent to the circle
from point Q .
Steps of construction:
(i) Take O is center and draw a circle of radius 2.5 cm.
(ii) Take point Q out side the circle at distance 5 cm from O.
(iii) Join Q and O .
(iv) Bisect $\overline{OQ}$ at C .
(v) Take C is a center and draw anther circle of radius $m\overline{OC}$, which the cut the first circle at two points A and B .
(vi) Join A and B to Q .
(vii) Thus $\overline{AQ}$ and $\overline{BQ}$ are the required tangents .

Q5: Radii of two circles are 2 cm and 3 cm and their centers are 8 cm apart. Draw direct common tangents to the circles.



Solution: Given:
Two circles of radii 2 cm and 3cm of centers O and P respectively. Also $\overline{OP} = 8cm$
Required: Draw two common tangent to given circles.
Steps of construction:
(i) Join O and P . $\overline{OP}$ cuts the circles at R and S respectively .
(ii) Bisect $\overline{OP}$ at point Q. With Q as center and $m\overline{QP}$ as radius draw a semi circle .
(iii) Take a point T on $\overline{OP}$ in the bigger circle such that $m\overline{RT} = m\overline{SP}$ = length of the radius of the smaller circle.

(iv) Take O is a center and draw an arc of radius $m\overline{OT}$ which cut the semicircle at point L.

(v) Draw $\overline{OL}$ to meet the bigger circle at M.

(vi) Draw $\overline{PN} \| \overline{OM}$.

(vii) Join M and N and produce it to both sides to get common tangent $\overline{MN}$ to the two given circles.

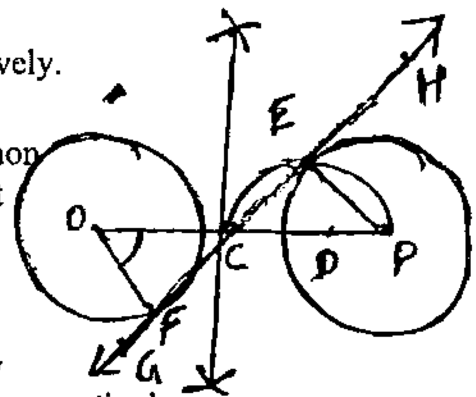
Q.6: Two congruent circles are of radius 4cm each. Their centers are 10 cm apart. Draw transverse common tangents to these circles.

Solution: Given:

Two equal circles with radii 4 cm and centers at O and P respectively.

Also $\overline{OP} = 10cm$

Required: Draw transverse common tangent or internal tangent to the given circles.



Steps of construction:

(i) Take O and P are centers Such that $\overline{OP} = 10cm$ and draw two circles each of radius 4 cm respectively.

(ii) Join O to P.

(iii) Bisect $\overline{OP}$ at C.

(iv) Bisect $\overline{CP}$ at D.

(v) Draw a semi – circle on $\overline{CP}$ diameter which cuts the given circle at E.

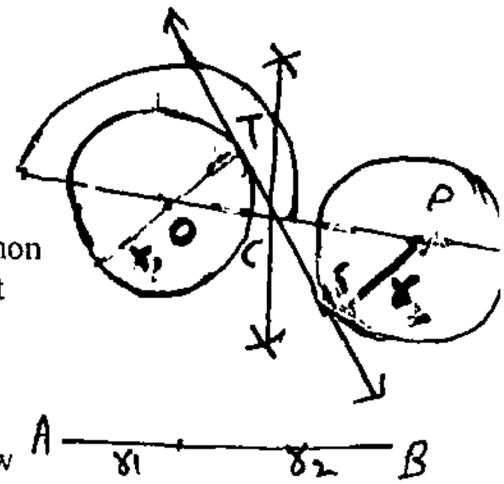
(vi) Join E and P.

(vii) Draw $\overline{OF} \| \overline{EP}$.

(viii) E join to F and extend the line to both directions.

(ix) Thus $\overline{GH}$ is the required tangents to both circles.

Q.7: Radii of two circles are 2 cm and 2.5cm respectively. Distance between their centers is 5.5 cm. Draw transverse common tangent to the circles.



Given: Two unequal circles with centers at O and P and having radii of $r_1 = 2cm$ and $r_2 = 2.5cm$. As $r_2 > r_1$ and $\overline{OP} = 5.5cm$

Required: Draw transverse common tangent or internal tangent to the given circles.

Steps of construction:

(i) Take O and P are centers Such that $\overline{OP} = 5.5cm$ and draw two circles of radii $r_1 = 2cm$ and $r_2 = 2.5cm$ respectively.

(ii) Join O to P and produce the line in both directions. Also draw a line segment $\overline{AB}$ such that $\overline{AB} = r_1 + r_2$.

(iii) with center O and radius equal to $\overline{AB}$ draw a Semi – circle .

(iv) Bisect $\overline{OP}$ at C . Then choosing C is a center and draw an arc of radius $m\overline{OC}$ which cuts the semi – circle at point Q .

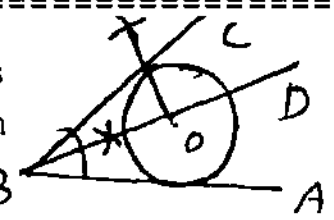
(v) Draw $\overline{OQ}$ cutting the bigger circle at T . Now from P, Draw $\overline{PS} \| \overline{OQ}$ in the opposite sense cutting the Smaller circle at S .

(vi) Join T and S and produce it in both directions. The line $\overline{TS}$ so obtained is the required transverse tangent Or internal tangent to the given unequal circles.

Q.8: If $< ABC$ of measure 60^o. Construct a circle having radius 2.5 cm and touching the arms of the angle.

Given: An angle $< ABC = 60^o$ whose vertex is B . And $\overline{AB}$ and $\overline{BC}$ are its arms . And a circle of radius 2.5 cm.

Required: To draw a circle which touches both arms $\overline{AB}$ and $\overline{BC}$ of the given angle.



Steps of construction:

(i) Draw $\overline{AB}$ and construct at point B, $< ABC = 60^o$.

(ii) Draw $\overline{BD}$, the bisector of < ABC .

(iii) Take any point O on $\overline{BD}$ and draw a perpendicular $\overline{OF}$ on $\overline{BC}$ such that $\overline{OF} = 2.5cm$

(iv) Taking O is a center and $m\overline{OF}$ as radius draw a circle .

(v) This is the required circle.

Review Exercise 13:

Q.1: At the end of each question , four circles are given . Fill in the correct circle only .

(i) The measure of the external angle of a regular hexagon is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) none of these

(ii) Tangents drawn at the end points of the diameter of circle are

(a) parallel (b) perpendicular
(c) intersecting (d) none of these

(iii) How many tangents can be drawn from a point out side the circle .

(a) 1 (b) 2 (c) 3 (d) 4

(iv) If the distance between the centers of two circles is equal to the sum of their radii , then the circles will

(a) intersect (b) do not intersect
(c) touch each other externally
(d) touch each other internally .

Answers: (i) d (ii) d (iii) d (iv) d

Q.2: Practically find the center of an arc ABC.

Solution: Given: Arc ABC .

Required : To locate the center of the arc ABC .

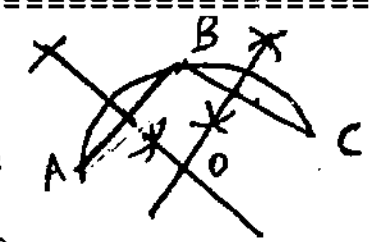
Steps of construction:

(i) Find the mid points of $\overline{AB}$ and $\overline{AC}$.

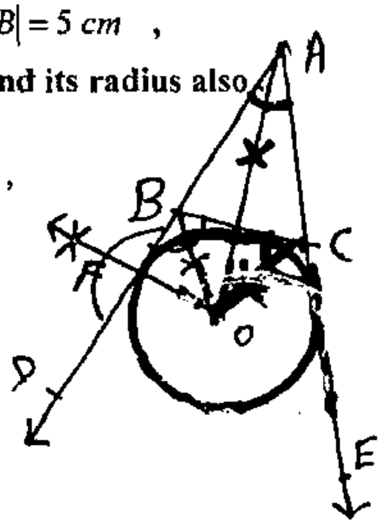
(ii) Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{AC}$.

(iii) These bisectors meet at point O.

(Iv) Thus O is the required center of the given arc ABC.



Q.3: Escribe a circle opposite to the vertex A to a triangle ABC with sides $|AB| = 5\ cm$, $|BC| = 4\ cm$, $|CA| = 3\ cm$. Find its radius also.



Solution: **Given:** A triangle ABC, $|AB| = 5\ cm$, $|BC| = 4\ cm$, $|CA| = 3\ cm$.

Required: Draw a triangle and and escribed circle to the opposite to vertex A.

Steps of construction:

(1) Draw $m\overline{BC} =$ 4 cm

(2) Take B is a center and draw an arc of length 5 cm.

(3) Take C is a center and draw an arc of length 3 cm.

(4) Which cut each other at point C. Join C with A and B.

(5) Thus triangle ABC is formed.

(6) Produce $\overline{AB}$ and $\overline{AC}$ to form two exterior angles < CBD and <BCE.

(7) Draw bisectors of < BAC, <CBA and < BCE which Intersect each other at point O.

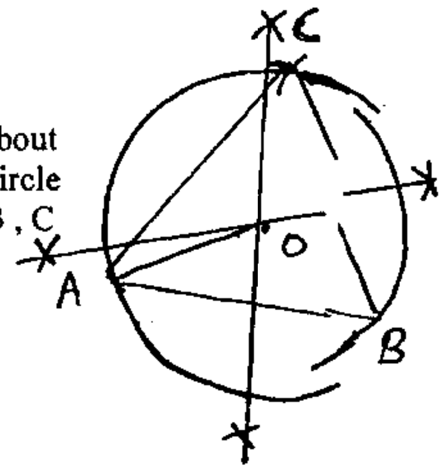
(8) O is the center of the required circle.

(9) Draw $\overline{OF} \perp \overline{AB}$

(10) Take O is the center and draw a circle of radius $m\overline{OF}$ which touches $\overline{BC}$, $\overline{BD}$ *and* $\overline{CE}$.

(11) This is the required circle. The radius is $m\overline{OF}$.

Q.4: Circumscribed a circle about a given triangle ABC with each side of length 5 cm .



Solution: Given: In triangle ABC .

$m\overline{AB} = 5$, $m\overline{BC} = 5, m\overline{AC} = 5$

Required : To circumscribe a circle about the given triangle .i.e. to construct a circle which passes through the vertex A , B , C of ABC .

Steps of construction:

(i) Draw $m\overline{AB} = 5cm$.

(ii) Take A is a center and draw an arc of length 5 cm .

(iii) Take B is a center and draw an arc of length 5 cm .

(iv) Which cut each other at point C . Join C with A and B .

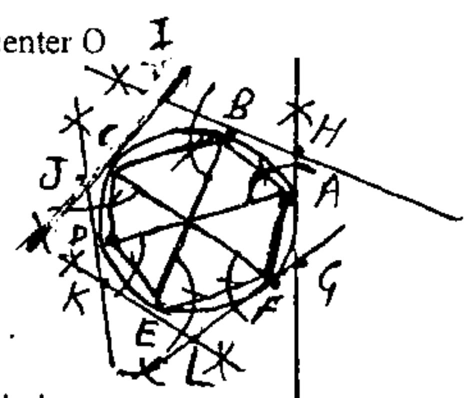
(v) Thus triangle ABC is formed .

(vi) Draw perpendicular bisectors of $\overline{AB}$ *and* $\overline{BC}$ which intersect each other at point O.

(vii) Take O is center and draw a circle of radius equal to $m\overline{OA}$ *or* $m\overline{OB}$ *or* $m\overline{OC}$.

(viii) This is the required circumscribe a circle about the given triangle .

Q.5: circumscribe a regular hexagon about a circle of radius 4 cm.



Solution: Given: A circle with center O and radius 4 cm .

Required: To circumscribe a regular hexagon about the given circle.

Steps of construction:

(i) Take O is a center and draw a circle of radius 4 cm .

(ii) Take any point A on the circumference of the given circle.

(iii) With A as a center and $m\overline{OA}$ as radius, draw two arcs which cut the circumference of the circle at B and F.

(iv) Through O, draw $\overline{AD}$, $\overline{BE}$ and $\overline{FC}$.

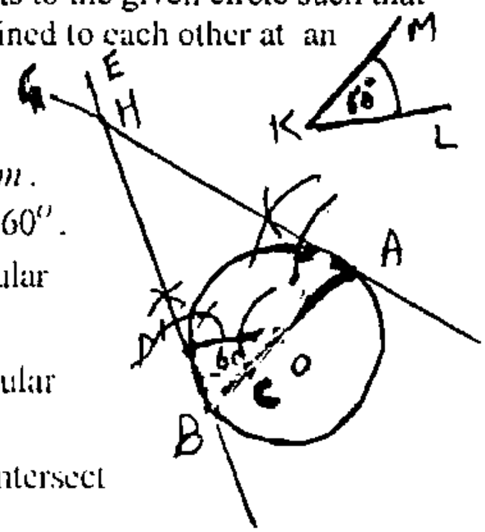
(v) Draw perpendiculars at the extremities A, B, C, D, E and F of the diameters $\overline{AD}$, $\overline{BE}$ and $\overline{FC}$ of the circle. These lines cut each other at points G, H, I, J, K and L.

(vi) GHIJKL is the required circumscribed hexagon.

Q.6: Construct a circle of radius 3 cm. Draw two tangents making angle of 60^o with each other.

Solution: Given: A circle with center O, radius 3 cm and angle 60^o. The vertex of the angle is k.

Required: To draw two tangents to the given circle such that the tangents are inclined to each other at an angle 60^o.



Step of construction:

(i) Draw a diameter $m\overline{AB} = 6cm$.

(ii) At O, construct $< BOD = 60^o$.

(iii) At D draw $\overline{DE}$ perpendicular to $\overline{OD}$.

(iv) At A draw $\overline{AG}$ perpendicular to $\overline{OA}$.

(v) These two perpendiculars intersect each other at H.

(vi) $\overline{DH}$ and $\overline{AH}$ are the required tangents inclined to each other at angle 60^o.

Q.7: Draw two equal circles are of radius 3.5 cm each. If the distance between their centers is 7 cm, then draw transverse tangents to these circles.

Solution: Given:

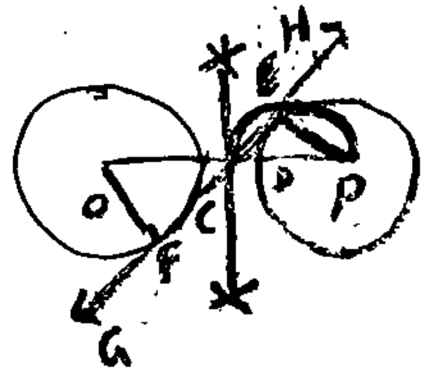
Two equal circles with radii 3.5cm and centers at O and P respectively.

Also $\overline{OP} = 7$ cm

Required: Draw transverse tangent given circles.

Steps of construction:

(i) Take O and P are centers Such that $\overline{OP} = 7cm$ and draw two circles each of radius 3.5 cm respectively.



(ii) Join O to P .

(iii) Bisect $\overline{OP}$ at C .

(iv) Bisect $\overline{CP}$ at D.

(v) Draw a semi – circle on $\overline{CP}$ diameter which cuts the given circle at E.

(vi) Join E and P .

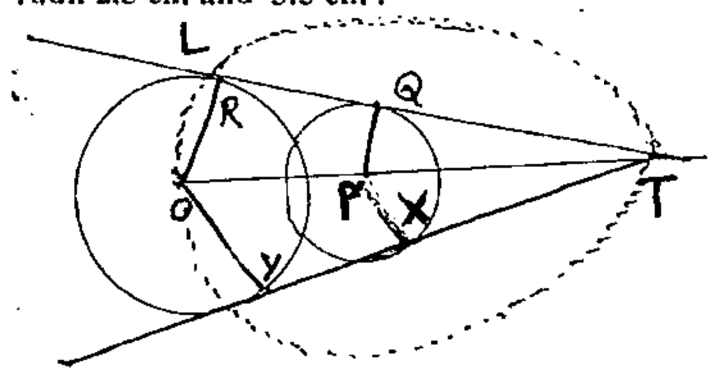
(vii) Draw $\overline{OF} \| \overline{EP}$.

(viii) E join to F and extend the line to both directions.

(ix) Thus $\overline{GH}$ is the required tangents to both circles.

Q.8: Draw two common tangents to two intersecting circles of radii 2.5 cm and 3.5 cm .

Solution:



Given: Two circles of radii 2.5 and 3.5cm of centers O and P intersecting each other .

Required: To draw two common tangents to the given circles .

Step of construction: (i) Join the centers O and P and produce it to right .

(ii) Draw $\overline{PQ}$ and $\overline{PX}$ as radii 2.5cm such that $\overline{PQ}$ and $\overline{PX}$ are not along $\overline{OP}$

(iii) Draw $\overline{OR}$ and $\overline{OY}$ as radii 3 cm such that $\overline{OR}$ and $\overline{OY}$ are not along $\overline{OP}$ such that $\overline{PQ} \parallel \overline{OR}$ and $\overline{PX} \parallel \overline{OY}$

(iv) Join R and Q and produced it to meet the line $\overline{OP}$ at S Also join X and Y and produced it to meet the line $\overline{OP}$ at S.

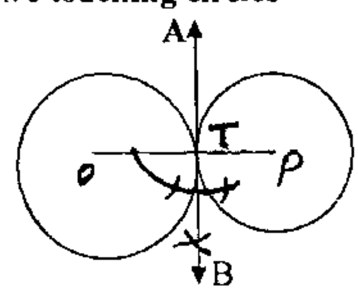
(v) Bisect the line segment $\overline{OS}$ at T and draw a circle of radius $\overline{OT}$ which passes through O and S and cut the circle of radius 3 at point L and Z.

(vi) Finally join S and L and produced to M.

(vii) Also join S and Z and produced to N.

Thus $\overline{SM}$ and $\overline{ZN}$ are the required tangents.

Q.9: Draw a common tangent to two touching circles of radii 3 cm and 4 cm.



Solution:

Given: Two circles of centers O and P of radii 3cm and 4 cm.

Required: To draw a tangent to the circles.

Step of construction:

(i) Join the centers O and P of the given circles. $\overline{OP}$ passes through T, the point of contact.

(ii) At T draw a line $\overline{ATB}$ perpendicular to the given circles.

(iii) $\overline{ATB}$ is the given tangent to the given circles.